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Subject: SRM - 3

Chapter: Unit 1

Category: Practice questions

1. CT6 April 2018 Q2
An insurance company has a portfolio of insurance policies. Claims arise according to a Poisson process, and claim amounts have a probability distribution with parameter q.

(i) State one assumption the insurance company is likely to make when modelling n aggregate claim amounts. [1]

(ii) Explain what the Maximum Likelihood Estimate (MLE) of q represents. [2]

(iii) State an alternative to using the MLE. [1]

(iv) Suggest two complications that may arise for the insurance company when it uses past claims data to determine the MLE of q. [2]
[Total 6]

2. CT6 September 2018 Q4
An insurance company has a portfolio of policies, where claim amounts follow a Pareto distribution with parameters $\alpha = 3$ and $\lambda = 100$. The insurance company has entered into an excess of loss reinsurance agreement with a retention of M, such that 90% of claims are still paid in full by the insurer.

(i) Calculate M. [4]

(ii) Calculate the average claim amount paid by the reinsurer, on claims which involve the reinsurer. [6]
[Total 10]

3. CT6 September 2018 Q8
For a portfolio of insurance policies, claims Xi are independent and follow a gamma distribution, with parameters $\alpha = 6$ and β, which is unknown.
A random sample of n claims, X1,..., Xn is selected, with mean $\bar{X}$. .

(i) Derive an expression for the estimator of β using the method of moments. [2]

(ii) Explain what the Maximum Likelihood Estimator (MLE) of β represents. [2]

(iii) Derive an expression for the MLE of β, commenting on the result. [5]

(iv) State the Moment Generating Function (MGF) of X. [1]

Let Y = 2nβ $\bar{X}$.

(v) Derive the MGF of Y, and hence its distribution, including statement of parameters. [5]
[Total 15]

4. CT6 September 2017 Q1
Claim amounts on a portfolio of insurance policies follow a Weibull distribution. The median claim amount is £1,000 and 90% of claims are less than £5,000.

Estimate the parameters of the Weibull distribution, using the method of moments. [4]

5. CT6 April 2017 Q3
(i) Explain why claim amounts from general insurance policies are typically modelled using statistical distributions with heavy tails. [2]

Claim amounts on a portfolio of insurance policies are assumed to follow a Weibull distribution. A quarter of losses are below 15 and a quarter of losses are above 80.

(ii) Estimate the parameters c, γ of the Weibull distribution that fit this data. [3]

(iii) Determine whether or not this Weibull distribution has a heavier tail than that of the exponential distribution with parameter c, by considering your estimate of γ. [2]
[Total 7]

6. CT6 April 2016 Q2

A portfolio of insurance policies has two types of claims:

- Loss amounts for Type I claims are exponentially distributed with mean 120.
- Loss amounts for Type II claims are exponentially distributed with mean 110.

25% of claims are Type I, and 75% are Type II.

(i) Calculate the mean and variance of the loss amount for a randomly chosen claim.

An actuary wants to model randomly chosen claims using an exponential distribution as an approximation.

(ii) Explain whether this is a good approximation.

7. CT6 April 2017 Q2

Claim amounts *Xi* from a portfolio of insurance policies follow a gamma distribution with parameters k and λ_i. Each λ_i.also follows a gamma distribution with parameters α and β.

(i) Show that the mixture distribution of losses is a generalised Pareto, with parameters α, β. k. [4]

Claim amounts are now assumed to be exponentially distributed with parameter λ_i

(ii) Show, using your answer to part (i), that the mixture distribution of losses is now a standard Pareto distribution with parameters α, β. [2]
[Total 6]

8. CT6 Oct 2015 Q9

A random variable X follows a gamma distribution with parameters α and λ.
(i) Derive the moment generating function (MGF) of X.

(ii) Derive the coefficient of skewness of X.

9. CT6 September 2016 Q5
(i) (a) Explain what is meant by a sequence of independent, identically distributed (I.I.D.) random variables.
(b) Give one example of a sequence of I.I.D. random variables. [3]

Claim amounts X_i from a portfolio of insurance policies are assumed to be I.I.D. and exponentially distributed, with parameter λ. In a given year there are n claims.

(ii) Show that the total claim amounts follow a gamma distribution, specifying its parameters. [2]

In practice the individual claim amounts are not I.I.D. but instead the exponential parameter λ_i varies between each claim. λ_i follows a gamma distribution with parameters α and β.

(iii) Show that the marginal distribution of claim amounts follows a Pareto distribution with parameters α and β. [5]
[Total 10]

10. CT6 April 2016 Q1
(i) Derive the median of a Pareto distribution with parameters α and λ. [3]

Let $\alpha = 2$ and $\lambda = 3$.

(ii) Comment on the skewness of this Pareto distribution. [3]
[Total 6]