Page No.:
Date:

Assingment (Calculus)

1] evaluate the following limit

$$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$$

$$= \frac{2(-3+0)^2 - 18}{0}$$

$$= \boxed{-12}$$

2] Determine if the following function is continous or discontinous at

(a) $x = 4$, (b) $x = 6$

$$g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

i) at $x = 4$

$g(x=4) = 2 \times 4 = 8$

$\boxed{g(x) \text{ is continous at } 4}$

ii) $x = 6$

$$\lim_{x \to 6^-} g(x) = 2 \times 6 = 12$$

$$\lim_{x \to 6^+} g(x) = 6 - 1 = 5$$

as $\lim_{x \to 6^-} g(x) \neq \lim_{x \to 6^+} g(x)$

$\therefore \boxed{g(x) \text{ is discontinous at } x = 6}$

3] Solve the following limit

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} = \frac{9-9}{3-\sqrt{9}} = \boxed{-6}$$

Scanned with CamScanner

Page No. :
Date :

4] Determine if the following function is continous or discontinous at

(a) $x = -1$ (b) $x = 0$ (c) $x = 3$

$$f(x) = \frac{4x+5}{9-3x}$$

a) $x = -1$ $f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$

$\therefore f(-1)$ exists

$\therefore$ $\boxed{f(x) \text{ is continous at } x = -1}$

b) $x = 0$ $f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\therefore f(0)$ exists

$\therefore$ $\boxed{f(x) \text{ is continous at } x = 0}$

c) at $x = 3$ $f(3) = \frac{4(3)+5}{9-3(3)} = \frac{12+5}{9-9} = \frac{17}{0} = \text{N.D}$

$\therefore f(3)$ doesn't exists

$\boxed{f(x) \text{ is discontinous at } x = 3}$

5] evaluate each of the following limits

$$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \lim_{x \to \infty} \frac{\cancel{e^{4x}}(6 - e^{-6x})}{\cancel{e^{4x}}(8 - e^{-2x} + 3e^{-6x})}$$

$$= \frac{6 - 1/e^{6(\infty)}}{8 - 1/e^{2(\infty)} + 3\frac{1}{e^{5(\infty)}}}$$

Scanned with CamScanner

Page No. :
Date :

$= \dfrac{6 - 1/\infty}{8 - 1/\infty + 3\,1/\infty} = \dfrac{6-0}{8-0+0} = \boxed{\dfrac{3}{4}}$

6] Given the function

$g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

compute the following limit

a] $\lim_{y \to 6} g(y) = \lim_{y \to 6} (1-3y) = 1-3(6) = \boxed{-17}$

b] $\lim_{y \to 2} g(y) = \lim_{y \to 2} (1-3y) = 1-3(2) = -5$

7] use the product rule to find the derivative o

$f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

derivative of $(4t^2-t)(t^3-8t^2+12)$

is $(24t^5 - 124t^3 + 8t - 24t^2 + 12)$.

8] use the quotient rule to find the derivative of

$R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$\dfrac{d}{dw}[R(w)] = \dfrac{(2w+1)\, d/dw\,(3w+w^4) - (3w+w^4)\, d/dw\,(2w+1)}{(2w+1)^2}$

$= \dfrac{(2w+1)(3+4w^3) + (3w+w^4)(2)}{(2w+1)^2}$

$= \dfrac{6w + 8w^4 + 3 + 4w^3 - 6w - 2w^4}{(2w+1)^2}$

Scanned with CamScanner

Page No.:
Date:

$$= \frac{6w^4 + 4w^3 + 3}{(2w+1)^2} = \frac{6w^4 + 4w^3 + 3}{4w^2 + 4w + 1}$$

9] Differentiate

$f(x) = 2e^x - 8x$

$$f'(x) = \frac{d}{dx}[2e^x - 8x] = \frac{d}{dx} 2e^x - \frac{d}{dx} 8x$$

$$= \boxed{2e^x - 8}$$

10] Differentiate

$y = z^5 - e^2 \ln(z)$

$$\frac{d}{dz} y = \frac{d}{dz} z^5 - \frac{d}{dz}(e^2 \ln(z))$$

$$\frac{dy}{dz} = 5z^4 - \left\{ e^2 \frac{d}{dz} \ln(z) + \ln(z) \frac{d}{dz} e^2 \right\}$$

$$= 5z^4 - \left[e^2 \frac{1}{z} + \ln(z) \cdot e^2 \right]$$

$$= 5z^4 - \frac{e^2}{z} \left[1 - z \ln(z) \right]$$

11] Using chain rule differentiate the following

a] $f(x) = (6x^2 + 7x)^4$

$$f'(x) = \frac{d}{dx}(6x^2 + 7x)^4 \cdot \frac{d}{dx}(6x^2 + 7x)$$

$$f'(x) = 4(6x^2 + 7x)^3 \cdot (12x + 7).$$

Page No. :
Date :

b] $f(t) = 5 + e^{4t} + t^7$

$= f'(t) = \frac{d}{dt}(5) + \frac{d}{dt} e^{4t} + \frac{d}{dt} t^7$

$= 0 + 4e^{t} + 7t^{6}$

$= 7t^6 + 4e^{4t}$.

12] Determine the second derivative of

a) $z = \ln(7 - x^3)$

$\frac{dz}{dx} = \frac{1}{(7-x^3)} \frac{d}{dx}(7-x^3)$

$\frac{dz}{dx} = \frac{1}{7-x^3}(-3x^2)$

$\frac{dz}{dx} = \frac{-3x^2}{-7x^3}$

$\therefore \frac{d^2z}{dx^2} = \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$

$= \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$

$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$

$= \frac{-3x^4 - 42x}{(7-x^3)^2}$

$\frac{d^2z}{dx^2} = \frac{-3x^4 - 42x}{(7-x^3)^2}$

Scanned with CamScanner

b) $\varphi(v) = \dfrac{2}{(6+2v-v^2)^4}$

$= 2(6-2v-v^2)$

$= \cancel{8}\ \varphi(v) = 2 - 4(6+2v-v^2)\dfrac{d}{dv}(6+2v-v^2)$

$= -8(6+2v-v^2)^{-5}.(2.2v)$

$\varphi''(v) = -8\left[(6+2v-v^2)^{-5}\dfrac{d}{dv}(2-2v) + (2-2v)\right.$

$\left.\dfrac{d}{dv}(6+2v-v^2)^{-5}\right]$

$= -8[(6+2v-v^2)(-2) + (2-2v) - 5(6+2v-v^2)^{-6}$
$(2-2v)]$

$\varphi''(v) = 16(16+2v-v^2)^{-5} + 40(6+2v-v^2)^{-6}(2-2v)^2$

c] $f(t) = \ln(1+t^2)$

$f(t) = \dfrac{1}{(1+t^2)}\dfrac{d}{dt}(1+t)^2$

$f'(t) = \dfrac{2t}{1+t^2}$

$\therefore f''(t) = \dfrac{(1+t^2)\dfrac{d}{dt}(2t) - (2t)\dfrac{d}{dt}(1+t^2)}{(1+t^2)^2}$

$= \dfrac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$

$= \dfrac{2+2t^2-4t^2}{(1+t^2)^2}$

$- \dfrac{2t^2+2}{(1+t^2)^2}$

Page No.:
Date:

$\therefore f''(t) = \frac{2t^2+2}{(1+t^2)^2}$

13] Find the maximum and minimum value of the function

$x^3 - 3x^2 - 9x + 12$

$= f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$

$= 3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

where $x = 3$

$f'(x) = 6 \times 3 - 9 = 18 - 9 = 9$

$f'(3) > 0$

$f(x)$ is maximum at $x = 3$

minimum value $= f(3)$

$= (3)^3 - 3(3)^2 - 9 \times 3 + 12$

$27 - 27 - 27 + 12$

$= -15$ - minimum value.

when $x = 1$

$f'(-1) = 6x(-1) - 9 = -6 - 9 = -15$

$f'(1) < 0$

$f(x)$ is maximum at $x = -1$

maximum value at $f(-1)$

Page No.:
Date:

$= (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17 =$ maximum value.

14] Find the maximum and minimum value of $4x^3 - 18x^2 + 24x - 7$

$f'(x) = 4 \times (3x^2) - 18x(2x) + 24 \times 1$

$f'(x) = 12x^2 - 36x + 24$

let $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$\cancel{x^2}\ (x-1)(x-2) = 0$

$x = 1$ & $x = 2$

$f'(x) = 12x^2 - 36x + 24$

$f'(x) = 12(2x) - 36 \times 1 + 0$

$f'(x) = 24x - 36$

$[f'(x)]_{[x=1]} = 14(1) - 36 = -12 < 0$

$\therefore$ at $x = 1$ $f(x)$ is maximum

$[f'(x)]_{[x=2]} = 24(2) - 36 = 12 > 0$

$\therefore$ at $x = 2$ $f(x)$ is minimum

$f(x=1) = 4(1)^3 - 18(1)^2 + 24(1) - 7 = 4 - 18 + 24 - 7 = 3$

$f(x=2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7 = 1$

maximum value $= 3$

minimum value $= 1$

Scanned with CamScanner

Page No. :
Date :

sketch the curve of the following function
$f(x) = x^2 \quad (x+3)$