INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: SRM 2

Chapter: Unit 3 & 4

Category: Assignment solutions

1.

(i) $M_X(t) = E(e^{tX}) = \int_0^\infty e^{tx} f(x)dx$

$$= \int_0^\infty 0.01^2 x e^{(t-0.01)x} dx$$

$$= \left[\frac{0.01^2 x e^{(t-0.01)x}}{t-0.01}\right]_0^\infty - \int_0^\infty \frac{0.01^2 e^{(t-0.01)x}}{t-0.01} dx$$

$$= 0 - 0 - \left[\frac{0.01^2 e^{(t-0.01)x}}{(t-0.01)^2}\right]_0^\infty \text{ provided that } t < 0.01$$

$$= \frac{0.01^2}{(t-0.01)^2} \text{ again provided that } t < 0.01$$

(ii) The adjustment coefficient is the unique positive solution of

$M_X(R) = 1 + 1.45E(X)R$

But $E(X) = M'_X(0) = \dfrac{d}{dt}\left[\dfrac{0.01^2}{(t-0.01)^2}\right]\Bigg|_{t=0}$

$$= \left.\frac{-2\times 0.01^2}{(t-0.01)^3}\right|_{t=0} = \frac{-2}{-0.01} = 200$$

So we need to solve $\dfrac{0.01^2}{(R-0.01)^2} = 1 + 290R$

i.e. $0.01^2 = (1 + 290R)\,(R - 0.01)^2 = (1 + 290R)\,(0.01^2 - 0.02R + R^2)$

i.e. $0.012 = 0.01^2 + 0.029R - 0.02R - 5.8R^2 + R^2 + 290R^3$

i.e. $290R^2 - 4.8R + 0.009 = 0$

$$R = \frac{4.8 \pm \sqrt{4.8^2 - 4\times 290\times 0.009}}{2\times 290}$$

i.e. $R = 0.00215578$ or $R = 0.0143959$

So taking the smaller root we have $R = 0.00215578$ since that is less than 0.01

The upper bound for the probability of ruin is given by Lundberg's inequality as

$\psi(U) \le e^{-RU} = e^{-0.00215578U}$

(iii) We want $\psi(U) \le e^{-0.00215578U} \le 0.01$

i.e. $-0.00215578U \le \log 0.01$

i.e. $U \ge \dfrac{\log 0.01}{-0.00215578} = 2136.20$

(iv) This time the adjustment coefficient is the solution to:

$e^{200R} = 1 + 290\text{R}$

So the question is whether $y = e^{200R}$ crosses the line $y = 1 + 290R$ before or after $y = 0.01^2(0.01 - R)^{-2}$ crosses the same line
But when $R = 0.00215578$ we have
$e^{200R} = e^{200\times 0.00215578} = 1.539 < 1 + 290R = 1.625.$

So $y = e^{200R}$ has not yet crossed the given line, and the second scenario has a larger adjustment coefficient that the first.

This means the second risk has a lower probability of ruin, which is to be expected since although the mean claim amounts are the same in each scenario, the claim amounts in the first scenario are more variable suggesting a greater risk.

2.

(i) Let $S(t)$ denote cumulative claims to time t. Let the annual rate of premium income be c and let the insurer's initial surplus be U=100.

Then the surplus at time t is given by:

$$U(t) = U + ct - S(t)$$

And the relevant probabilities are defined by:

$$\psi(100) = P(U(t) < 0 \text{ for some } t > 0)$$
$$\psi(100,1) = P(U(t) < 0 \text{ for some } t \text{ with } 0 < t \le 1)$$
$$\psi_1(100,1) = P(U(1) < 0)$$

(ii) The adjustment coefficient is the unique positive root of the equation

$$\lambda M_X(R) = \lambda + cR$$

Where λ is the rate of the Poisson process (i.e. 100) and X is the normal distribution with mean 30 and standard deviation 5.

(iii) In this case we have:

$$c = 100 \times 30 \times 1.2 = 3600$$

And

$$M_X(R) = \exp(30R + 12.5R^2)$$

So R is the root of

$$100\exp\left(30R + 12.5R^2\right) - 100 - 3600R = 0$$

Denote the left hand side of this equation by $f(R)$.

When $R = 0.0115$ we have

$$f(0.0115) = 100\exp(0.346653125) - 100 - 41.4 = 0.032604592 > 0$$

And when $R = 0.0105$ we have

$$f(0.0105) = 100\exp(0.316378125) - 100 - 37.8 = -0.585099862 < 0$$

Since the function changes sign between 0.0105 and 0.0115 the unique positive root must lie between these values and hence the root is 0.011 correct to 3 decimal places.

(iv) By Lundberg's inequality $\psi(100) < \exp(-100\times 0.011) = 0.33287$

Claims in the first year are approximately Normal, with mean $100\times 30 = 300$

And variance given by $100\times\left(25+30^2\right) = 92500$

So approximately

$$\psi_1(100,1) = P(100+3600-N(3000,92500)<0)$$

$$= P\left(N(3000,92500)>3700\right) = P\left(N(0,1) > \frac{3700-3000}{\sqrt{92500}}\right)$$

$$= P(N(0,1) > 2.302)$$

$$= 1-\left(0.98928\times 0.8 + 0.98956\times 0.2\right)$$

$$= 0.0107.$$

(v) The probability of ruin is much smaller in the first year than the long-term bound provided by Lundberg's inequality. This suggests that either the bound in Lundberg's inequality may not be that tight or that there is significant probability of ruin at times greater than 1 year.

3.

(i) The annual premium charged is $0.25 \times 150 \times 1.7 = 63.75$

(ii) Let X be an individual claim. Then

$$P(X < 200) = P\left(N\left(150, 30^2\right) < 200\right)$$

$$= P\left(N(0,1) < \frac{200-150}{30}\right)$$

$$= P(N(0,1) < 1.667)$$

$$= (0.95154 \times 0.3 + 0.7 \times 0.95254)$$

$$= 0.95224$$

(iii) We need to calculate:

$$p = \sum_{j=0}^{\infty} P(j \text{ claims}) \times P(\text{all claims below retention}) \, [1]$$

$$= \sum_{j=0}^{\infty} e^{-0.25} \frac{(0.25)^j}{j!} \times (0.95224)^j$$

$$= e^{-0.25} \times \sum_{j=0}^{\infty} \frac{(0.25 \times 0.95224)^j}{j!}$$

$$= e^{-0.25} \times e^{0.25 \times 0.95224}$$

$$= 0.9881$$

(iv) We need to first calculate the mean claim amount paid by the reinsurer. This is given by

$$I = \int_{200}^{\infty} (x-200) f(x) dx$$

Where $f(x)$ is the pdf of the Normal distribution with mean 150 and standard deviation 30.

Using the formula on p18 of the tables, we have:

$$I = \int_{200}^{\infty} xf(x) dx - 200P(X > 200)$$

$$= 150 \times \left[\Phi(\infty) - \Phi(1.667)\right] - 30 \times \left(\phi(\infty) - \phi(1.667)\right) - 200 \times (1 - 0.95224)$$

$$= 150(1 - 0.95224) - 30 \times (0 - 0.09942) - 200 \times 0.0.04776$$

$$= 0.5946$$

So the reinsurer charges $0.25 \times 0.5946 \times 2.2 = 0.32703$

(v) The direct insurers expected profit is given by:

$$63.75 - 0.32703 - 0.25 \times (150 - 0.5946) = 26.07$$

4.

Multiply the claim payments with the corresponding inflation factors given below:

Development year

2004	1.16757	1.11197	1.05400	1.00000
2005	1.11197	1.05400	1.00000	
2006	1.05400	1.00000		
2007	1.00000			

The resulting table is:

Development year

2004	478.70	905.14	227.66	79.00
2005	639.38	990.76	281.00	
2006	857.96	1066.00		
2007	1142.00			

The inflation adjusted accumulated claim payments in mid 2007 are given below:

Development year

year	0	1	2	3
2004	478.70	1383.84	1611.50	1690.50
2005	639.38	1630.14	1911.14	**2004.83**
2006	857.96	1923.96	**2248.66**	**2358.90**
2007	1142.00	**2853.75**	**3335.38**	**3498.88**

Note only the values of the last row are needed for the answer.

The bolded values show the completed table using the basic chain ladder approach.

The development factors are 2.4989, 1.1688, 1.0490.

For the answer we only need to work with the projected values at the last row as:

$(2853.75-1142.00)*1.08+(3335.38-2853.75)*1.08^2+(3498.88-3335.38)*1.08^3$
$= 2616.43$

$2616.43 * 5000 = £13{,}082{,}150$

5.

a. The development factors are given by

$R_1 = (136 + 156 + 130) / (96 + 100 + 120) = 1.335443$

$R_2 = (140 + 160) / (136 + 156) = 1.027397$

$R_3 = 168 / 140 = 1.2$

The fully developed table using the chain ladder is below:

Incident year	*0*	*1*	*2*	*3*
2005	96	136	140	168
2006	100	156	160	**192**
2007	120	130	**133.56**	**160.28**
2008	136	**181.62**	**186.60**	**223.92**
R	1.335443	1.027397	1.2	1
f	1.646436	1.232876	1.2	1

Reserve = (168 + 192 + 160.28 + 223.92) – (168 + 160 + 130 + 136) = 150.2

b. B-F method

Estimated loss ratio: 168/175 = 0.96

	2008	*2007*	*2006*	*2005*
F	1.646436	1.232876	1.2	1
1 – 1/f	0.392627	0.188888	0.1666667	0
IUL	188.16	182.4	173.76	168
Emerging liab.IUL(1 – 1/f)	73.87678	34.45325	28.96	0

Reserve is now = 73.87678 + 34.45325 + 28.96 = 137.29

6.

(i) The development ratio for development year 2 to development year 3 is given by 1862.3/1820 = 1.023242

Therefore $W = 1762 \times 1.023242 = 1803.0$

Because there is no claims development beyond development year 3 $X = 1803.0$ also.

The development factor from development year 1 to ultimate is given by 2122.5/1805 = 1.1759003

So the ratio from development year 1 to development year 2 is given by 1.1759003/1.023242 = 1.149190785

But under the definition of the chain ladder approach, this is calculated as:

$$1.149190785 = \frac{1762+1820}{Y+1485} = \frac{3582}{Y+1485}$$

So $Y = \dfrac{3582}{1.149190785} - 1485 = 1632.0$

(ii) We require the development ratio from year 0 to year 1; this is given by:

$$\frac{1485+1632+1805}{1001+1250+1302} = \frac{4922}{3553} = 1.385308$$

The development factor to ultimate is therefore

$$1.385308 \times 1.149190785 \times 1.023242 = 1.628984285$$

And so $Z = 2278.8 - 2500 \times 0.9 \times \left(1 - \dfrac{1}{1.628984285}\right) = 1410.0$

(iii) The outstanding claims reserve is

$$1862.3 + 2122.5 + 2278.8 - 1820 - 1805 - 1410 = 1228.6$$

7.

(i) The cumulative cost of claims is given by:

Accident year	*Development year* 0	1	2
2011	2,233	3,622	4222
2012	3,380	5,188	
2013	4,996		

Dividing by cumulative claim numbers:

Accident year	*Development year* 0	1	2
2011	15.950	17.842	18.848
2012	18.778	22.557	
2013	19.516		

using grossing up factors to estimate the ultimate average cost per claim for each accident year:

Accident year	*Development year* 0	1	2
2011	84.623% 15.950	94.663% 17.842	100% 18.848
2012	78.805% 18.778	94.663% 22.557	23.828
2013	81.714% 19.516		23.883

Taking the same approach for the claim numbers gives:

Accident year	*Development year* 0	1	2
2011	62.5% 140	90.625% 203	100% 224
2012	70.924% 180	90.625% 230	253.8
2013	66.712% 256		383.7

Total outstanding claims are therefore

$253.8 \times 23.828 + 383.7 \times 23.883 - 5188 - 4996$
$= \underline{5028.2}$ [7]

(ii) **Assumptions**

- The number of claims relating to each development year is a constant proportion of the total claim numbers from the relevant accident year.
- Claim amounts for each development year are a constant proportion of the total claim amount for the relevant accident year.
- Claims are fully run off after development year 2.

8. i) I

ii) a) Let S(t) denote the insurer's surplus at time t.
Then ψ(U) = Pr(S(t) < 0 for some value of t) i.e. the probability of ruin at some time
ψ(U,t) = Pr(S(k) < 0) for some k < t i.e. the probability that ruin occurs before time t.

b) Immediately before the payment of any claims, the insurer has cash reserves of 10,00,000 + 10,000+5,000 =10,15,000.

The distribution of S(1) is given by:

Deaths	S(1)	Prob	
None	1015000	=0.95*0.9 =	0.855
A Only	1015000-1700000=-685000	=0.9*0.05=	0.045
B only	1015000-400000=615000	=0.95*0.10=	0.095
A&B	1015000-1700000-400000=-1085000	=0.05*0.1=	0.005
Probability of Ruin is		=.045+.005	0.05

c) Assuming the surplus process ends if ruin occurs by time 1, then 2 possible values of S(2) are -6,85,000 and -10,85,000.
If there are no deaths in year 1, possible values of S(2) are
No deaths: 1015000 + 15000 = 10,30,000

Deaths	S(2) when no deaths in year 1	Prob	
None	10,30,000	=0.95*0.9 =	0.855
A Only	1030000-1700000=-670000	=0.9*0.05=	0.045
B only	1030000-400000=630000	=0.95*0.10=	0.095
A&B	1030000-1700000-400000=-1070000	=0.05*0.1=	0.005

If B dies in year 1, the possible values of S(2) are:
A lives: 615000 + 10000 = 6,25,000
A dies: 615000 + 10000 – 17,00,000 = -10,75,000

The probability of ruin within 2 years is given by:
0.05 + 0.855 × (0.05 × 0.9 + 0.05 × 0.1) + 0.095 × 0.05 = 0.0975

9. (i) The inflation figures for the year 2018 will be ignored in the calculation as this year claims payments are fully run-off.

(ii) The claim payouts given are as follows:

			Rs. In Crores	
	Claims paid in the year of Development			
Year of Accident	1	2	3	4
2018	275	115	62	22
2019	315	162	82	
2020	180	170		
2021	450			

The claim payouts have to be converted into 2021 inflation level. The converted payouts are as follows:

			Rs. In Crores	
	Claims paid in the year of Development			
Year of Accident	1	2	3	4
2018	315	126	66	22
2019	344	172	82	
2020	191	170		
2021	450			

Where 2018,1 = 275*1.05 * 1.03 *1.06 = 315
2018,2 = 115 * 1.03 * 1.06 = 126

2020,1= 180*1.06 (2)

Now, the above table has to be updated for cumulative claim payouts and the updated payouts are as follows:

			Rs. In Crores	
	Claims paid in the year of Development			
Year of Accident	1	2	3	4
2018	315	441	507	529
2019	344	516	598	
2020	191	361		
2021	450			

The development factors for the payouts are as follows:
Development Factor for Year 4 = (529/507) = 1.043
Development Factor for Year 3 = (507+598/441+516) = 1.544
Development Factor for Year 2 = (441+516+361/315+344+191) = 1.55

Using the above development factors, the run-off triangle is completed as follows:

	Claims paid in the year of Development			Rs. In Crores
Year of Accident	1	2	3	4
2018	315	441	507	529
2019	344	516	598	624
2020	191	361	417	435
2021	450	697	805	840

(1)

Now, to estimate the future inflation adjusted claims, we need to first find the estimated claims arising during each of the development years. The estimated claims arising during each of the development years are as follows:

	Claims paid in the year of Development			Rs. In Crores
Year of Accident	1	2	3	4
2018				
2019				26
2020			56	18
2021		247	108	35

Now, the projected claims have to be adjusted for future inflation and the table will adjusted as

	Claims paid in the year of Development			Rs. In Crores
Year of Accident	1	2	3	4
2018				
2019				28
2020			60	21
2021		265	123	44

Therefore, the outstanding claim reserve is = 28+60+21+265+123+44 = 541 Crores

(iii) Assumptions:

· The first year is fully run-off

· Claims in each development year are a constant proportion in real terms of total claims for each accident year

· Inflation is allowed for explicitly

· Past inflation and future inflation figures are correct.

10.

i) a) is false since there cannot be a claim until time 2

b) is false since the insurance company could be ruined at time 3 if there is a claim, if U is sufficiently small.
c) is false since the insurance company cannot be ruined in year 4, since by that stage it will have sufficient premiums to cover any loss.
d) is true since if it is not ruined by time 4, the insurance company cannot be ruined

ii) The premium charged will be: 1.3*0.2* (0.25*10Crores + 0.75*1Crore) = Rs. 0.845 Crores

iii) The possibilities are tabulated below, where N means not injured, R means injured but recovered and X means injured but career ending:

Year 1	Year 2	Probability	Ruin	Marks
N	N	0.8*0.8=0.64	No	
N	R	0.8*0.2*0.75=0.16	No	
N	X	0.8*0.2*0.25=0.04	Yes	1
R	N	0.2*0.75*0.8=0.16	No	
R	R	0.2*0.75*0.2*0.75=0.0225	No	
R	X	0.2*0.75*0.2*0.25=0.0075	Yes	1
X	N/A	0.2*0.25=0.05	Yes	1

Summing the cases where ruin occurs we have
ψ(1Crore, 2)=0.04+ 0.0075+0.0025=0.0975