INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: SRM - 2

Chapter: Unit 3

Category: Practice Questions

1) CT6 APRIL 2008 Q3

(i) X and Y are independent Poisson random variables with mean λ. Derive the moment generating function of X, and hence show that $X + Y$ also has a Poisson distribution. [4]

(ii) An insurer has a portfolio of 100 policies. Annual premiums of 0.2 units per policy are payable annually in advance. Claims, which are paid at the end of the year, are for a fixed sum of 1 unit per claim. Annual claims numbers on each policy are Poisson distributed with mean 0.18.

Calculate how much initial capital is needed in order to ensure that the probability of ruin at the end of the year is less than 1%.

2) CT6 APRIL 2008 Q6

A portfolio of general insurance policies is made up of two types of policies. The policies are assumed to be independent, and claims are assumed to occur according to a Poisson process. The claim severities are assumed to have exponential distributions.

For the first type of policy, a total of 65 claims are expected each year and the expected size of each claim is £1,200.

For the second type of policy, a total of 20 claims are expected each year and the expected size of each claim is £4,500.

(i) Calculate the mean and variance of the total cost of annual claims, S, arising from this portfolio.

The risk premium loading is denoted by θ, so that the annual premium on each policy is $(1+\theta)\times$expected annual claims on each policy. The initial reserve is denoted by u.

A normal approximation is used for the distribution of S, and the initial reserve is set by ensuring that $P(S < u + \text{annual premium income}) = 0.975$.

(ii) (a) Derive an equation for u in terms of θ.
(b) Determine the annual premium required in order that no initial reserve is necessary.

3) CT6 SEPT 2008 Q8

An insurer has issued two five-year term assurance policies to two individuals involved in a dangerous sport. Premiums are payable annually in advance, and claims are paid at the end of the year of death.

	Individual Annual Premium	*Sum Assured*	*Annual Prob (death)*
A	100	1,700	0.05
B	50	400	0.1

Assume that the probability of death is constant over each of the five years of the policy. Suppose that the insurer has an initial surplus of U.

(i) Define what is meant by $\psi(U)$ and $\psi(U,t)$.

(ii) Assuming $U = 1{,}000$

(a) Determine the distribution of $S(1)$, the surplus at the end of the first year, and hence calculate $\psi(U,1)$.

(b) Determine the possible values of $S(2)$ and hence calculate $\psi(U,2)$.

4) CT6 APRIL 2009 Q8

An insurer has an initial surplus of U. Claims up to time t are denoted by $S(t)$. Annual premium income is received continuously at a rate of c per unit time.

(i) Explain what is meant by the insurer's surplus process $U(t)$.

(ii) Define carefully each of the following probabilities:
(a) $\psi(U,t)$
(b) $\psi_h(U,t)$

(iii) Explain, for each of the following pairs of expressions, whether one of each pair is certainly greater than the other, or whether it is not possible to reach a conclusion.
(a) $\psi(10,2)$ and $\psi(20,1)$
(b) $\psi(10,2)$ and $\psi(5,1)$
(c) $\psi_{0.5}(10,2)$ and $\psi_{0.25}(10,2)$

5) CT6 SEPT 2009 Q2

An insurance company has a portfolio of two-year policies. Aggregate annual claims from the portfolio follow an exponential distribution with mean 10 (independently from year to year). Annual premiums of 15 are payable at the start of each year. The insurer checks for ruin only at the end of each year. The insurer starts with no capital.

Calculate the probability that the insurer is not ruined by the end of the second year.

6) CT6 APRIL 2010 Q10

Claims on a portfolio of insurance policies arrive as a Poisson process with annual rate λ. Individual claims are for a fixed amount of 100 and the insurer uses a premium loading of 15%. The insurer is considering entering a proportional reinsurance agreement with a reinsurer who uses a premium loading of 20%. The insurer will retain a proportion α of each risk.

(i) Write down and simplify the equation defining the adjustment coefficient R for the insurer.

(ii) By considering R as a function of α and differentiating show that:

$$\frac{(120\alpha - 5)dR}{d\alpha} + 120R = \left(100R + \frac{100\alpha dR}{d\alpha}\right)e^{100\alpha R}$$

(iii) Explain why setting $\frac{dR}{d\alpha} = 0$ and solving for α may give an optimal value for α.

(iv) Use the method suggested in part (iii) to find an optimal choice for α.

7) CT6 SEPT 2010 Q2

Claims on a portfolio of insurance policies follow a compound Poisson process with annual claim rate λ. Individual claim amounts are independent and follow an exponential distribution with mean μ. Premiums are received continuously and are set using a premium loading of θ. The insurer's initial surplus is U.

Derive an expression for the adjustment coefficient, R, for this portfolio in terms of μ and θ.
8)

8) Subject CT6 April 2011 Question 9

Claims on a portfolio of insurance policies arise as a Poisson process with parameter λ. Individual claim amount are taken from a distribution X and we define $m_i = E(X^i)$ for i = 1, 2…. The insurance company calculates premium using a premium loading of θ.

(i) Define the adjustment coefficient R. **(1)**

(ii) (a) Show that R can be approximated by $\frac{2\theta m_1}{m_2}$ by truncating the series expansion of $M_x(t)$

(b) Show that there is another approximation to R which is a solution of the equation $m_3 y^2 + 3m_2 y - 6\theta m_1 = 0$ **(6)**

Now suppose that X has an exponential distribution with mean 10 and that θ=0.3

(iii) Calculate the approximations to R in (ii) and (iii) and compare them to the value of R.
(6)