INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: SRM - 2

Chapter: Unit 3 & 4

Category: Assignment Solutions

1.

(i) Let $S(t)$ denote the total claims up to time t and suppose individual claim amounts follow a distribution X.

Then $U(t) = U + \lambda t(1 + \theta) E(X) - S(t)$.

(ii) $\psi(U, t) = \Pr(U(s) < 0 \text{ for some } s \in [0, t])$

$\psi(U) = \Pr(U(t) < 0 \text{ for some } t > 0)$

(iii) The probability of ruin by time t will increase as λ increases. This is because claims and premiums arrive at a faster rate, so that if ruin occurs it will occur earlier, which leads to an increase in $\psi(U, t)$.

The probability of ultimate ruin does not depend on how quickly the claims arrive. We are not interested in the time when ruin occurs as we are looking over an infinite time horizon.

2.

(i) $M_X(t) = E(e^{tX}) = \int_0^\infty e^{tx} f(x) dx$

$$= \int_0^\infty 0.01^2 xe^{(t-0.01)x} dx$$

$$= \left[\frac{0.01^2 xe^{(t-0.01)x}}{t-0.01} \right]_0^\infty - \int_0^\infty \frac{0.01^2 e^{(t-0.01)x}}{t-0.01} dx$$

$$= 0 - 0 - \left[\frac{0.01^2 e^{(t-0.01)x}}{(t-0.01)^2} \right]_0^\infty \text{ provided that } t < 0.01$$

$$= \frac{0.01^2}{(t-0.01)^2} \text{ again provided that } t < 0.01$$

(ii) The adjustment coefficient is the unique positive solution of

$M_X(R) = 1 + 1.45E(X)R$

But $E(X) = M'_X(0) = \frac{d}{dt}\left[\frac{0.01^2}{(t-0.01)^2}\right]\Bigg|_{t=0}$

$= \frac{-2\times 0.01^2}{(t-0.01)^3}\Bigg|_{t=0} = \frac{-2}{-0.01} = 200$

So we need to solve $\frac{0.01^2}{(R-0.01)^2} = 1 + 290R$

i.e. $0.01^2 = (1 + 290R)(R - 0.01)^2 = (1 + 290R)(0.01^2 - 0.02R + R^2)$

i.e. $0.012 = 0.01^2 + 0.029R - 0.02R - 5.8R^2 + R^2 + 290R^3$

i.e. $290R^2 - 4.8R + 0.009 = 0$

$R = \frac{4.8 \pm \sqrt{4.8^2 - 4\times 290\times 0.009}}{2\times 290}$

i.e. $R = 0.00215578$ or $R = 0.0143959$

So taking the smaller root we have $R = 0.00215578$ since that is less than 0.01

The upper bound for the probability of ruin is given by Lundberg's inequality as

$\psi(U) \le e^{-RU} = e^{-0.00215578U}$

(iii) We want $\psi(U) \le e^{-0.00215578U} \le 0.01$

i.e. $-0.00215578U \le \log 0.01$

i.e. $U \ge \dfrac{\log 0.01}{-0.00215578} = 2136.20$

(iv) This time the adjustment coefficient is the solution to:

$e^{200R} = 1 + 290\text{R}$

So the question is whether $y = e^{200R}$ crosses the line $y = 1 + 290R$ before or after $y = 0.01^2(0.01 - R)^{-2}$ crosses the same line
But when $R = 0.00215578$ we have
$e^{200R} = e^{200\times 0.00215578} = 1.539 < 1 + 290R = 1.625.$

So $y = e^{200R}$ has not yet crossed the given line, and the second scenario has a larger adjustment coefficient that the first.

This means the second risk has a lower probability of ruin, which is to be expected since although the mean claim amounts are the same in each scenario, the claim amounts in the first scenario are more variable suggesting a greater risk.

3.

(i) Let $S(t)$ denote cumulative claims to time t. Let the annual rate of premium income be c and let the insurer's initial surplus be U=100.

Then the surplus at time t is given by:

$$U(t) = U + ct - S(t)$$

And the relevant probabilities are defined by:

$$\psi(100) = P(U(t) < 0 \text{ for some } t > 0)$$
$$\psi(100,1) = P(U(t) < 0 \text{ for some } t \text{ with } 0 < t \leq 1)$$
$$\psi_1(100,1) = P(U(1) < 0)$$

(ii) The adjustment coefficient is the unique positive root of the equation

$$\lambda M_X(R) = \lambda + cR$$

Where λ is the rate of the Poisson process (i.e. 100) and X is the normal distribution with mean 30 and standard deviation 5.

(iii) In this case we have:

$$c = 100 \times 30 \times 1.2 = 3600$$

And

$$M_X(R) = \exp(30R + 12.5R^2)$$

So R is the root of

$$100\exp\left(30R + 12.5R^2\right) - 100 - 3600R = 0$$

Denote the left hand side of this equation by $f(R)$.

When $R = 0.0115$ we have

$$f(0.0115) = 100\exp(0.346653125) - 100 - 41.4 = 0.032604592 > 0$$

And when $R = 0.0105$ we have

$$f(0.0105) = 100\exp(0.316378125) - 100 - 37.8 = -0.585099862 < 0$$

Since the function changes sign between 0.0105 and 0.0115 the unique positive root must lie between these values and hence the root is 0.011 correct to 3 decimal places.

(iv) By Lundberg's inequality $\psi(100) < \exp(-100 \times 0.011) = 0.33287$

Claims in the first year are approximately Normal, with mean $100 \times 30 = 300$

And variance given by $100 \times (25 + 30^2) = 92500$

So approximately

$$\psi_1(100,1) = P(100 + 3600 - N(3000,92500) < 0)$$

$$= P(N(3000,92500) > 3700) = P\left(N(0,1) > \frac{3700-3000}{\sqrt{92500}} \right)$$

$$= P(N(0,1) > 2.302)$$

$$= 1 - (0.98928 \times 0.8 + 0.98956 \times 0.2)$$

$$= 0.0107.$$

(v) The probability of ruin is much smaller in the first year than the long-term bound provided by Lundberg's inequality. This suggests that either the boun in Lundberg's inequality may not be that tight or that there is significant probability of ruin at times greater than 1 year.

4.

(i) The annual premium charged is $0.25 \times 150 \times 1.7 = 63.75$

(ii) Let X be an individual claim. Then

$$P(X < 200) = P\left(N\left(150, 30^2\right) < 200\right)$$

$$= P\left(N(0,1) < \frac{200-150}{30}\right)$$

$$= P(N(0,1) < 1.667)$$

$$= (0.95154 \times 0.3 + 0.7 \times 0.95254)$$

$$= 0.95224$$

(iii) We need to calculate:

$$p = \sum_{j=0}^{\infty} P(j \text{ claims}) \times P(\text{all claims below retention}) \; [1]$$

$$= \sum_{j=0}^{\infty} e^{-0.25} \frac{(0.25)^j}{j!} \times (0.95224)^j$$

$$= e^{-0.25} \times \sum_{j=0}^{\infty} \frac{(0.25 \times 0.95224)^j}{j!}$$

$$= e^{-0.25} \times e^{0.25 \times 0.95224}$$

$$= 0.9881$$

(iv) We need to first calculate the mean claim amount paid by the reinsurer. This is given by

$$I = \int_{200}^{\infty} (x - 200) f(x) dx$$

Where $f(x)$ is the pdf of the Normal distribution with mean 150 and standard deviation 30.

Using the formula on p18 of the tables, we have:

$$I = \int_{200}^{\infty} xf(x) dx - 200P(X > 200)$$

$$= 150 \times \left[\Phi(\infty) - \Phi(1.667)\right] - 30 \times \left(\phi(\infty) - \phi(1.667)\right) - 200 \times (1 - 0.95224)$$

$$= 150(1 - 0.95224) - 30 \times (0 - 0.09942) - 200 \times 0.0.04776$$

$$= 0.5946$$

So the reinsurer charges $0.25 \times 0.5946 \times 2.2 = 0.32703$

(v) The direct insurers expected profit is given by:

$$63.75 - 0.32703 - 0.25 \times (150 - 0.5946) = 26.07$$

5.

A is false since there cannot be a claim until time 2 [2]

B is false since the insurance company could be ruined at time 3 if there is a claim, if U is sufficiently small. [2]

C is false since the insurance company cannot be ruined in year 4, since by that stage it will have sufficient premiums to cover any loss. [2]

D is true since if it is not ruined by time 4, the insurance company cannot be ruined. [2]

6.

Multiply the claim payments with the corresponding inflation factors given below:

Development year

2004	1.16757	1.11197	1.05400	1.00000
2005	1.11197	1.05400	1.00000	
2006	1.05400	1.00000		
2007	1.00000			

The resulting table is:

Development year

2004	478.70	905.14	227.66	79.00
2005	639.38	990.76	281.00	
2006	857.96	1066.00		
2007	1142.00			

The inflation adjusted accumulated claim payments in mid 2007 are given below:

Development year

year	0	1	2	3
2004	478.70	1383.84	1611.50	1690.50
2005	639.38	1630.14	1911.14	**2004.83**
2006	857.96	1923.96	**2248.66**	**2358.90**
2007	1142.00	**2853.75**	**3335.38**	**3498.88**

Note only the values of the last row are needed for the answer.

The bolded values show the completed table using the basic chain ladder approach.

The development factors are 2.4989, 1.1688, 1.0490.

For the answer we only need to work with the projected values at the last row as:

$(2853.75-1142.00)*1.08+(3335.38-2853.75)*1.08^2+(3498.88-3335.38)*1.08^3$
$= 2616.43$

2616.43 * 5000 = £13,082,150

7.

a. The development factors are given by

$R_1 = (136 + 156 + 130) / (96 + 100 + 120) = 1.335443$

$R_2 = (140 + 160) / (136 + 156) = 1.027397$

$R_3 = 168 / 140 = 1.2$

The fully developed table using the chain ladder is below:

Incident year	*0*	*1*	*2*	*3*
2005	96	136	140	168
2006	100	156	160	**192**
2007	120	130	**133.56**	**160.28**
2008	136	**181.62**	**186.60**	**223.92**
R	1.335443	1.027397	1.2	1
f	1.646436	1.232876	1.2	1

Reserve = (168 + 192 + 160.28 + 223.92) – (168 + 160 + 130 + 136) = 150.2

b. B-F method

Estimated loss ratio: 168/175 = 0.96

	2008	*2007*	*2006*	*2005*
F	1.646436	1.232876	1.2	1
1 − 1/f	0.392627	0.188888	0.1666667	0
IUL	188.16	182.4	173.76	168
Emerging liab.IUL(1 − 1/f)	73.87678	34.45325	28.96	0

Reserve is now = 73.87678 + 34.45325 + 28.96 = 137.29

8.

(i) The development ratio for development year 2 to development year 3 is given by 1862.3/1820 = 1.023242

Therefore $W = 1762 \times 1.023242 = 1803.0$

Because there is no claims development beyond development year 3 $X = 1803.0$ also.

The development factor from development year 1 to ultimate is given by 2122.5/1805 = 1.1759003

So the ratio from development year 1 to development year 2 is given by 1.1759003/1.023242 = 1.149190785

But under the definition of the chain ladder approach, this is calculated as:

$$1.149190785 = \frac{1762+1820}{Y+1485} = \frac{3582}{Y+1485}$$

So $Y = \dfrac{3582}{1.149190785} - 1485 = 1632.0$

(ii) We require the development ratio from year 0 to year 1; this is given by:

$$\frac{1485+1632+1805}{1001+1250+1302} = \frac{4922}{3553} = 1.385308$$

The development factor to ultimate is therefore

$$1.385308 \times 1.149190785 \times 1.023242 = 1.628984285$$

And so $Z = 2278.8 - 2500 \times 0.9 \times \left(1 - \dfrac{1}{1.628984285}\right) = 1410.0$

(iii) The outstanding claims reserve is

$$1862.3 + 2122.5 + 2278.8 - 1820 - 1805 - 1410 = 1228.6$$

9.

First accumulate claims:

Cumulative Claims		Development year 0	1	2
	2010	2,328	3,812	4,196
Accident year	2011	1,749	2,937	
	2012	2,117		

DY1 = (3,812 + 2,937) / (2,328 + 1,749) = 1.655 384
DY2 = 4,196 / 3,812 = 1.100 735

Now complete lower half of table:

Cumulative Claims		Development year 0	1	2
	2009	2,328	3,812	4,196
Accident year	2011	1,749	2,937	3,232.86
	2012	2,117	3,504.45	3,857.47

So estimated amount of outstanding claims is:

(3,232.86 – 2,937) + (3,857.47 – 2,117) = 2,036.3.

10.

(i) The cumulative cost of claims is given by:

Accident year	*Development year* 0	1	2
2011	2,233	3,622	4222
2012	3,380	5,188	
2013	4,996		

Dividing by cumulative claim numbers:

Accident year	*Development year* 0	1	2
2011	15.950	17.842	18.848
2012	18.778	22.557	
2013	19.516		

using grossing up factors to estimate the ultimate average cost per claim for each accident year:

Accident year	*Development year* 0	1	2
2011	84.623% 15.950	94.663% 17.842	100% 18.848
2012	78.805% 18.778	94.663% 22.557	 23.828
2013	81.714% 19.516		 23.883

Taking the same approach for the claim numbers gives:

Accident year	*Development year* 0	1	2
2011	62.5% 140	90.625% 203	100% 224
2012	70.924% 180	90.625% 230	253.8
2013	66.712% 256		383.7

Total outstanding claims are therefore

$$253.8 \times 23.828 + 383.7 \times 23.883 - 5188 - 4996$$
$$= \underline{5028.2}$$ [7]

(ii) **Assumptions**

- The number of claims relating to each development year is a constant proportion of the total claim numbers from the relevant accident year.
- Claim amounts for each development year are a constant proportion of the total claim amount for the relevant accident year.
- Claims are fully run off after development year 2.

[3]
[Total 10]