INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject: SRM 3

Chapter:

Category: Assignment 2

1. A sports analytics firm is working on the form of sportspersons for an upcoming world cup tournament. A sportsman could be in either out-of-form ("O") or in-form ("I"). Probability of going in-form while being out-of-form in (t, t+dt) is 0.25dt +o(dt). If a sportsman is in- form, going out-of-form has probability 0.75 dt + o(dt) in (t, t+ dt).

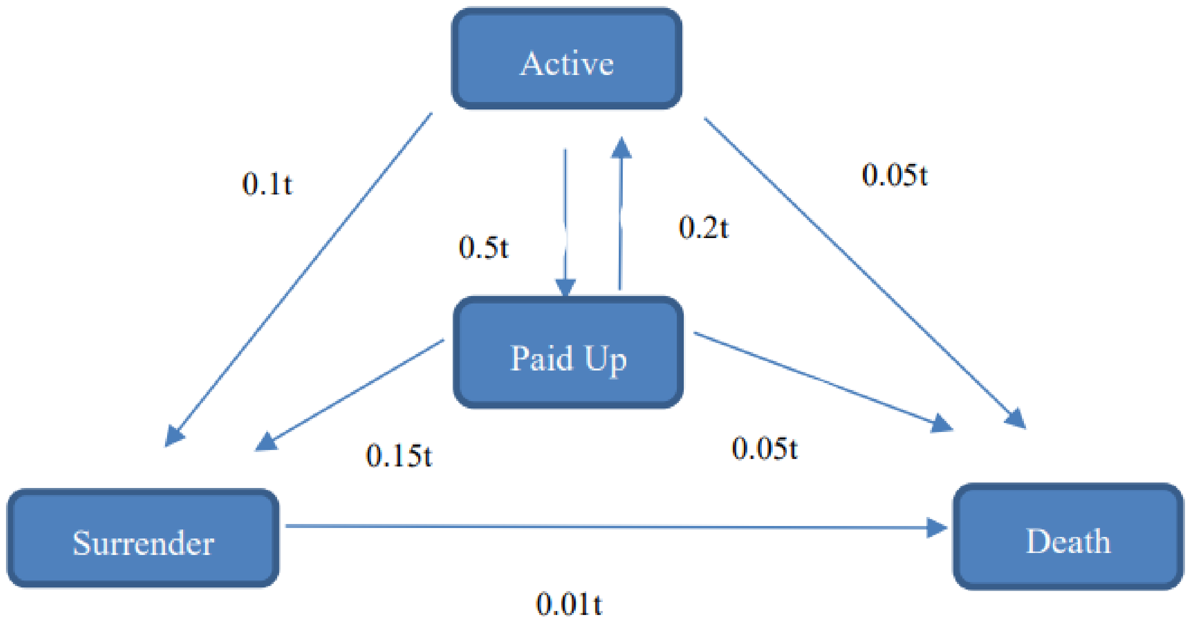
Notation	Probability that sportsman is:	Given sportsperson is at t=0:
$P_{OI}(t)$	In-Form	Out-of-Form
$P_{OO}(t)$	Out-of-Form	Out-of-Form
$P_{IO}(t)$	Out-of-Form	In-Form
$P_{II}(t)$	In-Form	In-Form

i) Write down the generator matrix based on the above information.

ii) Explain whether the sportsperson's form may or may not be considered as a Markov jump.

iii) Solve the Kolmogorov Forward equation for dPOO (t) /dt.

2. Consider the following time-inhomogeneous Markov jump process {X(t) : t >_ 0} of a life insurance company with policy states defined as active [A], surrender [S], paid up [P] and death [D] statuses. Transition rates as shown below:



i) Write down the Chapman-Kolmogrov equations and differentiate it to obtain the forward and backward equations.

ii) State Kolmogorov's forward differential equation for probability P_{PS} (s,t) and P_{SS} (s,t).

iii) A policyholder aged 40 surrendered his policy at 3rd policy anniversary. Calculate the probability that the policyholder is still alive at the 5th policy anniversary.

3. A pet mouse is kept in an artificial mouse hole which is made up of three large balls, each connected to other through pipes. The overall arrangement is triangular in shape with balls at nodes and pipes as sides. The mouse keeps moving at a very fast speed between the balls and randomly changing the direction while in the ball. The mouse cannot change direction in the pipes. Let each node be considered as state of continuous-time process with three states observed from time 0 up until the time of the 20th transition. The results may be summarised as follows:

			Number of transition from State I to:		
State, i	No. of visits to state i	Time spent in state i	State 1	State 2	State 3
1	16	96	0	6	10
2	8	320	2	0	6
3	16	480	14	2	0

i) Describe the stages of model fitting and model verification in the modelling process.

ii) Suppose that a Markov jump process model is to be fitted to the data set above. List all the parameters of the model and discuss the assumptions made when such a model is fitted to a data set.

iii) Estimate the parameters of the model in (ii) above and write down the estimated generator matrix.

4. A large bank in a developed country is envisaging implementing blockchain technology for its accounting system and in particular the payment system as current system requires too much of efforts in reconciliation. This will also help in

the reduction of transaction costs for international payments. Blockchain network is a collection of high-end servers (nodes) that make cryptographic calculations. Each node tries to outperform each other by performing the calculations as fast as possible and node owner gets a small fee for providing their computing power to the network depending on certain success factors.

Under this system, the transactions originating anywhere are added to a queue which is managed by a queuing server. The transaction at the front of the queue is shared with all network nodes attached to the blockchain network. These nodes perform certain cryptographic calculations and determine the validity of the transaction. Once a node validates a transaction it sends a message across network which is known as "consensus". The queuing server does not share the transaction at the front of queue with network for processing until the previous transaction is validated and added to the blockchain, i.e. queuing server has received the required number of consensuses for previous transactions.

As per current requirements, four consensus are required to add a transaction to the blockchain. The first four network nodes from whom the consensus was received will receive the fee for consensus. There is no delay between arrival of the last consensus for previous transaction and issue of new transaction by the queuing server.

The time taken by various nodes to solve cryptographic problem follows a random process and consensus messages received by queuing server follows a Poisson process with a rate of β per minute.

i) Explain how the number of consensus received by queuing server for the current transaction can be modelled as Markov jump process.

Write down, for this process:

ii) The Generator Matrix

iii) Kolmogorov's forward equations in component form

iv) Calculate the expected time a blockchain node that has sent the consensus will have to wait until the previous transaction is added to the blockchain and new transaction from queue is received for computing.

The bank felt that the average number of consensus should depend on size of the risk i.e. transaction amount. It performed certain risk analysis of the transactions and arrived at a conclusion that transactions with amount 100K and below will be accepted at Three consensus whereas those above 100k will require Six consensus for addition of that transaction to the blockchain. All transactions have equal probability of being ‘more than’ or ‘less than and equal to’ 100k and they arrive randomly at the queuing server.

v) Write down the transition matrix of the Markov jump chain describing the number of consensus received by queuing server for the current transaction after this rule change.

vi) Calculate the expected waiting time for a blockchain node that has sent the consensus until the current transaction is added to the blockchain and new transaction from queue is received for computing after this rule change and compare this with your answer to part (iii).

5. The Candy maker has a single machine that is used to prepare different shapes of candies in children’s amusement park. The machine has a tendency to break down, at which point it must be repaired. The time until breakdown and the time required to effect repairs both follow the exponential distribution. Let P1i(t), i = 0, 1, be the probability that at time t (t > 0) there are i candy machines working, given that the candy maker machine is working at time t = 0.

i) Derive the Kolmogorov forward differential equations for P1i(t) i = 0, 1 in terms of:

σ where $1/\sigma$ is the mean time to breakdown for a machine; and

ρ, where $1/\rho$ is the mean time to repair a machine

ii) Show that $P10(t) = \sigma/(\sigma+\rho)*(1-\exp-(\sigma+\rho)*t)$ deduce the value of $P11(t)$.

iii) The candy maker is considering adding a second identical machine, though there is only one repair team to work on the machines in the event that both are out of action simultaneously. Assuming that a second machine is added and operates independently of the first one:

a) Write down the generator matrix of the Markov jump process Xt which counts the number of working candy maker machines at time t.

b) Derive the Kolmogorov forward differential equations for pi(t), i = 0, 1, 2, (the probability that i ticket machines are working)

c) Given that, for some t,

$$P0\,(t) = 2\sigma^2/(2\sigma^2 + 2\rho\sigma + 2\rho^2)$$
$$P1\,(t) = 2\rho\sigma/(2\sigma^2 + 2\rho\sigma + 2\rho^2)$$
$$P2\,(t) = \rho^2)/(2\sigma^2 + 2\rho\sigma + 2\rho^2)$$

Show that $d/dt\ P_i(t) = 0$ for $i = 0,1,2$

d) State what conclusions you draw from part (c).

6. In a historic city in India, an industrialist wants to make money by offering the denizens a first-of-its-kind experience of viewing the historic places in the city from a helicopter. A brand-new 3-seater helicopter is employed for the ride. As it's a 3-seater helicopter, a group of 3 passengers are gathered at the boarding center, before they are allowed to board it. Hence, the helicopter would not fly until all the 3 seats are full. Passengers arrive at the center according to a Poisson process with the rate of $\lambda = 1/15$ per minute.

i) Calculate the expected waiting time until the first helicopter takes off. (2)

ii) What is the probability that helicopter does not take off in the first two hours, assuming that there are no other hindrances? (2)

iii) Today, it has been informed by the authorities that the weather conditions will not be conducive after three hours. However, the operator wants to fly at least 3 rides today before weather conditions become poor. What is the probability that the operator completes at least 3 rides in three hours? (3) [7]