

PUSASQF403
Financial Engineering 1

Time: 2 hours
Total Marks: 60 marks

Note:

- 1) The candidate has option to either attempt question 4A or question 4B. Rest all questions are mandatory.
- 2) Numbers to the right indicate full marks.
- 3) The candidates will be provided with the formula sheet and graph papers (if required) for the examination.
- 4) Use of approved scientific calculator is allowed.

Q1

A.

5 Marks

Suppose that the price of a given stock, X_t at time t , follows a geometric Brownian motion with parameters μ and σ i.e. $dX_t = \mu X_t dt + \sigma X_t dW_t$, where W_t is Wiener process. Let $Z_t = X_t^\lambda$, $0 < \lambda < 1$

Using Ito's lemma, show that Z_t follows geometric Brownian motion with respect to the filtration on W_t .

B.

5 Marks

Explain why vega is positive for options, theta is negative for options and rho is positive for a call and negative for a put option.

C.

5 Marks

The stochastic process X follows the SDE given by $dX = 0.25 dt + \sigma dW_t$ where W is a standard Brownian motion. Consider a new process Y defined by $Y_t = f(t, X_t)$ where $f(t, x) = \exp(-t) * x^2$.

- (i) Write an expression for dY_t
- (ii) Under what condition will the process be a martingale?

Q2.

A.

5 Mark

A certain share pays a dividend every quarter which is 1% of the share price immediately before the ex-dividend date. Immediately before one such ex-dividend date, the share price is Rs 100. The risk-free interest rate is 5% per annum. Deduce whether it might be advantageous to exercise either of following American put options immediately:

- (a) expiry date in one month, exercise price=Rs 150.
- (b) expiry date in three months, exercise price=Rs 140.

B.

5 Mark

An investor bought a 5-year forward contract on 1 June 2004 to buy Rs.400 nominal of a stock that pays coupons of 5 % pa payable half yearly on 31 March, 30 September. The stock is also expected to pay out a lump sum of Rs.50% of nominal value on 1 June 2008. The stock is expected to yield 5.5% pa effective if purchased on 1 June 2004 and held forever. Calculate the forward price for the contract, given that the risk free rate of interest is 6% per annum and the current price of the stock is Rs. 133.33.

C.

5 Mark

The bank balance of a young Actuarial student follows the Weiner process with a drift of 100 per month and a variance of 4000 per month.

- i) At the beginning of the year, the bank balance is Rs. 200. What is the probability of the bank balance being greater than zero at the end of the year?
- ii) What should be the starting bank balance so that the probability of a positive bank balance is 99%?

Q3.

A.

5 Marks

Derive an expression for the put-call parity of a European option that has a dividend payable prior to the exercise date.

B.

5 Marks

Using the standard Black-Scholes call option pricing formula, calculate the price of an European call on a non-dividend paying stock with the following features: current price of the underlying stock : 200, strike price : 200, time to expiration : 1 year, risk free rate : 6% pa [continuously compounded], volatility : 25% pa. State assumptions, if any.

C.

5 Marks

The price of a stock is currently Rs. 500. Over each of the next two 3-month periods the stock price is expected to go up by 6% or down by 5%. The risk free interest is 5% per annum with continuous compounding.

- i) What is the value of a six-month European call option with a strike price of Rs.510?
- ii) What is the value of a six-month European put option with a strike price of Rs.510?

Q4

A.

15 Marks

i) Define the formula used for pricing a European call option under the Black-Scholes framework, defining all the terms used.

A stock following Geometric Brownian Motion is currently priced Rs. 300, risk free interest rate is 6% per annum, and the volatility is 30% per annum.

ii) An investor decides to buy a 2-month European put option assuming the stock is non-dividend paying and with a current strike price of Rs. 300. Calculate how much the investor should be prepared to pay.

iii) He later identifies that a dividend of Rs. 9 is expected in a month's time. Calculate the price the investor will be expected to pay.

iv) The investor attempts to find the price of the put option on the basis of initial assumption using a two-step binomial tree. Derive the price he will calculate, stating your assumptions.

B.

15 Marks

i) State what is meant collectively by the Greeks of an option?

ii) Define delta for a derivative.

iii) Assume that the assumptions underlying the Black-Scholes model hold.

Derive a formula for the delta of a European call option for a non-dividend paying stock.

Evaluate the delta of an at-the-money call, when the risk free rate is zero, time to expiry is one year, and volatility is 20% p.a.

iv) One investor, A, holds £1 million of equity. Another, B, holds £1 million in cash, and the call option evaluated in (ii), over £1 million of the same equity. The equity market rises instantly by 20%. Explain which of these investors now owns greater value.