



Class: MSc

Subject : Statistics and Risk Modelling - 2

Chapter: Unit 4 Chapter 7

Chapter Name: Methods and Models of Loss reserving

Agenda

- 1.1 Introduction
- 1.2 Types of Reserves
- 1.3 Techniques for estimating unpaid losses
- 1.4 Role of Rating factor & exposure
- 1.5 Definitions
- 1.6 Experience rating
- 1.7 Calculating Premium

1.1 Introduction

General Insurance Company:

Sells policies and collects premiums

(-) Pays claims

= Surplus Cash

Should we distribute this as profit?



NO

Reason: A delay in claims settlement or claims reporting.

For this we need to maintain reserves.

1.2 Types of Reserves

1) IBNR : Incurred But Not Reported Reserves

Example: Accident has occurred but have not yet been reported to the insurance company due to lack of awareness.

2) Outstanding Reported Claims Reserves

Example: Claim is reported but settlement is fully or partially delayed due to final loss assessment pending.

1.2 Types of Reserves

To calculate these reserves, we use run off triangles.

1. Chain-ladder Method
2. Average cost per claim Method
3. Bornheutter Ferguson Method

1.3 Techniques for estimating unpaid losses

1) The chain-ladder or loss development triangle method

- In the chain-ladder method, the actuary estimates the age-to-age development factors based on history, and uses these to develop immature claims to their ultimate cost.
- The chain-ladder method projects future payments purely off past payments.
- For a claim with a long time to settlement, the cumulative amount paid usually grows from year to year. This growth is called *loss development*.
- An *age-to-ultimate development factor* is the ratio of the ultimate payment to the cumulative payment at any age.

1.3 Techniques for estimating unpaid losses

1) The chain-ladder or loss development triangle method

ASSUMPTIONS

- The first accident year is fully run off.
- Claims in each development year are a constant proportion in monetary terms of total claims for each accident year.
- Inflation is not allowed for explicitly, rather it is allowed for implicitly as a weighted average of past inflation.

1.3 Techniques for estimating unpaid losses

1) The chain-ladder or loss development triangle method

STEPS

1. Write down the cumulative claims paid/ incurred in a triangle format.
2. Calculate development factors using the data.
3. Estimate future claims amount using development factors.

1.3 Techniques for estimating unpaid losses



EXAMPLE:

The table below shows the numbers of household insurance claims reported in each development year for accident years from 2009 to 2012. Use the basic chain ladder method to estimate the total ultimate number of claims arising from accidents occurring between 1 January 2009 and 31 December 2012.

Number of claims reported	Development Year			
	0	1	2	3
2009	17500	5000	2250	750
2010	21000	6200	2750	
2011	18800	5500		
2012	21300			

Question



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The cumulative incurred claims for an insurance company for the last four accident years are given in the following table:

Accident Year	Development Year			
	0	1	2	3
2005	96	136	140	158
2006	100	156	160	
2007	120	130		
2008	136			

It can be assumed that claims are fully Run off after three years. The premium received for each year from 2005 to 2008 are 175, 181, 190 and 196 respectively.

Calculate the reserves at the end of year 2008 using basic chain ladder method.

1.3 Techniques for estimating unpaid losses

1) The chain-ladder method – Adjusted for Inflation

1. Past inflation to adjust given data.
2. Future inflation to get more accurate reserves.

1.3 Techniques for estimating unpaid losses

1) The chain-ladder method – Adjusted for Inflation

STEPS

1. Incremental claims paid/ incurred.
2. Multiply past data with relevant inflation index to adjust to real terms.
3. Write down inflation adjusted cumulative claims data.
4. Calculate development factors.
5. Estimate future claims amount.
6. Write down incremental claims paid/ incurred.
7. Calculate future incremental claims amount using inflation index.

1.3 Techniques for estimating unpaid losses



EXAMPLE:

Accident Year	Development Year				
	0	1	2	3	4
2008	786	624	806	224	79
2009	904	671	904	281	
2010	995	819	1066		
2011	1220	922			
2012	1182				

Suppose that the annual claim payments inflation rates over the 12 months up to the middle of the given year are as follows:

2009	2010	2011	2012
5.1%	6.4%	7.3%	5.4%

1.3 Techniques for estimating unpaid losses

1) The chain-ladder method – Adjusted for Inflation

MODEL CHECK:

To check whether estimated claims figures using development factors are appropriate to given data.

1.3 Techniques for estimating unpaid losses

2) Average cost per claim method

- This method requires development tables for both total claim amounts and claim numbers.
- A third development table, of the average claim amounts, is then formed by dividing the figures in the corresponding cells of the first two tables.
- Finally, the projected ultimate claims can be calculated by multiplying together for each accident year the projected figures for the average claim amounts and claim numbers.
- A reserve can then be calculated by subtracting all payments to date in respect of claims relating to the data in the table.

1.3 Techniques for estimating unpaid losses

2) Average cost per claim method

ASSUMPTIONS

- The first accident year is fully run off.
- The average cost per claim in each development year is a constant proportion in monetary terms of the ultimate average cost per claim for each accident year.
- The number of claims in each development year is a constant proportion in of the ultimate number of claims for each accident year.
- Inflation is not allowed for explicitly, rather it is allowed for implicitly as a weighted average of past inflation.

1.3 Techniques for estimating unpaid losses

2) Average cost per claim method

STEPS

1. Write down cumulative average claims incurred/ paid.
2. Using grossing up factors, calculate estimated ultimate average claim and ultimate claim numbers.
3. Future claim amount by multiplying the above 2.

1.3 Techniques for estimating unpaid losses



EXAMPLE:

Cumulative Number of claims reported, by year of accident and reporting development.

Accident Year	Development Year						
	0	1	2	3	4	5	Ult
1	414	460	482	488	492	494	494
2	453	506	526	536	539		
3	494	548	572	582			
4	530	588	615				
5	545	605					
6	557						

1.3 Techniques for estimating unpaid losses

Cumulative incurred claims data, by year of accident and reporting development.

Accident Year	Development Year						
	0	1	2	3	4	5	Ult
1	2777	3264	3452	3594	3719	3717	3717
2	3252	3804	3973	4321	4319		
3	3725	4404	4779	4946			
4	4521	5422	5676				
5	5369	6142					
6	5818						

Question



The number of claims settled

Accident Year	Development Year		
	0	1	2
2006	442	151	50
2007	623	111	
2008	681		

The cost of settled claims during each year (in 000's):

Accident Year	Development Year		
	0	1	2
2006	6321	1901	701
2007	7012	2237	
2008	7278		

Claims are fully runoff after year 2. Calculate the outstanding claims reserve using the average cost per claim method with grossing up factors. Inflation can be ignored.

1.3 Techniques for estimating unpaid losses

3) The Bornhuetter-Ferguson method

- The Bornhuetter-Ferguson method is a compromise between the loss ratio method and the chain ladder method.
- With this method higher past payments do not reduce future payments, as would occur with the loss ratio method. Nor do they raise future payments, as would occur with the chain ladder method. (However, if future link ratios are increased, that will lower the proportion paid so far and thus will still increase the reserve.)
- As with the chain-ladder method, this method may be used on cumulative payment triangles to generate the total reserve or on incurred loss triangles to generate the IBNR reserve.

1.3 Techniques for estimating unpaid losses

3) The Bornhuetter-Ferguson method

ASSUMPTIONS

- The first accident year is fully run off.
- The loss ratio is correct.
- Claims in each development year are a constant proportion in monetary terms of total claims for each accident year.
- Inflation is not allowed for explicitly, rather it is allowed for implicitly as a weighted average of past inflation.

1.3 Techniques for estimating unpaid losses

3) The Bornhuetter-Ferguson method

STEPS

1. Calculate ultimate claims using loss ratio and earned premium.
2. Write down cumulative claims paid/ incurred.
3. Calculate the ratio (r) and then development factors (f).
4. Estimate future claims liability using the formula:
5. $\text{Ultimate Claim} * (1 - 1/f)$
6. Calculate revised ultimate claims using given data and estimated future liability.

1.3 Techniques for estimating unpaid losses

EXAMPLE

The table below shows the cumulative costs of incurred claims. The claims are assumed to be fully run-off by the end of Development Year 2.

<i>£000s</i>	<i>Development Year</i>		
<i>Accident Year</i>	<i>0</i>	<i>1</i>	<i>2</i>
2011	2,670	3,290	4,310
2012	2,850	3,420	
2013	3,030		

Annual premiums written were:

Year	Premiums (£000s)
2011	5390
2012	5600
2013	6030

The ultimate loss ratio has been estimated at 80% and the total amount of claims paid to date is £5,720,000. Calculate the outstanding claims reserve using the Bornhuetter-Ferguson method.

1.4 Role of Rating factor & exposure

Exposure

Indication of measure of risk present in the policy.
It should allow for both frequency and severity of claims.
It should be practical.
Not open to manipulation.
Used in calculation of premium.

Example:

Life Insurance – Sum assured years
Employers liability insurance – Payroll
Motor third party liability insurance – Vehicle year
Aviation liability – Passenger kilometers
Public liability/ Product liability – Turnover

1.5 Definitions



Risk factors:

Factors that have bearing on and of risk.

Rating factors:

Objectively measurable risk factors or factors that can be used as reliable proxies for risk factors.

They are used to classify risk into homogenous groups

Since exposure measure cannot generally account for all the factors determining risk.

Example:

Employer liability – Type of industry

Property damage insurance – Location/ Age of building

Motor Insurance – Number of miles driven

1.6 Experience Rating



Rating system in which premium depends on post actual claims experience.

Types:

1. Prospective versus retrospective basis
2. Number based
3. Cost based

1.6 Experience Rating

1) Prospective versus retrospective



Prospective uses past claim experience to calculate future premiums

Example –

NCD system in motor insurance

Retrospective uses current period claim experience to adjust current policy premium

Example –

Employer liability/ Product liability – Deposit premium is paid at start of period and then it is adjusted/ refunded based on claim experience.

1.6 Experience Rating

2) Number based

-  Premium adjustments are made based on number of claims and not amount of claims (for small individual risk).

Example – No claim bonuses in Health insurance.

3) Cost based system

-  Premium adjustments are made based on amount of claims in the policy period.

Example – Employers liability, Marine fleet insurance

1.7 Premium Calculation

Calculation of premiums using Pure premium method and loss ratio method.

1) Pure premium method

$$\begin{aligned}\text{Pure premium} &= \frac{\text{Loss}}{\text{Exposure}} \\ &= \frac{\text{Claim numbers}}{\text{Exposure}} * \frac{\text{Loss}}{\text{Claim numbers}} \\ &= \text{Frequency} * \text{Severity}\end{aligned}$$

1.7 Premium Calculation

To calculate Office Premium:

Premium = Losses + Expenses + Underwriting profit

= Losses + Fixed Expenses + Variable Expenses + Underwriting profit

Premium – Variable Expenses = Losses + Fixed Expenses + Underwriting profit

Premium(1- V- Q) = Losses + Fixed Expenses

Premium = (Losses + Fixed Expenses)/ (1- V- Q)

$$\text{Premium rate} = \frac{\text{Premium}}{\text{Exposure}} = \frac{\frac{\text{Loss}}{\text{Exposure}} + \frac{\text{Fixed Expense}}{\text{Exposure}}}{1-V-Q}$$

$$= \frac{\text{Pure Premium} + \frac{\text{Fixed Expense}}{\text{Exposure}}}{1-V-Q}$$

Question



Determine the indicated rate per exposure unit, given in the following information:

- Frequency per exposure unit = 0.25
- Severity = \$100
- Fixed expense per exposure unit = \$10
- Variable expense factor = 20%
- Profit and contingencies factor = 5%

1.7 Premium Calculation

2) Loss Ratio method

Used to calculate change in rates instead of setting rates.

i. (Ignoring Expenses)

Premium = Losses + Underwriting profit

Underwriting profit = Premium - Losses

$(\text{Underwriting profit}) / \text{Premium} = 1 - \text{Losses} / \text{Premium}$

$Q = 1 - \text{Loss Ratio}$

1.7 Premium Calculation

For a new target Underwriting profit: Q_{target}

$$Q_{target} = 1 - \text{Loss} / (\text{ICF} * \text{Premium})$$

(ICF = Indicated Change Factor)

$$\text{ICF} = \frac{\text{Loss Ratio}}{1 - Q_{target}}$$

Example-

$$\text{LR} = 85\% , Q_{target} = 20\%$$

$$\text{ICF} = 0.85 / (1 - 0.2) = 1.0625$$

ie. 6.25% increase in premiums.

1.7 Premium Calculation

2) Loss Ratio method

Used to calculate change in rates instead of setting rates.

ii. (With Expenses)

Underwriting profit = Premium – Losses + Fixed expense – Variable expense

$$Q = 1 - (\text{LR} + \text{Fixed expense ratio}) - V$$

For a new target Underwriting profit: Q_{target}

$$Q_{target} = 1 - (\text{Loss} + \text{Fixed}) / (\text{ICF} * \text{Premium})$$

$$\text{ICF} = \text{ICF} = \frac{\text{Loss Ratio} + \text{Fixed}}{1 - V - Q_{target}}$$