



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject: Statistical
Techniques and Risk
Management

Chapter: Unit 1&2

Category: Practice
Questions

Extreme Value Theory

1. Subject CS2 September 2020 Question 2

- (i) Explain why Extreme Value Theory (EVT) models can be useful.
A sports scientist is interested in analysing the probability that the javelin world record may be broken next year and is intending to use EVT to do this. The sports scientist has obtained data for the distances of all javelin throws from all javelin competitions in 2019. The total number of throws recorded was 3,000. The sports scientist has carried out an EVT analysis using the Generalised Pareto Distribution by selecting only those throws that exceeded 50 metres. This resulted in the longest 150 throws being selected for the analysis. The following parameters were obtained from the EVT analysis:
 $\beta = 15$,
 $\gamma = 3$.
- (ii) Determine the percentage of javelin throws that would be expected to exceed 70 metres next year.
- (iii) Comment on the limitations of this analysis.

2. Subject CS2 September 2021 Question 1

An Analyst is assessing the risks of an equity portfolio and wishes to estimate the probability that the portfolio will incur at least one daily loss exceeding 5% next month.

Explain how a generalised extreme value distribution and the block maxima method could be used to estimate this probability.

Copulas

3. Subject CS2 April 2019 Question 7

- (i) Write down Sklar's theorem.
- (ii) Explain, in words, the meaning of the following copula expression: $C(u_1, u_2, u_3)$

The Gumbel copula has a generating function: $\text{Gumbel}\psi_\alpha [F(x)] = (-\ln F(x))^\alpha$

- (iii) Derive an expression for the Gumbel (Hougaard) copula for the case where there are three variables.

A student has fitted a Gumbel copula to investment returns from three developing markets, and has calculated a value for the dependency parameter, α , of 4.0. She has separately determined that the probability of making a loss over the next calendar year (i.e. the probability that the return is less than 0%) in each of the three markets is 5%, 7.5% and 10% respectively.

- (iv) Calculate the probability that all three markets have returns of less than 0% over the next calendar year.
- (v) State what type of copula is equivalent to a Gumbel copula if $\alpha = 1.0$.
- (vi) Calculate the probability that all three markets have returns of less than 0% over the next calendar year, assuming that each of the markets were independent.

4. Subject CS2 April 2021 Question 1

The Frank copula, C_F , for a bivariate distribution is defined as:

$$C_F(u, v) = -\frac{1}{\alpha} \ln \left(1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{(e^{-\alpha} - 1)} \right), \alpha > 0$$

- (i) Determine the probability that two jointly distributed random variables, X and Y , are both less than or equal to their median values where X and Y follow the Frank copula, C_F , with $\alpha = 1$. [2]
 - (ii) Determine the revised value of the probability in part (i) when $\alpha = 0.1$. [1]
 - (iii) Determine the probability that two jointly distributed random variables, X and Y , are both less than or equal to their median values where X and Y follow the product copula. [1]
 - (iv) Comment on your answers to parts (i), (ii) and (iii) with reference to the sign and level of dependence exhibited by the Frank copula. [2]
- [Total 6]

5. Subject CS2 September 2020 Question 1

Consider a two-dimensional Gaussian copula function, $C_{\text{Gauss}}(u_1, u_2)$, with parameter $\rho = 0$:

- (i) Give the solution to the copula function $C_{\text{Gauss}}(1, 1)$. [1]
- (ii) Give the solution to the copula function $C_{\text{Gauss}}(1, 0.2)$. [1]
- (iii) Give the solution to the copula function $C_{\text{Gauss}}(0.2, 0.2)$. [1]
- (iv) Outline how your answers to parts (i), (ii) and (iii) would change if $\rho = 1$. [2]

An insurer uses copulas to model the dependencies between various types of claims. A statistical analysis of the insurer's claims shows that a Gumbel copula is a better representation of historic claims data than a Gaussian copula.

- (v) Discuss why the use of a Gaussian copula in a claims model could result in solvency issues for this insurer. [3]

[Total 8]