INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Statistical & Risk Modelling 3

Chapter: Unit 2

Category: Practice Questions

1. CS1A IAI November 2019 Q3
The number of claims X, on an insurance policy over a year follows a Poison distribution with unknown parameter θ. The number of claims observed in the previous n years are $x1, x2 \ldots xn$ Prior distribution for θ has gamma distribution with parameters α and λ, as defined in the actuarial tables.

i) Derive the posterior distribution of θ given $x1, x2 \ldots xn$

ii) Show that the Bayesian estimate of θ under quadratic loss is equal to $\frac{\alpha+\sum x_i}{\lambda+n}$

iii) Show that the mean of the posterior distribution can be written in the form $Z * (sample\ mean) + (1 - Z) * (mean\ of\ prior\ distribution)$, defining Z as appropriate.

2. CS1A IAI November 2019 Q4
The investment department of a life insurance company feels that the chances of the economy moving into a high level of financial stress over the next month are 80%.

Based on an alternate study of macro-economic variables, it has been found that the relative position of credit spreads vis-à-vis their long-term historical average at the beginning of a month is a leading indicator of the level of financial stress in the economy over the following month. The studies indicate that the economy may face high levels of stress when there is a spike in credit spreads.

Based on data gathered for the past 10 years, it has been observed that:
 75% of the times when the economy ends up in high level of financial stress, it is preceded by high credit spreads; and
 40% of the times when the economy ends up in a low level of financial stress, it is preceded by high credit spread.

Compute the following:
i) Prior probability of the economy moving into a high level of financial stress. (1)

ii) Find the conditional probability of the credit spreads being high in the beginning of the month given that the level of financial stress was high over the following month. (2)

iii) Calculate the posterior probability that the financial stress index will be high given that the credit spreads are high in the beginning of the month. (3) [6]

3. CT6 September 2018 Q3

(i) State the fundamental difference between Bayesian estimation and Classical estimation. [2]

(ii) State three different loss functions which may be used under Bayesian estimation, indicating for each its link to the posterior distribution. [3]

The proportion, Θ, of the population of a particular country who use online banking is being estimated. Of a sample of 500 people, 326 do use online banking.

An actuary is estimating Θ using a suitable uniform distribution as a prior.

(iii) (a) Determine the posterior distribution of Θ.
(b) Calculate an estimate of Θ using the loss function that minimises the mean of the posterior distribution. [4]
[Total 9]

4. CT6 April 2018 Q3

An insurance company has collected data on the number of claims arising from certain risks over the last n years. The number of claims from the ith risk in the jth year is denoted by X_{ij} for $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, n$.

The distribution of X_{ij} depends on an unknown parameter Θ_i. The X_{ij} are independent identically distributed random variables given □*i*.

(i) Describe briefly what is meant by each of the following: $m(\theta)$, $s^2(\theta)$, $E(s^2(\theta))$, $\text{var}(m(\theta))$, and Z, when using Empirical Bayes Credibility Theory (EBCT) Model 1. [5]

(ii) Explain how the value of Z depends on the following factors: n, $E(s^2(\theta))$, $\text{var}(m(\theta))$. [5]
[Total 10]

5. CT6 April 2017 Q7

An actuary is assessing three different insurance companies, A, B and C. Corresponding claim amounts and number of policies are shown in the data below.

	Company A		Company B		Company C	
	$m	Policies	$m	Policies	$m	Policies
2013	1.16	85	0.85	68	1.48	110
2014	1.18	88	1.02	82	1.52	132
2015	1.14	85	0.96	70	1.78	143
2016	1.32	92	0.87	80	1.92	165
Total	**4.8**	**350**	**3.7**	**300**	**6.7**	**550**

Company C has 180 policies to insure in 2017.

(i) Calculate its expected claim amount, using the assumptions underlying Empirical Bayes Credibility Theory (EBCT) Model 2. [11]

(ii) Discuss why it might be preferable to use EBCT Model 2 rather than EBCT Model 1 for this purpose. [2]
[Total 13]

6. CT6 September 2016 Q3
The table below shows aggregate annual claim statistics for four risks over a period of six years. Annual aggregate claims for risk *i* in year *j* are denoted by *Xij*.

Risk, i	$\overline{X_i} = \frac{1}{6}\sum_{j=1}^{6} X_{ij}$	$S_i^2 = \frac{1}{5}\sum_{j=1}^{6}\left(X_{ij} - \overline{X_i}\right)^2$
$i=1$	46.8	1227.4
$i=2$	30.2	1161.4
$i=3$	74.5	1340.3
$i=4$	60.7	1414.7

(i) Calculate the credibility premium of each risk under the assumptions of Empirical Bayes Credibility Theory (EBCT) Model 1. [7]

(ii) Comment on why the credibility factor is relatively low in this case. [2]
[Total 9]

7. CT6 September 2015 Q5

Claims X each year from a portfolio of insurance policies are normally distributed with mean θ and variance τ^2. Prior information is that θ is normally distributed with known mean μ and known variance σ^2.

Aggregate claims over the last n years have been xi for i = 1 to n, and you should assume that these are independent.

(i) Derive the posterior distribution of θ. [5]

(ii) Write down the Bayesian estimate of θ under quadratic loss. [1]

(iii) Show that the estimate in your answer to part (ii) can be expressed in the form of a credibility estimate, including statement of the credibility factor Z.
[2]
[Total 8]

8. CT6 April 2015 Q5

An insurance company has for five years insured three different types of risk. The number of policies in the jth year for the ith type of risk is denoted by Pij for i = 1, 2, 3 and j = 1, 2, 3, 4, 5. The average claim size per policy over all five years for the ith type of risk is denoted by $\underline{X_i}$. The values of Pij and $\underline{X_i}$ are tabulated below.

	Number of policies					*Mean claim size*
Risk type i	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	$\bar{X}_i$
1	17	23	21	29	35	850
2	42	51	60	55	37	720
3	43	31	62	98	107	900

The insurance company will be insuring 30 policies of type 1 next year and has calculated the aggregate expected claims to be 25,200 using the assumptions of Empirical Bayes Credibility Theory Model 2.

Calculate the expected annual claims next year for risks 2 and 3 assuming the number of policies will be 40 and 110 respectively. [9]

9. CT6 September 2014 Q6

For three years an insurance company has insured buildings in three different towns against the risk of fire damage. Aggregate claims in the *j*th year from the *i*th town are denoted by X_{ij} for $i = 1, 2, 3$ and $j = 1, 2, 3$. The data is given in the table below.

Town i	*Year j*		
	1	*2*	*3*
1	8,130	9,210	8,870
2	7,420	6,980	8,130
3	9,070	8,550	7,730

Calculate the expected claims from each town for the next year using the assumptions of Empirical Bayes Credibility Theory model 1. [10]

10. CT6 September 2013 Q10

The number of service requests received by an IT engineer on any given day follows a Poisson distribution with mean μ. Prior beliefs about μ follow a gamma distribution with parameters α and λ. Over a period of *n* days the actual numbers of service requests received are 1 2 , , .

(i) Derive the posterior distribution of μ . [3]

(ii) Show that the Bayes estimate of μ under quadratic loss can be written as a credibility estimate and state the credibility factor. [2]

Now suppose that α =10 , λ = 2 and that the IT worker receives 42 requests in 6 days.

(iii) Calculate the Bayes estimate of μ under quadratic loss. [1]