Lecture 1

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: SY BSc

Subject : Statistical & Risk Modelling - 2

Chapter: Unit 1 Chapter 6

Chapter Name: Theory of Decision Making

Agenda

1.0 Introduction

In complex choices and games, it is not always straightforward to make decisions .

We will use game theory and statistical criterion to make these decisions.

1.1 Zero–Sum 2 game players

Example:

	A		
B		H	T
	H	+1	-1
	T	-1	+1

1.2 Strategies

1) Domination:

(in Rs. 100 Crores)

	Investor				
Government		RIL	TCS	HUL	HDFC BANK
	Remove	2	0	1	4
	Reduce	1	2	2	3
	Maintain	4	1	3	2

1.2 Strategies

2) Minimax Strategies:

Strategy that minimizes max loss and maximizes minimum gain.

Example:

	Player A			
Player B		I	II	III
	1	6	2	1
	2	4	3	5
	3	-3	1	3

1.2 Strategies

3) Saddle Points:

- Equilibrium point in a strategy.
- Element that is the largest in its column and smallest in its row.
- If saddle point exists, minimax strategy is spy-proof.

Example:

	I	II	III
1	4	1	-4
2	3	2	3
3	-4	1	4

If B decides to switch to Strategy I

Not all matrices have saddle points.

Question

For what values of x and y does the matrix:

2	7	x
y	3	1
6	8	5

Have a saddle point?

1.2 Strategies

3) Randomized strategy:

Example:

	I	II
1	7	-6
2	1	5

Assume the worst outcome in every case and minimize the total max loss.

1.3 Statistical games

Game between nature - that controls features of a population, and a statistician, trying to make a decision about the population

Example:

- Coin is either balanced or two- headed.
- 1 coin toss will be done.
- Statistician has to decide whether it is balanced or 2 headed.
- If correct then no reward, but if incorrect then a penalty of 1.

	X (2 H)	Y (H T)
A (2 H)	0	1
B (H T)	1	0

Consider all types of decisions and calculate Expected loss for both scenarios A & B.

Question

A statistician is observing values from a Bin(2,p) distribution. He knows that p is either equal to ¼ or ½ , and he is trying to choose between these two values. He observes a single value x from the distribution. He proposes to use one of the following four decision functions:

$d_1(x)$:	set p =1/4	when x=0
	set p =1/2	when x=1 or 2
$d_2(x)$:	set p =1/4	when x=0 or 1
	set p =1/2	when x=2
$d_3(x)$:	set p =1/4	when x=0 , 1 or 2
$d_4(x)$:	set p =1/2	when x=0, 1 or 2

If he incorrectly concludes that p =1/4, he suffers a loss of 1. if he incorrectly concludes that p =1/2 , he suffers a loss of 2.

Find the risk function for each decision function, and find the decision function that minimises the maximum expected loss.

1.4 Decision criteria

Minimax criteria:
Seen earlier.

Bayes criteria:
Decision making using additional knowledge of prior distributions.

Example:
If probability of 2H is 2/3, 1/6, 1/3, then which is the most optimal strategy as per Bayes criterion?

Question

The statistician in the previous example has a prior feeling that it is about equally likely that p will be equal to ¼ or ½. Calculate the Bayes risk for each of the four decision functions. Which decision function has the smallest Bayes risk?

Question

A consumer has to decide whether to take out a travel insurance policy which covers him for all trips over the next year or to purchase a separate policy each time he makes a trip he isn't sure how many trips we will make.

An annual policy would cost 95.

A separate policy for each trip cost 30.

He estimates that the probability distribution of the number of trips he will take over the next year, X, is as follows:

Number of trips, x	P(X=x)
1	0.1
2	0.2
3	0.3
4	0.3
5	0.1

Determine the minimax and Bayes decision.