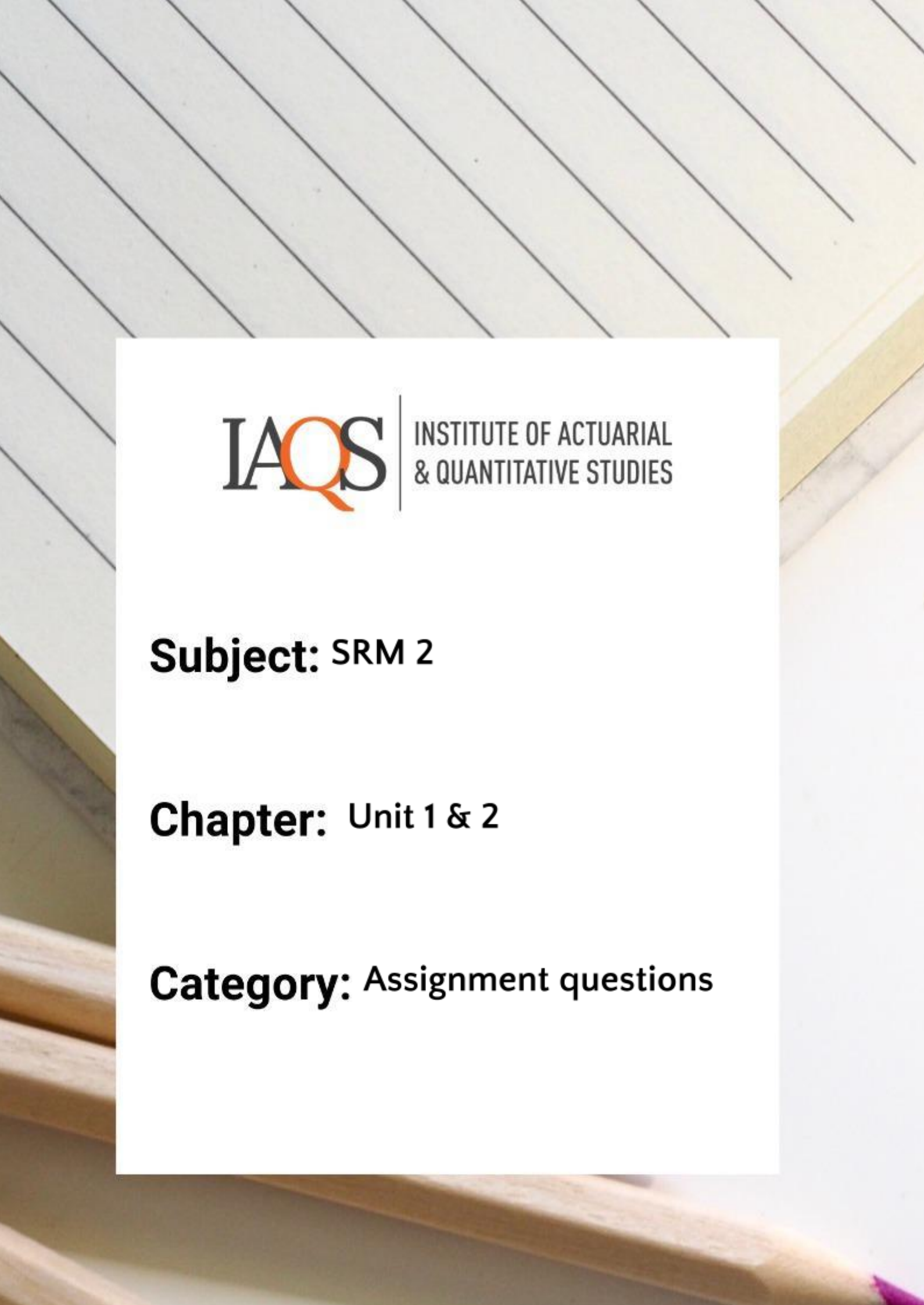

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: SRM 2

Chapter: Unit 1 & 2

Category: Assignment questions

1. Claim amounts arising from a certain type of insurance policy are believed to follow a Lognormal distribution. One thousand claims are observed and the following summary statistics are prepared:
mean claim amount 230
standard deviation 110
lower quartile 80
upper quartile 510

(i) Fit a Lognormal distribution to these claims using:
(a) the method of moments.
(b) the method of percentiles.

(ii) Compare the fitted distributions from part (i).

2. An underwriter has suggested that losses on a certain class of policies follow a Weibull distribution. She estimates that the 10th percentile loss is 20 and the 90^{th} percentile loss is 95.

(i) Calculate the parameters of the Weibull distribution that fit these percentiles.

(ii) Calculate the 99.5th percentile loss.

3. The total number of claims N on a portfolio of insurance policies has a Poisson distribution with mean λ. Individual claim amounts are independent of N and each other, and follow a distribution X with mean μ and variance σ^2. The total aggregate claims in the year is denoted by S. The random variable S therefore has a compound Poisson distribution.

(i) Derive an expression for the moment generating function of S in terms of the moment generating function of X.

(ii) Derive expressions for the mean and variance of S in terms of λ, μ and σ.

For a particular type of policy, individual losses are exponentially distributed with mean 100. For losses above 200 the insurer incurs an additional expense of 50 per claim.

(iii) Calculate the mean and variance of S for a portfolio of such policies with $\lambda = 500$

4. An insurance company has a portfolio of 1,000 car insurance policies. Claims arise on individual policies according to a Poisson process with annual rate μ . The insurance company believes that μ follows a gamma distribution with parameters $\alpha = 2$ and $\lambda = 8$.

(i) (a) Show that the average annual number of claims per policy is 0.25.

(b) Show that the variance of the number of annual claims per policy is 0.28125.

Individual claim amounts follow a gamma distribution with density

$f(x) = \frac{x}{1{,}000{,}000} e^{-(\frac{x}{1000})}$ for x > 0

(ii) Calculate the mean and variance of the annual aggregate claims for the whole portfolio.

The insurance company has agreed an aggregate excess of loss reinsurance contract with a retention of £0.55m (this means that the reinsurance company will pay the excess above £0.55m if the aggregate claims on the portfolio in a given year exceed £0.55m).

(iii) Calculate, using a Normal approximation, the probability of aggregate claims exceeding the retention in any year.

For each of the last three years, the total claim amount has in fact exceeded the retention.

(iv) Comment on this outcome in light of the calculation in part (iii).

5. A portfolio of insurance policies contains two types of risk. Type I risks make up 80% of claims and give rise to loss amounts which follow a normal distribution with mean 100 and variance 400. Type II risks give rise to loss amounts which are normally distributed with mean 115 and variance 900.

(i) Calculate the mean and variance of the loss amount for a randomly chosen claim.

(ii) Explain whether the loss amount for a randomly chosen claim follows a normal distribution.

The insurance company has in place an excess of loss reinsurance arrangement with retention 130.

(iii) Calculate the probability that a randomly chosen claim from the portfolio results in a payment by the reinsurer.

(iv) Calculate the proportion of claims involving the reinsurer that arise from Type II risks.

6. The total claim amount, *S*, on a portfolio of insurance policies has a compound Poisson distribution with Poisson parameter 50. Individual loss amounts have an exponential distribution with mean 75. However, the terms of the policies mean that the maximum sum payable by the insurer in respect of a single claim is 100.

(i) Find *E*(*S*) and Var (*S*).

(ii) Use the method of moments to fit as an approximation to *S*:
(a) a normal distribution
(b) a log-normal distribution

(iii) For each fitted distribution, calculate $P(S > 3{,}000)$.

7. The annual number of claims on an insurance policy within a certain portfolio follows a Poisson distribution with mean μ. The parameter μ varies from policy to policy and can be considered as a random variable that follows an exponential distribution with mean 1/λ.

Find the unconditional distribution of the annual number of claims on a randomly chosen policy from the portfolio.

8. For a claim distribution 'C' with LogNormal ($\mu = 6$, $\sigma^2 = 4$), calculate reinsurer's expected payout per claim to insurer where the reinsurer pays:

$$P = \begin{cases} 0 \text{ if } C < 900 \\ C - 900 \text{ if } 900 < C < 1700 \\ 800 \text{ if } C > 1700 \end{cases}$$

9. Five years ago, an insurance company began to issue insurance policies covering medical expenses for dogs. The insurance company classifies dogs into three risk categories: large pedigree (category 1), small pedigree (category 2) and non-pedigree (category 3).
The number of claims n_{ij} in the i^{th} category in the j^{th} year is assumed to have a Poisson distribution with unknown parameter θ_i. Data on the number of claims in each category over the last 5 years is set out as follows:

Category	Year					$\sum_{j=1}^{5} n_{ij}$	$\sum_{j=1}^{5} n_{ij}^2$
	1	2	3	4	5		
1	28	41	47	54	62	232	11434
2	35	48	55	57	65	260	14028
3	26	29	20	39	31	145	4399

Prior beliefs about θ1 are given by a gamma distribution with mean 50 and variance 25.
(i) Find the Bayes estimate of θ1 under quadratic loss.

(ii) Calculate the expected claims for year 6 of each category under the assumptions of Empirical Bayes Credibility Theory Model 1.

(iii) Explain the main differences between the approach in part (i) and that in part (ii).

(iv) Explain why the assumption of a Poisson distribution with a constant parameter may not be appropriate and describe how each approach might be generalised.

10. The losses on certain type of insurance policy X follow Burr distribution with density function:

$f(x) = \frac{0.5*\alpha*0.4^{\alpha}*x^{0.5}}{(0.4+x^{0.4})^{\alpha+1}}$ where 'α' is an unknown constant.

An insurer has an excess of loss reinsurance arrangement with retention limit of 234.

Claims observed are as below:

· Claims below 234 (10 claims) : 201, 230, 158, 198, 199, 205, 141, 155, 197, 217

· Claims above 234 (7 claims): Amounts unknown

(i) Estimate 'α' using method of maximum likelihood.

(ii) Estimate 'α' applying the method of percentiles to the median.

(iii) Explain briefly as to why do you find the estimate of α different under (i) and (ii) above.

11. An Indian life insurer has written a large book of policies which provides the Sum Assured on diagnosis of major stage cancer. The claim frequency per mille (number of claims per 1,000 policies) arising on this book over the past 3 years is as below:

Year	Claim frequency (per mille)
2019-20	16.4
2020-21	17.3
2021-22	16.7

The age composition and other aspects of the book have been relatively unchanged over this period.

The claim frequency per mille is modelled as following a Poisson distribution with an unknown parameter λ.

The Pricing Actuary models λ as following a Gamma (A, B) distribution, with A = 15 and B = 1.

The Bayesian credibility factor is given by n/(n + B), where n is the number of years for which data is available.

(i) Calculate the Bayesian credibility estimate for the number of claims per 1,000.

The Pricing Actuary had chosen the prior distribution of Gamma (15, 1) based on inputs from a global reinsurer, who expected the claim frequency to be 15 per mille.

The Appointed Actuary disagrees with the Pricing Actuary's choice of Gamma parameters, as he believes the reinsurer's experience may not be very relevant to the Indian market. He therefore suggests using Gamma (3, 0.2) as the prior distribution.

(ii) Briefly explain, by general reasoning, how the suggested parameters reflect the greater uncertainty regarding the relevance of the reinsurer's data.

(iii) Calculate the revised Bayesian credibility estimate.

12. Individual claim amounts on a particular insurance policy can take the values 150, 200 or 250. There is at most one claim in a year. Annual premiums are 60. The insurer must choose between three reinsurance arrangements:

1. No Reinsurance
2. individual excess of loss with retention 150 for a premium of 10
3. proportional reinsurance of 25% for a premium of 20

(i) Complete following table of loss

Loss Table	Reinsurer's loss			Insurer's Loss		
Claims	1	2	3	1	2	3
0						
150						
200						
250						

(ii) Determine whether any of the reinsurance arrangements is dominated from the viewpoint of the insurer.

(iii) Determine the minimax solution for the insurer.

13. An analyst is working on renewals of health insurance policies. He has estimated following aggregate claim statistics for four policies over last 3 years:

Policy Number	$\sum_{j=1}^{3} X_j$	$\sum_{j=1}^{3} (X_j - \bar{X})$
1	12,183	12,504
2	13,098	12,718
3	12,822	12,432
4	13,453	12,242

(i) Using EBCT Model 1, compute the credibility premium of Policy Number 4 for the upcoming renewal. Students may refer actuarial table for the formula.

(ii) The medical inflation in last few years is very high and increasing at a rate of 20% per year on average. Describe the adjustment to be made in the above estimation (as determined in sub part i) in order to incorporate the medical inflation. Also, state any additional data that will be required.

(iii) Explain, without performing any further calculation, what change is expected to the credibility factor (determined in sub-part i), if Policy Number 1 is excluded.

Unit 1 & 2

ASSIGNMENT QUESTIONS

14. An insurance company has a sample of 500 motor insurance policies with claims. Say X denotes the number of policies where the annual claim amount exceeded INR 10,000 and X follows Binomial distribution with parameters (5000, p).

(i) Assuming annual total claim amounts per policy are independent and identically distributed, estimate the unknown proportion "p" of claims higher than 10,000 per year by deriving its maximum likelihood.

(ii) If there is a prior knowledge about p follows Uniform distribution with parameters (0, 1) then derive the density of the Bayesian posterior distribution of p in terms of n and X. Also state the name of the posterior distribution that p follows along with its parameters.

If 200 out of 500 policies with claims from the sample have a claim above INR 10,000 then

(iii) Estimate p using the MLE in (i).

(iv) Estimate p using the Bayesian estimator under quadratic loss, based on the posterior distribution derived in part (ii).

(v) Comment on the difference between the values of p estimated in (iii) and (iv)

(vi) From part (iv), express the Bayesian estimator in the form of credibility factor and determine the value of credibility factor.