UNIT 1 CHAPTER 4 – EXTRA QUESTIONS

Q1.

A claim size distribution is modeled using a simple distribution with density of the form

$$f(x)=\begin{cases} k(50-x), & 0\le x\le 50 \\ 0\ , & otherwise \end{cases}$$

a) Find k
b) Determine the mean of this claim size distribution.
c) Calculate the probability that an individual claim size is greater than 25
d) Calculate the probability that an individual claim size is less than 30 given that it is greater than 25

Ans –

3.a) $\int_0^{50} k(50-x)dx = 1 \Rightarrow k = \frac{1}{1250}$

b) $\int_0^{50} \frac{1}{1250}x(50-x)dx = \frac{1}{1250}\left[\frac{50x^2}{2}-\frac{x^3}{3}\right]_0^{50}$

$$= \frac{1}{1250}\left[\frac{50\times(50)^2}{2}-\frac{50^3}{3}\right]$$

$$= 16.67$$

c) $\int_{25}^{50} \frac{1}{1250}(50-x)dx = \frac{1}{1250}\left[50x-\frac{x^2}{2}\right]_{25}^{50}$

$$= 0.25$$

d) $P(X<30/X>25) = \frac{P(25<X<30)}{P(X>25)}$

$$= \frac{\int_{25}^{30} f(x)dx}{\int_{25}^{50} f(x)dx}$$

$$= \frac{0.09}{0.25} = 0.36$$

Q2.

Let $X_1, X_2, \dots, X_n$ be a random sample from the following density function

$$f(x;\theta) = \frac{kx}{\theta^2}; \quad 0 < x < \theta,\ \theta > 0$$

Find k such that above is a valid density function

Ans –

a. $\int_0^\theta \frac{kx}{\theta^2} dx$

$$= \frac{k}{\theta^2} \int_0^\theta x dx$$

$$= \frac{k}{\theta^2} [\frac{x^2}{2}]_0^\theta$$

$$= \frac{k}{\theta^2} * \frac{\theta^2}{2}$$

$$= \frac{k}{2}$$

For the above integral to be 1, k should be 2.

Q3.

Let $Y = e^{2X} + 5$.
Find $F_Y(y)$, expressed in terms of $F_X[g(y)]$, where $g(y)$ is some function solely of the variable y.

Ans –

$Y = e^{2X} + 5$ implies that

$F_Y(y)$

$= \Pr(Y \leq y)$

$= \Pr(e^{2X} + 5 \leq y)$

$= \Pr(e^{2X} \leq y-5)$

$= \Pr(2X \leq \ln(y-5))$

$= \Pr(X \leq \frac{1}{2}\ln(y-5)\ )$

$= F_X[\frac{1}{2}\ln(y-5)]$

$= F_X[g(y)], \quad where\ g(y) = \frac{1}{2}\ln(y-5).$

Q4.

A continuous random variable has the probability density function $f_X(x) = ke^{-2x}$, for $x > 0$. Calculate k, and $P(X < 5.27)$.

Ans –

If X is a random variable, then $\int f_X(x)\,dx = 1$.

Here:

$$\int_0^{\infty} ke^{-2x}\,dx = 1 \Rightarrow \left[-\frac{k}{2}e^{-2x}\right]_0^{\infty} = 1 \Rightarrow \frac{k}{2} = 1 \Rightarrow k = 2$$

$$P(X < 5.27) = \int_0^{5.27} ke^{-2x}\,dx = \left[-\frac{k}{2}e^{-2x}\right]_0^{5.27}$$

But since $k = 2$:

$$P(X < 5.27) = -e^{-10.54} + 1 = 0.99997$$

Q5.

A random variable X has PDF:

$$f_X(x) = \frac{1}{9}x^2 \qquad 0 < x < 3$$

Calculate $E(2X+1)$.

Ans –

Using $E[g(X)] = \int g(x) f(x)\, dx$, we obtain:

$$E(2X+1) = \int (2x+1) f(x)\, dx$$

$$= \int_0^3 (2x+1)\frac{1}{9}x^2\, dx = \int_0^3 \frac{2}{9}x^3 + \frac{1}{9}x^2\, dx = \left[\frac{2}{36}x^4 + \frac{1}{27}x^3\right]_0^3 = 5.5$$