INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: SRM - 2

Chapter:

Category: Assignment Questions

1. Claims arising on a particular type of insurance policy are believed to follow a Pareto distribution. Data for the last several years shows the mean claim size is 170 and the standard deviation is 400.

(i) Fit a Pareto distribution to this data using the method of moments.

(ii) Calculate the median claim using the fitted parameters and comment on the result.

2. Claim amounts arising from a certain type of insurance policy are believed to follow a Lognormal distribution. One thousand claims are observed and the following summary statistics are prepared:
mean claim amount 230
standard deviation 110
lower quartile 80
upper quartile 510

(i) Fit a Lognormal distribution to these claims using:
(a) the method of moments.
(b) the method of percentiles.

(ii) Compare the fitted distributions from part (i).

3. An underwriter has suggested that losses on a certain class of policies follow a Weibull distribution. She estimates that the 10th percentile loss is 20 and the 90^{th} percentile loss is 95.

(i) Calculate the parameters of the Weibull distribution that fit these percentiles.

(ii) Calculate the 99.5th percentile loss.

4. The total number of claims N on a portfolio of insurance policies has a Poisson distribution with mean λ. Individual claim amounts are independent of N and each other, and follow a distribution X with mean μ and variance σ^2. The total aggregate claims in the year is denoted by S. The random variable S therefore has a compound Poisson distribution.

(i) Derive an expression for the moment generating function of S in terms of the moment generating function of X.

(ii) Derive expressions for the mean and variance of S in terms of λ, μ and σ.

For a particular type of policy, individual losses are exponentially distributed with mean 100. For losses above 200 the insurer incurs an additional expense of 50 per claim.

(iii) Calculate the mean and variance of S for a portfolio of such policies with $\lambda = 500$

5. An insurance company has a portfolio of 1,000 car insurance policies. Claims arise on individual policies according to a Poisson process with annual rate μ . The insurance company believes that μ follows a gamma distribution with parameters α = 2 and λ = 8.
(i) (a) Show that the average annual number of claims per policy is 0.25.

(b) Show that the variance of the number of annual claims per policy is 0.28125.

Individual claim amounts follow a gamma distribution with density

$f(x) = \frac{x}{1{,}000{,}000}e^{-\left(\frac{x}{1000}\right)}$ for x > 0

(ii) Calculate the mean and variance of the annual aggregate claims for the whole portfolio.

The insurance company has agreed an aggregate excess of loss reinsurance contract with a retention of £0.55m (this means that the reinsurance company will pay the excess above £0.55m if the aggregate claims on the portfolio in a given year exceed £0.55m).

(iii) Calculate, using a Normal approximation, the probability of aggregate claims exceeding the retention in any year.

For each of the last three years, the total claim amount has in fact exceeded the retention.

(iv) Comment on this outcome in light of the calculation in part (iii).

6. (i) (a) Explain why an insurance company might purchase reinsurance.

(b) Describe two types of reinsurance.

The claim amounts on a particular type of insurance policy follow a Pareto distribution with mean 270 and standard deviation 340.

(ii) Determine the lowest retention amount such that under excess of loss reinsurance the probability of a claim involving the reinsurer is 5%.

7. A portfolio of insurance policies contains two types of risk. Type I risks make up 80% of claims and give rise to loss amounts which follow a normal distribution with mean 100 and variance 400. Type II risks give rise to loss amounts which are normally distributed with mean 115 and variance 900.

(i) Calculate the mean and variance of the loss amount for a randomly chosen claim.

(ii) Explain whether the loss amount for a randomly chosen claim follows a normal distribution.

The insurance company has in place an excess of loss reinsurance arrangement with retention 130.

(iii) Calculate the probability that a randomly chosen claim from the portfolio results in a payment by the reinsurer.

(iv) Calculate the proportion of claims involving the reinsurer that arise from Type II risks.

8. The number of claims N on a portfolio of insurance policies follows a binomial distribution with parameters n and p. Individual claim amounts follow an exponential distribution with mean $1/\lambda$. The insurer has in place an individual excess of loss reinsurance arrangement with retention M.

(i) Derive an expression, involving M and λ, for the probability that an individual claim involves the reinsurer.

Let I_i be an indicator variable taking the value 1 if the ith claim involves the reinsurer and 0 otherwise.

(ii) Evaluate the moment generating function $M_{I_i}(t)$.

Let K be the number of claims involving the re-insurer so that $K = I_1 + \ldots + I_N$.

(iii) (a) Find the moment generating function of K.
(b) Deduce that K follows a binomial distribution with parameters that you should specify.

9. The total claim amount, S, on a portfolio of insurance policies has a compound Poisson distribution with Poisson parameter 50. Individual loss amounts have an exponential distribution with mean 75. However, the terms of the policies mean that the maximum sum payable by the insurer in respect of a single claim is 100.

(i) Find $E(S)$ and $\mathrm{Var}(S)$.

(ii) Use the method of moments to fit as an approximation to S:
(a) a normal distribution
(b) a log-normal distribution

(iii) For each fitted distribution, calculate $P(S > 3{,}000)$.

10. The annual number of claims on an insurance policy within a certain portfolio follows a Poisson distribution with mean μ. The parameter μ varies from policy to policy and can be considered as a random variable that follows an exponential distribution with mean $1/\lambda$.

Find the unconditional distribution of the annual number of claims on a randomly chosen policy from the portfolio.