

**Class:** SY MSc

**Subject :** Statistical & Risk Modelling - 2

**Chapter:** Unit 4 Chapter 1

**Chapter Name:** Methods and models of loss reserving

# Agenda

1. Reserves
  1. Types of reserves
2. Claims Data
3. Projections using development factors
4. The Chain Ladder Method (BCL)
5. The Inflation adjusted chain ladder method
6. The Average cost per claim method (ACPC)
7. The Bornheutter-Ferguson Method (BF)

# 0 Introduction

## The origins of run-off triangles

Run-off triangles (or delay triangles) are an important topic in the practical work of actuaries working in general insurance who make use of spreadsheets and other computer packages to forecast future claim numbers and amounts.

Run-off triangles (delay triangles) usually arise in types of insurance (particularly non-life insurance) where it may take some time after a loss until the full extent of the claims which have to be paid is known. It is important that the claims are attributed to the year in which the policy was written.

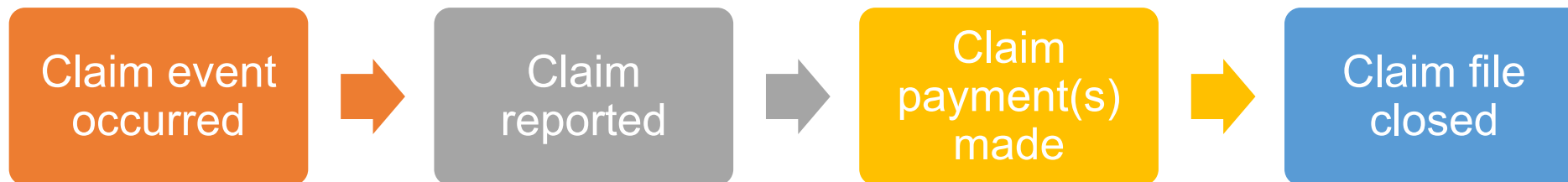
Although the insurance company does not know the exact figure for total claims each year, it must try to estimate that figure with as much confidence and accuracy as possible.

# 1 Reserves

## Reserving

General insurers need to be able to estimate the ultimate cost of claims for several purposes. For example, they need to know the full cost of paying claims in order to set future premium rates. They also need to set up reserves in their accounts to make sure that they have sufficient assets to cover their liabilities.

The normal steps involved in settling a general insurance claim are shown in the diagram:



## 1.1 Types of reserves

### Types of reserves

- An IBNR (pronounced 'I.B.N.R.') claims reserve is required in respect of claims that have been incurred but not reported, reported but the claim event has occurred, but the claim has not yet been reported to the insurer.
- An outstanding reported claims reserve is required in respect of claims that have been reported but have not yet been closed.

## 2 Claims data

### Presentation of claims data

There are several ways of presenting claims data, which emphasize different aspects of the data. Here they will be presented as a triangle, which is the most used method.

- The year in which the incident happened, and the insurer was on risk is called the accident year.
- The number of years until a payment is made is called the delay, or development period.
- The claims data is divided up by the accident year and the development year.

## 2 Claims Data

### Presentation of claims data

The following table is an example of claims data referenced by accident year and development year. In some types of insurance, it might be relevant to look at development of claims by month or quarter, but the principles are unchanged.

| Accident<br>Year | Cumulative claim payments |       |       |       |       |
|------------------|---------------------------|-------|-------|-------|-------|
|                  | Development Year          |       |       |       |       |
|                  | 0                         | 1     | 2     | 3     | 4     |
| 2008             | 786                       | 1,410 | 2,216 | 2,440 | 2,519 |
| 2009             | 904                       | 1,575 | 2,515 | 2,796 |       |
| 2010             | 995                       | 1,814 | 2,880 |       |       |
| 2011             | 1,220                     | 2,142 |       |       |       |
| 2012             | 1,182                     |       |       |       |       |

## 3

# Projections using development factors

## Run-off patterns

The basic assumption made in estimating outstanding claims concerns the run-off pattern. The simplest assumption is that payments will emerge in a similar way in each accident year.

The proportionate increases in the known cumulative payments from one development year to the next can then be used to calculate the expected cumulative payments for future development years.

The ratios that are used to project future claims are known as development factors or link ratios. A development factor may describe the ratio between cumulative claim amounts in consecutive years or between years over a longer period.



## 4 The chain ladder method

This section explains how you would carry out the calculations for completing the run-off triangle using the basic chain ladder method.

The name given to this method presumably arises from the ladder-like operations which are chained over the development years. The development factors for the chain ladder technique can be found for each development year by adding the appropriate number of terms.

After calculating the development factors, we will do projection for further claims.

Note that no projection can be done for the first accident year because it is not possible to project beyond the highest development year. It is therefore assumed that all claims from this cohort have completely run off.

## 4 The chain ladder method

### Assumptions:

- The first accident year is fully run off.
- Claims in each development year are a constant proportion in monetary terms of total claims for each accident year.
- Inflation is not allowed for explicitly, rather it is allowed for implicitly as a weighted average of past inflation.

# Question

CT6 April 2013 Q5

The following table shows incremental claims data from a portfolio of insurance policies for the accident years 2010, 2011 and 2012. Claims from this type of policy are fully run off after the end of development year two.

|                      |      | <i>Incremental<br/>Claims</i> |          |          |
|----------------------|------|-------------------------------|----------|----------|
|                      |      | <i>Development year</i>       |          |          |
|                      |      | <i>0</i>                      | <i>1</i> | <i>2</i> |
| <i>Accident year</i> | 2010 | 2,328                         | 1,484    | 384      |
|                      | 2011 | 1,749                         | 1,188    |          |
|                      | 2012 | 2,117                         |          |          |

Estimate the total claims outstanding using the basic chain ladder technique.

# Solution

First accumulate claims:

|                      |             | <i>Cumulative</i>       |          |          |
|----------------------|-------------|-------------------------|----------|----------|
|                      |             | <i>Claims</i>           |          |          |
|                      |             | <i>Development year</i> |          |          |
|                      |             | <i>0</i>                | <i>1</i> | <i>2</i> |
| <i>Accident year</i> | <i>2010</i> | 2,328                   | 3,812    | 4,196    |
|                      | <i>2011</i> | 1,749                   | 2,937    |          |
|                      | <i>2012</i> | 2,117                   |          |          |

$$DY1 = (3,812 + 2,937) / (2,328 + 1,749) = 1.655\ 384$$

$$DY2 = 4,196 / 3,812 = 1.100\ 735$$

# Solution

Now complete lower half of table:

| <i>Cumulative<br/>Claims</i> |             | <i>Development year</i> |          |          |
|------------------------------|-------------|-------------------------|----------|----------|
|                              |             | <i>0</i>                | <i>1</i> | <i>2</i> |
| <i>Accident year</i>         | <i>2009</i> | 2,328                   | 3,812    | 4,196    |
|                              | <i>2011</i> | 1,749                   | 2,937    | 3,232.86 |
|                              | <i>2012</i> | 2,117                   | 3,504.45 | 3,857.47 |

So estimated amount of outstanding claims is:

$$(3,232.86 - 2,937) + (3,857.47 - 2,117) = 2,036.3.$$

## 5 The inflation adjusted chain ladder method

### Dealing with past inflation

Claims inflation will affect the payments in the run-off triangle by calendar year of payment.

The inflation adjusted chain ladder method works by adjusting the figures in the triangle to allow for the effects of inflation. Here, it will be assumed that claims inflation is at the same annual rate for all claims within a particular calendar year of payment.

When adjusting for inflation, it is the payments in each calendar year which need to be considered, rather than cumulative totals.

The first step is to calculate incremental payments from the cumulative totals, by differencing along each row. For simplicity, it is also assumed that payments are made in the middle of each calendar year. Then it is straightforward to form a table of inflation adjusted cumulative payments to which the chain ladder technique can be applied.

## 5 The inflation adjusted chain ladder method

### Dealing with future inflation

The predictions of cumulative payments do not, however, take account of future inflation. In order to forecast the actual payments, an assumed rate of future inflation will be needed. Again, it is necessary to convert to non-cumulative data rather than the cumulative totals before adjusting these for future inflation in a similar way to that used when dealing with past inflation.

## 5 The inflation adjusted chain ladder method

### Steps:

1. Apply past inflation factors to incremental data so that all the claims data in the table is expressed in the monetary terms of the most recent accident year.
2. Accumulate the data and calculate development factors.
3. Use these development factors to project the future cumulative claims (note that these will still be expressed in the monetary terms of the most recent accident year).
4. Disaccumulate the data to make it incremental.
5. Apply future inflation assumptions to convert the outstanding claim payments into the amounts relating to each future year.



## 5 The inflation adjusted chain ladder method

### Assumptions:

- The first accident year is fully run off.
- Claims in each development year are a constant proportion in real terms of total claims for each accident year.
- Inflation is allowed for explicitly and we assume that both the past and future inflation assumptions are correct

# Question

CT6 April 2012 Q8

The table below shows claims paid on a portfolio of general insurance policies. You may assume that claims are fully run off after three years.

| <i>Underwriting year</i> | <i>Development Year</i> |          |          |          |
|--------------------------|-------------------------|----------|----------|----------|
|                          | <i>0</i>                | <i>1</i> | <i>2</i> | <i>3</i> |
| 2008                     | 450                     | 312      | 117      | 41       |
| 2009                     | 503                     | 389      | 162      |          |
| 2010                     | 611                     | 438      |          |          |
| 2011                     | 555                     |          |          |          |

Past claims inflation has been 5% p.a. However, it is expected that future claims inflation will be 10% p.a.

Use the inflation adjusted chain ladder method to calculate the outstanding claims on the portfolio.

# Solution

The claims uplifted to 2011 prices are as follows:

| <i>Underwriting<br/>Year</i> | <i>Development Year</i> |          |          |          |
|------------------------------|-------------------------|----------|----------|----------|
|                              | <i>0</i>                | <i>1</i> | <i>2</i> | <i>3</i> |
| 2008                         | 520.93                  | 343.98   | 122.85   | 41       |
| 2009                         | 554.56                  | 408.45   | 162      |          |
| 2010                         | 641.55                  | 438      |          |          |
| 2011                         | 555                     |          |          |          |

Accumulating gives:

| <i>Underwriting<br/>Year</i> | <i>Development Year</i> |          |          |          |
|------------------------------|-------------------------|----------|----------|----------|
|                              | <i>0</i>                | <i>1</i> | <i>2</i> | <i>3</i> |
| 2008                         | 520.93                  | 864.91   | 987.76   | 1028.76  |
| 2009                         | 554.56                  | 963.01   | 1125.01  |          |
| 2010                         | 641.55                  | 1079.55  |          |          |
| 2011                         | 555                     |          |          |          |

# Solution

Hence the development factors are given by:

$$DF_{0,1} = \frac{864.91 + 963.01 + 1079.55}{520.93 + 554.56 + 641.55} = 1.693304$$

$$DF_{1,2} = \frac{987.76 + 1125.01}{864.91 + 963.01} = 1.155833$$

$$DF_{2,3} = \frac{1028.76}{987.76} = 1.041508$$

The completed triangle of cumulative claims is:

| <i>Underwriting<br/>year</i> | <i>Development Year</i> |          |          |          |
|------------------------------|-------------------------|----------|----------|----------|
|                              | <i>0</i>                | <i>1</i> | <i>2</i> | <i>3</i> |
| 2008                         | 520.93                  | 864.91   | 987.76   | 1028.76  |
| 2009                         | 554.56                  | 963.01   | 1125.01  | 1171.70  |
| 2010                         | 641.55                  | 1079.55  | 1247.78  | 1299.57  |
| 2011                         | 555.00                  | 939.78   | 1086.23  | 1131.32  |

# Solution

Dis-accumulating gives (in 2011 prices):

| <i>Underwriting<br/>year</i> | <i>Development Year</i> |          |          |          |
|------------------------------|-------------------------|----------|----------|----------|
|                              | <i>0</i>                | <i>1</i> | <i>2</i> | <i>3</i> |
| 2008                         |                         |          |          |          |
| 2009                         |                         |          |          | 46.70    |
| 2010                         |                         |          | 168.23   | 51.79    |
| 2011                         |                         | 384.78   | 146.45   | 45.09    |

Inflating for future claims growth gives:

| <i>Underwriting<br/>year</i> | <i>Development Year</i> |          |          |          |
|------------------------------|-------------------------|----------|----------|----------|
|                              | <i>0</i>                | <i>1</i> | <i>2</i> | <i>3</i> |
| 2008                         |                         |          |          |          |
| 2009                         |                         |          |          | 51.37    |
| 2010                         |                         |          | 185.05   | 62.67    |
| 2011                         |                         | 423.26   | 177.20   | 60.01    |

And the outstanding claims are:

$$51.37 + 62.67 + 60.01 + 185.05 + 177.20 + 423.26 = 959.56$$

## 6

# The average cost per claim method

This method, considers separately the two key elements of total claim amounts, ie the number of claims and the average amounts of the claims.

## Description of method

- This method requires development tables for both total claim amounts and claim numbers. We normally use cumulative figures, as before.
- A third development table, of the average claim amounts, is then formed by dividing the figures in the corresponding cells of the first two tables.
- The next stage is the projection of figures in the average claims and number of claims tables, using either grossing-up factors or development factors.
- **A grossing-up factor is not very different from a development factor. A grossing-up factor gives the proportion of the ultimate claim amount that has been paid so far.**

## 6

## The average cost per claim method

### Steps:

1. Divide the entry in each cell in the cumulative claims table by the entry in the corresponding cell of the claim number table. This gives the average cost per claim.
2. Calculate grossing-up factors for the average claim amounts. Use these to estimate the final average for each accident year.
3. Repeat the last step for the claim number table.
4. For each accident year, multiply together the figures from the ACPC and claim number tables.
5. Sum over all accident years to obtain the total projected loss estimate.

## 6

# The average cost per claim method

## Assumptions:

- The first accident year is fully run off.
- The average cost per claim in each development year is a constant proportion in monetary terms of the ultimate average cost per claim for each accident year.
- The number of claims in each development year is a constant proportion in of the ultimate number of claims for each accident year.
- Inflation is not allowed for explicitly, rather it is allowed for implicitly as a weighted average of past inflation.



# Question

## CT6 September 2017 Q5

The table below shows the cost of claims settled per calendar year for a set of car insurance policies, with figures in €000s.

| <i>Accident Year</i> | <i>Development Year</i> |          |          |
|----------------------|-------------------------|----------|----------|
|                      | <i>0</i>                | <i>1</i> | <i>2</i> |
| 2014                 | 5,419                   | 908      | 239      |
| 2015                 | 6,234                   | 1,088    |          |
| 2016                 | 7,719                   |          |          |

The corresponding number of settled claims is as follows:

| <i>Accident Year</i> | <i>Development Year</i> |          |          |
|----------------------|-------------------------|----------|----------|
|                      | <i>0</i>                | <i>1</i> | <i>2</i> |
| 2014                 | 760                     | 98       | 37       |
| 2015                 | 819                     | 93       |          |
| 2016                 | 881                     |          |          |

# Question

CT6 September 2017 Q5

- (i) Calculate the outstanding claims reserve for this portfolio, using the average cost per claim method with grossing up factors. [7]
- (ii) State four key assumptions made in part (i). [2]  
[Total 9]

# Solution

(i) Cumulative amounts & claims

| <i>Accident Year</i> |          |          |          |
|----------------------|----------|----------|----------|
|                      | <b>0</b> | <b>1</b> | <b>2</b> |
| 2014                 | 5,419    | 6,327    | 6,566    |
| 2015                 | 6,234    | 7,322    |          |
| 2016                 | 7,719    |          |          |

| <i>Accident Year</i> |          |          |          |
|----------------------|----------|----------|----------|
|                      | <b>0</b> | <b>1</b> | <b>2</b> |
| 2014                 | 760      | 858      | 895      |
| 2015                 | 819      | 912      |          |
| 2016                 | 881      |          |          |

# Solution

Average cost per claim (cumulative)

| <i>Accident Year</i> | Development Year     |                       |                               |
|----------------------|----------------------|-----------------------|-------------------------------|
|                      | 0                    | 1                     | 2                             |
| 2014                 | 97.191%<br>7.130 263 | 100.515%<br>7.374 126 | 100%<br>7.336 313             |
| 2015                 | 95.297%<br>7.611 722 | 100.515%<br>8.028509  | <b>100%</b><br><b>7.987 3</b> |
| 2016                 | 96.244%<br>8.761 635 |                       | <b>100%</b><br><b>9.103 5</b> |

Claim numbers (cumulative)

| <i>Accident Year</i> |                |                |                               |
|----------------------|----------------|----------------|-------------------------------|
|                      | 0              | 1              | 2                             |
| 2014                 | 84.916%<br>760 | 95.866%<br>858 | 100%<br>895                   |
| 2015                 | 86.090%<br>819 | 95.866%<br>912 | <b>100%</b><br><b>951.33</b>  |
| 2016                 | 85.503%<br>881 |                | <b>100%</b><br><b>1,030.4</b> |

Total claims  $6,566 + 7.98734 * 951.329 + 9.103564 * 1030.373 = 23,545$

Claims to date 21,607 so reserve €1,938k

# Solution

- (ii) Claims fully developed after DY2.  
The proportions of claim numbers relating to each DY remain constant in different AYs.  
Cost of claims settled equals amount actually paid out.  
The average cost per claim figures relating to each DY remain constant in different AYs.  
Inflation has been allowed for.

## 7 Loss Ratios

The ratio of incurred claims to earned premiums over a defined period is called the loss ratio.

$$\text{Loss ratios} = \frac{\text{incurred claims}}{\text{earned premium}}$$

Investigation of the loss ratios for each of several different origin years would normally show some consistency, provided that there have not been any distortions, and in particular no significant change in premium rates.

## 7 The Bornhuetter-Ferguson method

The Bornhuetter-Ferguson method combines the estimated loss ratio with a projection method. Here, 'projection method' refers to methods such as the basic chain ladder method which are based on past claim amounts and/or numbers. It therefore improves on the crude use of a loss ratio by taking account of the information provided by the latest development pattern of the claims, whilst the addition of the loss ratio to a projection method serves to add some stability against distortions in the development pattern.

The concepts behind the method are:

- That whatever claims have already developed in relation to a given origin year, the future development pattern will follow that experienced for other origin years.
- The past development for a given origin year does not necessarily provide a better clue to future claims than the more general loss ratio. In other words, it is a compromise that combines the loss ratios with the development pattern.

## 7 The Bornhuetter-Ferguson method

In its simplest form the concept leads to the following approach to calculations:

1. Determine the initial estimate of the total ultimate claims from each origin year using premiums and loss ratios.
2. Divide these estimates by projection factors (f) determined, in a normal manner, from a claims development table. These are effectively estimates of the claims that should have developed to date.
3. Subtract these amounts from the corresponding total ultimate claims figures to give an estimate of the amount of claims that are yet to develop.

Clearly, the three stages could be combined and expressed as:

$$\text{Future claims development} = \text{Premium} \times \text{Estimated Loss Ratio} \times (1 - 1/f)$$



## 7 The Bornhuetter-Ferguson method

### Steps:

1. Decide on the amount of the loss ratio.
2. Calculate development factors (as in BCL method).
3. Calculate the cumulative development factors  $f$ . For each accident year that is not fully run-off:
4. Multiply the earned premium by the loss ratio to obtain the initial estimate of the ultimate loss.
5. Use the initial estimate and the cumulative development factors to determine the expected amount paid out so far.
6. Use this to see how much is expected to be paid in the future (emerging liability).
7. The revised estimate of the ultimate loss is the reported liability (last known figure) plus the emerging liability.
8. Finally, sum the revised estimates of the ultimate losses for each accident year to obtain an estimate of the total liability.

## 7 The Bornhuetter-Ferguson method

### Assumptions:

- The first accident year is fully run off.
- The loss ratio is correct.
- Claims in each development year are a constant proportion in monetary terms of total claims for each accident year.
- Inflation is not allowed for explicitly, rather it is allowed for implicitly as a weighted average of past inflation.

# Question

CT6 April 2017 Q8

The table below shows the cumulative incurred claim amounts on a portfolio of insurance policies.

| <i>Underwriting Year</i> | <i>Development Year</i> |          |          |
|--------------------------|-------------------------|----------|----------|
|                          | <i>0</i>                | <i>1</i> | <i>2</i> |
| 2014                     | 3,215                   | 6,847    | 10,078   |
| 2015                     | 2,986                   | 7,123    |          |
| 2016                     | 4,167                   |          |          |

Claims are assumed to fully run off after Development Year 2. The estimated loss ratio of both 2015 and 2016 is 91% and the respective premium income in each year is:

|      | <i>Premium Income</i> |
|------|-----------------------|
| 2014 | 11,365                |
| 2015 | 12,012                |
| 2016 | 12,867                |

Contd.

# Question

CT6 April 2017 Q8

The total of claim amounts paid to date is 21,186 from policies written in 2014 to 2016.

Calculate the outstanding claim reserve for this portfolio using the Bornheutter-Ferguson method.

## Solution

DF from year 2 to year 3 is  $10078 / 6847 = 1.471885$

DF from year 1 to year 2 is  $(7123 + 6847) / (3215 + 2986) = 2.252862$

For AY 2015, expected ultimate loss is  $0.91 * 12012 = 10931$

Expected loss to date is  $10931 / 1.471885 = 7,426$

So the adjusted ultimate loss is  $10931 - (7426 - 7123) = 10627$

For AY 2016, expected ultimate loss is  $0.91 * 12867 = 11709$

Expected loss to date is  $11709 / (1.471885 * 2.252862) = 3531$

So the adjusted ultimate loss is  $11709 - (3531 - 4167) = 12345$

So the reserve is  $10078 + 10627 + 12345 - 21186 = 11864$