Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: MSc

Subject : Statistical & Risk Modelling - 2

Chapter: Unit 1 Chapter 3

Chapter Name: Reinsurance arrangements in insurance risk management

Agenda

1. Reinsurance
2. Role of Reinsurance
3. Functions of Reinsurance
4. Types of Reinsurance
5. Coverage Modifications
6. Truncated or censored Data
7. Effects of Inflation on losses

1.0 Reinsurance

The purpose of risk sharing is to spread the risk among those involved. The principal, or direct, insurer may pass on some of the risk to another insurance company, which, in this role, is called the reinsurer.

1.1 Role of Reinsurance

- There is a reduction in the mean amount paid out by the direct insurer on claims.

- There is a reduction in the variability of the amount paid out by the direct insurer on claims.

- There is a reduction in the probability that the direct insurer will face a “very large” payout on any particular claim (or collection of claims).

- In other words, reinsurance “stabilizes” the direct insurer’s payouts on claims.

2.0 Why Reinsurance

- To protect itself from unusually large claims.
- To reduce mean and variance of claim amounts paid.
- It stabilizes the direct insurer’s payout.
- Helps in multiplication of volume/ insurance that can be written. [Easing of new business strain]
- Assurance of claim settlement.

3.0 Functions of Reinsurance

- It helps the main insurer to grow or multiply in terms of volume of premium.
- It protects the main insurer from catastrophe to occur.
- It increases the capacity to assume more risks & to issue to more policies.
- It provides a great stability to the profits of insurance business.
- Distribution of risk to big players.
- Assurance of claim settlement from big players.

4.0 Types of Reinsurance

REINSURANCE

1) Proportional

(Quota share treaty)

Equal sharing of

Premium claims

Example: Retained proportion is 70%, $\alpha = 70\%$

ie. 70% of claim is paid by insurer.

30% paid by reinsurer

Example: 30% proportional $(1-\alpha)$ or 25% $(1-\alpha)$ quota share treaty

Proportion paid by reinsurer

2) Excess of Loss [XOL]

M- Retention limit

(maximum amount that an insurer pays)

4.0 Types of Reinsurance

1) Proportional Reinsurance:

Y and *Z*, the amounts paid by the direct insurer and the reinsurer, respectively, are defined simply as follows:

$Y = \alpha X$

$Z = (1 - \alpha)X$

where X is the claim amount. As before, we have $X = Y + Z$

$E(Y) =$

$E(Z) =$

$Var(Y) =$

$Var(Z) =$

4.0 Types of Reinsurance

1) Proportional Reinsurance:

- Accordingly, Proportional reinsurance helps in reducing the variability of impact of claims for the direct insurer and hence, helps reduce risk.
- Both the direct insurer and the reinsurer are involved in paying each claim, and both have unlimited liability (unless there is a cap on the claim amount).
- However, direct insurer is still exposed to very large claim amounts (risky tail) and hence, proportional reinsurance doesn't help in dealing with such extreme cases.

4.0 Types of Reinsurance

2) Excess Of Loss Reinsurance:

The amounts paid by the direct insurer and the reinsurer, *Y* and *Z*, respectively, are defined as follows:

$$Y = \begin{cases} X & \text{if } X \leq M \\ M & \text{if } X > M \end{cases}$$

$$Z = \begin{cases} 0 & \text{if } X \leq M \\ X-M & \text{if } X > M \end{cases}$$

Or

$Y = \min(X, M)$

$Z = \max(0, X-M)$

where M is the retention limit.

4.0 Types of Reinsurance

2) Excess Of Loss Reinsurance:

$$Y = \begin{cases} X & ; X \leq M \\ M & ; X > M \end{cases}$$

$$E(Y) = \int_0^M xf(x)dx + \int_M^\infty Mf(x)dx$$

$$= \int_0^M xf(x)dx + M.P(X > M)$$

$$E(Y) = \int_0^M xf(x)dx + M[1 - F_X(M)]$$

$$E(Y^2) = \int_0^M x^2 f(x)dx + M^2[1 - F_X(M)]$$

$$Var(Y) = E(Y^2) - E(Y)^2$$

4.0 Types of Reinsurance

2) Excess Of Loss Reinsurance:

$$Z = \begin{cases} 0 & ; X \leq M \\ X-M & ; X > M \end{cases}$$

$$E(Z) = \int_M^\infty (X - M) f(x) dx$$

$$E(Z^2) = \int_M^\infty (X - M)^2 f(x) dx$$

$$Var(Z) = E(Z^2) - E(Z)^2$$

$$E(Y) = \int_0^M x f(x) dx + \int_M^\infty M f(x) dx$$

$$= \int_0^M x f(x) dx + \int_M^\infty x f(x) dx - \int_M^\infty x f(x) dx + \int_M^\infty M f(x) dx = E(X) - \int_M^\infty (X - M) f(x) dx$$

$$E(Y) = E(X) - E(Z)$$

4.0 Types of Reinsurance

2) Excess Of Loss Reinsurance:

E(Y)

Var(Y)

E(Z)

Var(Z)

Question

Find E(Y) when X has a Pareto distribution with parameters $\lambda = 150\ and\ \alpha = 5,\ M = 80$.

Question

An insurance company has a portfolio of policies, where claim amounts follow a Pareto distribution with parameters $\alpha = 3$ and $\lambda = 100$. The insurance company has entered into an excess of loss reinsurance agreement with a retention of M, such that 90% of claims are still paid in full by the insurer.

(i) Calculate M. [4]

(ii) Calculate the average claim amount paid by the reinsurer, on claims which involve the reinsurer. [6]

[Total 10]

Solution

(i) $P(X < M) = 1 - \left(\frac{100}{100+M}\right)^3 = 0.9$ [1½]

$\left(\frac{100}{100+M}\right) = 0.1^{1/3} \Rightarrow M = \frac{100 - 100 * 0.1^{1/3}}{0.1^{1/3}} = 115.4$ [2½]

Solution

(ii) Let Y be the claim amount paid by the reinsurer, so that

$$Y = \begin{cases} 0 & X <= M \\ X - M & X > M \end{cases}$$

$$E(Y \mid X > M) = \frac{E(Z)}{P(X > M)} \qquad [1]$$

$$E(Z) = \int_M^\infty (x - M) f(x) dx = \int_M^\infty (x - M) \frac{3 * 100^3}{(100 + x)^4} dx \qquad [1]$$

$$u = (x - M); \frac{dv}{dx} = \frac{3 * 100^3}{(100 + x)^4} \qquad [1]$$

$$E(Z) = \left[-(x - M) \frac{100^3}{(100 + x)^3} \right]_M^\infty + \int_M^\infty \frac{100^3}{(100 + x)^3} dx \qquad [1]$$

$$= 0 + \left[\frac{-100^3}{2(100 + x)^2} \right]_M^\infty = \left(\frac{100^3}{2(100 + M)^2} \right) = 10.772 \qquad [1]$$

$$E(Y \mid X > M) = \frac{10.772}{0.1} = 107.7 \qquad [1]$$

5.0 Coverage Modifications

1) Deductibles:

- The policyholder agrees to bear the first amount, say D, of any loss (and so only submits a claim when the loss exceeds D). In this arrangement D is called a deductible or the policy excess.

- The overall reduction in the number and average amount of potential claims to be settled opens up the possibility of a lowering of premiums, with consequent market advantages.

- In this case, claim payment for the insurer are identical to that of a reinsurer in an excess of loss arrangement.

- Example: Private motor insurance policy.

5.0 Coverage Modifications

2) Limits:

- The opposite of a deductible is a policy limit. The typical policy limit arises in a contract where for losses below *u* the insurance pays the full loss but for losses above *u* the insurance pays only *u*.

- For the insurer, the claim payments are identical to that of an insurer in an Excess of loss arrangement.

- Example: Health insurance policy with a specified cover.

5.0 Coverage Modifications

3) Coinsurance:

- The insurance company pays a proportion, α, of the loss and the policyholder pays the remaining fraction.
- For the insurer, the claim payments are identical to a proportional reinsurance case.
- Example: Property damage insurance.

6.0 Truncated or Censored Data

INCOMPLETE SAMPLES

1) Censored

Example- no. of observation = 10

8 observations with full information

(exact values)

2 with partial information

2) Truncated

No information for a particular range of variable

Example: Cases of X< M for reinsurer are truncated.

6.0 Truncated or Censored Data

Insurer- $X_1, X_2, M, X_4, X_5, X_6, M$ (Censored)

Reinsurer (Truncated)

Truncated pdf = $\frac{\text{original pdf}}{\text{P(not truncated)}}$

Censored:

L= $\prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} f(x_i)[P(X > M)^k]$

For n observation = M = full info. , k = partial info.

Truncated:

W – claim amount paid by reinsurer when he is actually involved

W= $\frac{X-M}{X>M}$

6.0 Truncated or Censored Data

Reinsurer's distribution: (truncated pdf)

Using CDF Method

$$F_w(W) = P(W \leq w)$$
$$= P\left(X - M \leq \frac{w}{X>M}\right)$$
$$= P\left(X \leq \frac{w+M}{X>M}\right)$$

Example: X ~ Exp(λ)

W~ ?

6.0 Truncated or Censored Data

Claim amounts from a portfolio have the distribution with PDF $f(x) = 2cxe^{-cx^2}, x \geq 0$. An individual excess of loss reinsurance arrangement with retention limit M=3 is in force. A sample of the reinsurer's non-zero payment amounts gives the following values:

n=9

$$\sum w_i = 8.1$$

$$\sum w_i^2 = 90$$

Where the units are millions of pounds. Find the maximum likelihood estimate of c.

7.0 Effects of Inflation on losses

Inflation rate = j%

Inflation factor = (1+j)% = k

If M is constant ,

Y = kX ; $kX \leq M$ $i.e.\ \ X \leq \frac{M}{k}$

= M ; Kx > M $i.e.\ \ X > \frac{M}{k}$

E(Y) = $\int_0^{M/k} kxf(x)dx + \int_{M/k}^{\infty} Mf(x)dx$

If M remains constant,

Mean claim amount paid by insurer → Increases by a factor less than k

Mean claim amount paid by reinsurer → Increases by a factor more than k

7.0 Effects of Inflation on losses

Claims from a portfolio are believed to have a Pareto(α, λ) distribution. In year 0, $\alpha = 5$, $\lambda = 800$. An excess of loss reinsurance arrangement is in force, with a retention limit of 500. Inflation is constant 10% pa.

i) Find the distribution of the insurer's claim payments in Years 1 and 2 before reinsurance.

ii) Find the percentage increase in the insurer's mean net claims payout in each year.