INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Statistical and Risk Modelling - 2

Chapter: Unit-1

Category: Practice Question

1. CT6 September 2010 Q3

An underwriter has suggested that losses on a certain class of policies follow a Weibull distribution. She estimates that the 10th percentile loss is 20 and the 90th percentile loss is 95.

(i) Calculate the parameters of the Weibull distribution that fit these percentiles.

(ii) Calculate the 99.5th percentile loss.

Answer: (i) γ = 1.9795337 and c = 0.000280056, (ii) 99.5th percentile loss =144.73

2. CT6 September 2010 Q7

An insurance company has a portfolio of policies under which individual loss amounts follow an exponential distribution with mean $1/\lambda$. There is an individual excess of loss reinsurance arrangement in place with retention level 100.

In one year, the insurer observes:

- 85 claims for amounts below 100 with mean claim amount 42; and
- 39 claims for amounts above the retention level.

(i) Calculate the maximum likelihood estimate of λ.

(ii) Show that the estimate of λ produced by applying the method of moments to the distribution of amounts paid by the insurer is 0.011164.

Answer: (i) λ = 0.011379

3. CT6 September 2010 Q10

An insurance company has a portfolio of 10,000 policies covering buildings against the risk of flood damage.

(i) State the conditions under which the annual number of claims on the portfolio can be modelled by a binomial distribution B(n, p) with n = 10,000. [3]

These conditions are satisfied and p = 0.03. Individual claim amounts follow a normal distribution with mean 400 and standard deviation 50. The insurer wishes to take out proportional reinsurance with the retention α set such that the probability of aggregate payments on the portfolio after reinsurance exceeding 120,000 is 1%.

(ii) Calculate α assuming that aggregate annual claims can be approximated by a normal distribution.

This reinsurance arrangement is set up with a reinsurer who uses a premium loading of 15%.

(iii) Calculate the annual premium charged by the reinsurer.

As an alternative, the reinsurer has offered an individual excess of loss reinsurance arrangement with a retention of M for the same annual premium. The reinsurer uses the same 15% loading to calculate premiums for this arrangement.

(iv) Show that the retention M is approximately 358.50.

[You may wish to use the following formula which is given on page 18 of the Tables:

$$\int_L^U xf(x)dx = \mu[\Phi(U') - \Phi(L')] - \sigma[\varphi(U') - \varphi(L')]$$

Where $L' = \frac{L-\mu}{\sigma}$ and $U' = \frac{U-\mu}{\sigma}$

Here $\Phi(z)$ is the cumulative density function of the N(0,1) distribution and $\varphi(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}$]

Answer: (ii) α = 88.2%, (iii) 16,284.

4. CT6 April 2011 Q10

The number of claims on a portfolio of insurance policies has a Poisson distribution with mean 200. Individual claim amounts are exponentially distributed with mean 40.

The insurance company calculates premiums using a premium loading of 40% and is considering entering into one of the following re-insurance arrangements:

A. No reinsurance.
B. Individual excess of loss insurance with retention 60 with a reinsurance company that calculates premiums using a premium loading of 55%.
C. Proportional reinsurance with retention 75% with a reinsurance company that calculates premiums using a premium loading of 45%.

(i) Find the insurance company's expected profit under each arrangement.

(ii) Find the probability that the insurer makes a profit of less than 2000 under each of the arrangements using a normal approximation.

Answer: (i) E(ZA) = 3200, E(ZB) = 2218.227, E(ZC) = 2300,
(ii) P(A) = 0.06681, P(B) = 0.34082, P(C) = 0.30854

5. CT6 September 2011 Q3

Loss amounts under a class of insurance policies follow an exponential distribution with mean 100. The insurance company wishes to enter into an individual excess of loss reinsurance arrangement with retention level M set such that 8 out of 10 claims will not involve the reinsurer.

(i) Find the retention M.

For a given claim, let X_I denote the amount paid by the insurer and X_R the amount paid by the reinsurer.

(ii) Calculate $E(X_I)$ and $E(X_R)$.

Answer: (i) M = 160.9437912, (ii) E(XI) = 100 and E(XR) = 20

6. CT6 September 2011 Q7

A portfolio of insurance policies contains two types of risk. Type I risks make up 80% of claims and give rise to loss amounts which follow a normal distribution with mean 100 and variance 400. Type II risks give rise to loss amounts which are normally distributed with mean 115 and variance 900.

(i) Calculate the mean and variance of the loss amount for a randomly chosen claim.

(ii) Explain whether the loss amount for a randomly chosen claim follows a normal distribution.

The insurance company has in place an excess of loss reinsurance arrangement with retention 130.

(iii) Calculate the probability that a randomly chosen claim from the portfolio results in a payment by the reinsurer.

(iv) Calculate the proportion of claims involving the reinsurer that arise from Type II risks.

Answer: (i) E(X) = 103 and Var(X) = 536, (ii) No, the loss distribution is not Normal. (iii) 0.115156, (iv) 53.6%

7. CT6 April 2012 Q3

Claim amounts on a certain type of insurance policy follow a distribution with density

$$f(x) = 3cx^2e^{-cx^3} \qquad \text{for } x>0$$

where c is an unknown positive constant. The insurer has in place individual excess of loss reinsurance with an excess of 50.

The following ten payments are made by the insurer:

- Losses below the retention: 23, 37, 41, 11, 19, 33
- Losses above the retention: 50, 50, 50, 50

Calculate the maximum likelihood estimate of c.

Answer: c = 8.8775 x 10^-6

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8. CT6 April 2012 Q7

The numbers of claims on three different classes of insurance policies over the last four years are given in the table below.

	Year 1	Year 2	Year 3	Year 4	Total
Class 1	1	4	5	0	10
Class 2	1	6	4	6	17
Class 3	5	6	4	9	24

The number of claims in a given year from a particular class is assumed to follow a Poisson distribution.

(i) Determine the maximum likelihood estimate of the Poisson parameter for each class of policy based on the data above.

Answer: (i) $\lambda 1 = 2.5$, $\lambda 2 = 4.25$, $\lambda 3 = 6$

9. CT6 September 2012 Q4

Claims arising on a particular type of insurance policy are believed to follow a Pareto distribution. Data for the last several years shows the mean claim size is 170 and the standard deviation is 400.

(i) Fit a Pareto distribution to this data using the method of moments.

(ii) Calculate the median claim using the fitted parameters and comment on the result.

Answer: (i) $\alpha = 2.441$ and $\lambda = 244.95$, (ii) 80.44

10. CT6 September 2012 Q6

Individual claim amounts from a particular type of insurance policy follow a normal distribution with mean 150 and standard deviation 30. Claim numbers on an individual policy follow a Poisson distribution with parameter 0.25. The insurance company uses a premium loading of 70% to calculate premiums.

(i) Calculate the annual premium charged by the insurance company.

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The insurance company has an individual excess of loss reinsurance arrangement with a retention of 200 with a reinsurer who uses a premium loading of 120%.

(ii) Calculate the probability that an individual claim does not exceed the retention.

(iii) Calculate the probability for a particular policy that in a given year there are no claims which exceed the retention.

(iv) Calculate the premium charged by the reinsurer.

(v) Calculate the insurance company's expected profit.

Answer: (i) 63.75, (ii) 0.95224, (iii) 0.9881, (iv) 0.32703, (v) 26.07

11. CT6 September 2012 Q8

An insurer classifies the buildings it insures into one of three types. For Type 1 buildings, the number of claims per building per year follows a Poisson distribution with parameter λ. Data are available for the last five years as follows:

Year	1	2	3	4	5
Number of type 1 buildings covered	89	112	153	178	165
Number of claims	15	23	29	41	50

Determine the maximum likelihood estimate of λ based on the data above.

Answer: $\lambda = 0.226686$

12. CT6 September 2013 Q2

Claim amounts on a certain type of insurance policy follow an exponential distribution with mean 100. The insurance company purchases a special type of reinsurance policy so that for a given claim X the reinsurance company pays

0	if $0 < X < 80$;
$0.5X - 40$	if $80 < X \le 160$;

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X – 120 if X ≥ 160

Calculate the expected amount paid by the reinsurance company on a randomly chosen claim.

Answer: 32.56.

13. CT6 September 2013 Q8

The number of claims per month Y arising on a certain portfolio of insurance policies is to be modelled using a modified geometric distribution with probability density given by

$$p(\alpha) = \frac{\alpha^{y-1}}{(1+\alpha)^y} \qquad y = 1, 2, 3, \ldots\ldots$$

where α is an unknown positive parameter. The most recent four months have resulted in claim numbers of 8, 6, 10 and 9.

Derive the maximum likelihood estimate of α

Answer: α = 7.25.

14. CT6 Oct 2015 Q9

A random variable X follows a gamma distribution with parameters α *and* λ.

(i) Derive the moment generating function (MGF) of X.

(ii) Derive the coefficient of skewness of X.

15. CT6 April 2016 Q2

A portfolio of insurance policies has two types of claims:

- Loss amounts for Type I claims are exponentially distributed with mean 120.
- Loss amounts for Type II claims are exponentially distributed with mean 110.

25% of claims are Type I, and 75% are Type II.

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(i) Calculate the mean and variance of the loss amount for a randomly chosen claim.

An actuary wants to model randomly chosen claims using an exponential distribution as an approximation.

(ii) Explain whether this is a good approximation.

Answer: (i) Mean = 112.5 and Variance = 12,694

16. CT6 April 2017 Q2

Claim amounts Xi from a portfolio of insurance policies follow a gamma distribution with parameters k and λ_i. Each λ_i.also follows a gamma distribution with parameters α and β.

(i) Show that the mixture distribution of losses is a generalised Pareto, with parameters α, β. k.

Claim amounts are now assumed to be exponentially distributed with parameter λ_i

(ii) Show, using your answer to part (i), that the mixture distribution of losses is now a standard Pareto distribution with parameters α, β.

17. CT6 April 2017 Q3

(i) Explain why claim amounts from general insurance policies are typically modelled using statistical distributions with heavy tails.

Claim amounts on a portfolio of insurance policies are assumed to follow a Weibull distribution. A quarter of losses are below 15 and a quarter of losses are above 80.

(ii) Estimate the parameters c, γ of the Weibull distribution that fit this data.

(iii) Determine whether or not this Weibull distribution has a heavier tail than that of the exponential distribution with parameter c, by considering your estimate of γ.

Answer: (ii) γ = 0.9394 and c = 0.0226

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18. CT6 September 2017 Q1

Claim amounts on a portfolio of insurance policies follow a Weibull distribution. The median claim amount is £1,000 and 90% of claims are less than £5,000.

Estimate the parameters of the Weibull distribution, using the method of moments.

Answer: γ = 0.74594 and c = 0.004009

19. CT6 September 2018 Q8

For a portfolio of insurance policies, claims Xi are independent and follow a gamma distribution, with parameters α = 6 and β, which is unknown.
A random sample of n claims, X1,…, Xn is selected, with mean X. .

(i) Derive an expression for the estimator of β using the method of moments.

(ii) Explain what the Maximum Likelihood Estimator (MLE) of β represents.

(iii) Derive an expression for the MLE of β, commenting on the result.

(iv) State the Moment Generating Function (MGF) of X.

Let Y = 2nβ X.

(v) Derive the MGF of Y, and hence its distribution, including statement of parameters.

20. CT6 September 2018 Q4

An insurance company has a portfolio of policies, where claim amounts follow a Pareto distribution with parameters α = 3 and λ = 100. The insurance company has entered into an excess of loss reinsurance agreement with a retention of M, such that 90% of claims are still paid in full by the insurer.

(i) Calculate M.

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(ii) Calculate the average claim amount paid by the reinsurer, on claims which involve the reinsurer.

Answer: (i) M = 115.4, (ii) 107.7

21. CT6 April 2018 Q2

An insurance company has a portfolio of insurance policies. Claims arise according to a Poisson process, and claim amounts have a probability distribution with parameter q.

(i) State one assumption the insurance company is likely to make when modelling n aggregate claim amounts.

(ii) Explain what the Maximum Likelihood Estimate (MLE) of q represents.

(iii) State an alternative to using the MLE.

(iv) Suggest two complications that may arise for the insurance company when it uses past claims data to determine the MLE of q.

22. CS2A April 2022 Q7

The annual aggregate claim amount, S, arising on a short-term insurance portfolio follows a compound Poisson distribution with parameter 5. Individual claim amounts follow a two-parameter Pareto distribution with parameters α and λ. A sample of individual claim amounts was taken and the sample mean and standard deviation were 10,000 and 15,000, respectively.

(i) Estimate the parameters of the Pareto distribution of the individual claim amounts using the method of moments.

(ii) Determine the variance and the third central moment of S, using the estimated Pareto parameters from part (i).

Answer: (i) α = 3.6 and λ = 26,000, (ii) var[S] = 1.625 * 10^9,
Third central moment of S = 2.1125 * 10^14

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23. CS2A September 2023 Q8

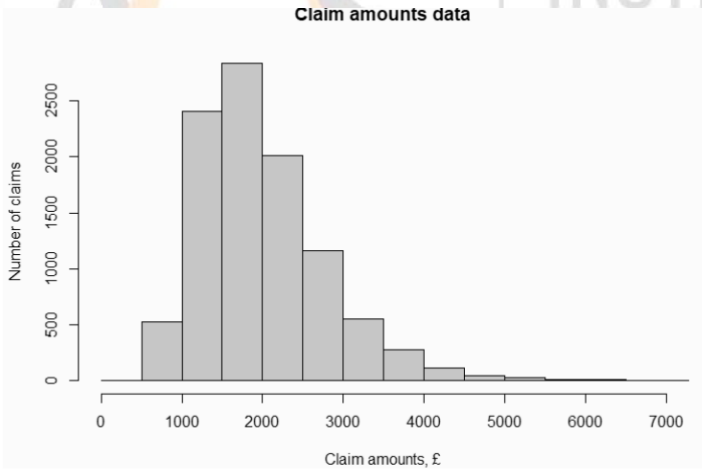
An insurer has incurred 10,000 claims under a portfolio of home insurance policies. These claims have a mean size of £2,000 and a standard deviation of £800. One hundred of these claims have exceeded the excess of loss limit on a reinsurance policy that the insurer has in place.

(i) Using a Lognormal distribution, estimate the excess of loss limit on this reinsurance policy. State your assumptions and show all your working clearly.

(ii) Calculate the number of claims that would be expected to be less than £1,000.

It has been proposed by the insurance regulator that statutory solvency calculations should be based on modelling claims using a suitable Normal distribution.

(iii) Comment on the appropriateness of using a Normal distribution under various conditions. (You may use the information about the portfolio of policies above to illustrate your answer.)



(iv) Comment briefly on which of the following alternative distributions should be considered, in addition to the Lognormal distribution, when fitting a suitable model to these claims, given the histogram showing the claims data in the figure above, and how you may decide which distribution to include in your final model.

gamma

exponential

weibull.

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Answer: (i) 2.33, (ii) 541 claims

24. CS2A April 2024 Q1

Suppose that the size of an insurance claim, X, has the following density function:

$$f(x) = \frac{1}{3x\sqrt{2\Pi}} e^{-z^2/2}$$

for $x > 0$ where $z = \frac{\ln(x)-2.6}{3}$.

The insurance coverage pays claims subject to a deductible of £2,000 per claim.

(i) State the distribution of X and its parameters.

(ii) Calculate the expected claim amount paid by the insurer per claim assuming that the claims occur at the end of the year with inflation of 20% p.a.

Answer: (i) Lognormal distribution with mu = 2.6 and sigma = 3, (ii) E(Z) = £1227.314

25. CS2A September 2024 Q1

An insurance company has a portfolio of policies where the loss amounts follow an exponential distribution (with parameter λ). The company has an individual excess of loss reinsurance arrangement with a retention level $12,000.

The following twelve claim amounts (all in $ and net of reinsurance) are observed:

8,760	10,510	7,800	6,900	11,000	9,000
9,500	11,570	10,400	12,000	12,000	12,000

(i) Calculate the maximum likelihood estimate of parameter λ.

The insurance company wishes to replace the current reinsurance treaty with a proportional reinsurance arrangement such that the population mean claim amount after retention arising from the new treaty is the same as that of the current treaty.

(ii) Derive the retained percentage for the proportional reinsurance arrangement.

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Answer: (i) $\lambda = 0.0000741$, (ii) 58.91%

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