Shrunkhala Kambale 89

classmate

Date 30/4/22

Page

PnS&Q Assignment 1

Q1) $X \sim Poi(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$$

$x = 200$

$$\therefore \theta = x \pm 1.96(\sqrt{\theta/n})$$

we also have

$$var\left[\frac{(n-1)s^2}{\sigma^2}\right] = 2(n-1) \Rightarrow var[s^2] = \frac{\sigma^4}{(n-1)^2} \times 2(n-1)$$

$$= \frac{2\sigma^4}{(n-1)}$$

Q2) $X_i = 1, 2, \ldots, n$

mean $= \mu$

var $= \sigma^2$

$\bar{X} = \sum_1^n X_i$ = sample mean

$S^2 = \frac{1}{n-1} \sum_1^n (X_i - \bar{X})^2$ = sample variance

(i) S.T mean of $\bar{X} = \mu$ & var $= \frac{\sigma^2}{n}$

→ Since it is a sequence of independent, i.d.r.vs with mean μ and variance σ^2 then the distribution of $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ approaches standard normal distribution, $N(0,1)$

since $n \to \infty$.

$\therefore E(\bar{X}) = \frac{E(\Sigma X_i)}{n} = \frac{n\mu}{n} = \mu$

$\therefore V(\bar{X}) = \frac{Var(\Sigma X_i)}{n} = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$

(ii) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ $\quad \therefore E(S^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2$

$\therefore \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ are approximately distributed as $N(0,1)$

$\therefore \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\therefore$ Mean $= \underline{\mu}$, Var $= \underline{\frac{\sigma^2}{n}}$ & $E(S^2) = \sigma^2$

(iii) S.T $var(s^2) = \frac{2\sigma^4}{(n-1)}$

→ Using χ^2 result,

$E\left[\frac{(n-1)S^2}{\sigma^2}\right] = n-1 \Rightarrow E[S^2] = \frac{\sigma^2}{n-1}(n-1) = \sigma^2$

and

Given :- $n = 10$; $\sigma^2 = 100$

(iv) $P(50 < s^2 < 150) = P\left(\frac{9 \times 50}{100} < \chi^2_9 < \frac{9 \times 150}{100}\right)$

$= P\left(\frac{9}{2} < \chi^2_9 < \frac{27}{2}\right)$

$= 4.5 , 13.5$

Q3) Binomial. $n=4$ and p. ∴ $Bin(4,p)$ no. of policies = 180

→ (i) $L(P) = [(1-p)^n]^y \ldots [x p^x (1-p)^n]^y$

$$= \prod_{i=0}^{4} f(x_i \cdot \theta)$$

$$= [(1-p)^4]^{86} \cdot [4p(1-p)^3]^{75} \cdot [6p^2(1-p)^2]^{16} \cdot [4p^3(1-p)]^2 \cdot p^4$$

$$= (1-p)^{344} \cdot p^{75}(1-p)^{225} \cdot p^{32}(1-p)^{32} \cdot p^6(1-p)^2 \cdot p^4$$

$$= (1-p)^{603} \cdot p^{117}$$

$$\ln L(P) = 603 \ln(1-p) + 117 \ln P$$

$$\therefore \frac{d}{dp} \ln L(P) = \frac{-603}{1-p} + \frac{117}{p} = 0$$

$$\therefore 117 - 117P - 603p$$

$$\therefore q = 0.8375 \quad \therefore p = 0.1625$$

(ii) $P(X=0) = (1-p)^4 = 0.492$

$P(X=1) = 4P(1-P)^3 = 0.3818$

$P(X=2) = 6P^2(1-P)^2 = 0.1111$

$P(X=3) = 4P^3(1-P) = 0.0144$

$P(X=4) = 0.0007$

We have

	0	1	2	3	4
Oi	86	75	16	2	1
Ei	88.56	68.72	19.98	2.59	0.126

By clubbing,

			2
Oi	86	75	19
Ei	88.56	68.72	22.71

	0	1	2
$c_i = \frac{\Sigma(O_i - E_i)^2}{E_i}$	0.0757	0.5739	0.6061

$\Sigma c_i = 1.2557 \sim \chi^2_1$ ∴ $1.2557 < 3.841$

∴ Do not reject H_0.

Q5) since $U(a,b)$.

(i) Given that $E(X) = \frac{a+b}{2}$; $Var(X) = \frac{(b-a)^2}{12}$

show that $\hat{a} = \bar{x} - \sqrt{3}s$ where $\bar{x}$ = sample mean
s = sample S.D.

Having $a = 2\bar{x} - b$

$\therefore a = b - 2\sqrt{3}s$

$\therefore 2\bar{x} - b = b - 2\sqrt{3}s$

$\therefore b = \bar{x} + \sqrt{3}s$

$\therefore a = \bar{x} - \sqrt{3}s$

Hence proved.

(ii) $b = \bar{x} + \sqrt{3}s$

(iii) let sample = 2, 4, 6, 8, 10

$\therefore \bar{x} = 6$

$s = 3.16$

$\therefore a = 6 - \sqrt{3}(3.16)$

$= 0.527$

$\therefore b = 6 + \sqrt{3}(3.16)$

$= 11.47$

(iv) $U(a,b)$ $\therefore f(x) = \frac{1}{b}$

$\therefore L = \frac{1}{b^n}$

$\log L = -n \log b$

$\therefore \frac{d}{dL} \log L = \frac{-n}{b}$ $\therefore b \to \alpha$

Q7)

(i) $\bar{X}$ public = 30

s^2 public = 64.3125

$\bar{X}$ private = 35.056

s^2 private = 67.403

(ii) we want $H_0: \mu_{pub} = \mu_{pri}$

$H_1: \mu_{pub} \neq \mu_{pri}$

$\therefore \dfrac{(30 - 35.056)}{\sqrt{8.123}\sqrt{\frac{2}{9}}} = -1.407$

$\therefore$ Do not reject H_0.

(iii) Private hospitals do not result in higher claim size at 95% level.

Q9] (i) $t_k = \dfrac{N(0,1)}{\sqrt{\frac{t_k}{k}}}$; $\dfrac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

(ii) $\mu(t_k) = 0$; $Var(t_k) = \dfrac{k}{k-2}$

(iii) $n = 10$; $\bar{x} = 50$; $s^2 = 48.667$; C.I = 99%

a) $t_{0.005,9} = 3.25$

$\therefore \mu = \bar{X} \pm t_{0.5\%,9} \dfrac{S}{\sqrt{n}}$

$= 50 \pm 3.25 \sqrt{\dfrac{48.667}{10}}$

$= 57.17, 42.83$

b) $t_{0.005,9} = 2.5758$

$\therefore \mu = 50 \pm 2.5758\left(\sqrt{\dfrac{48.667}{10}}\right) = 55.68, 44.32$

Q8) $\hat{\theta}$ is an estimator of parameter θ.

i) Unbiased Estimator:-

Mean value equals the true parameter value

$$E(\hat{\theta}) = \theta$$

ii) Bias:-

Bias of an estimator is the difference betn this estimator's expected value and the true value of the parameter being estimated. $E(\hat{\theta}) - \theta = 0$

iii) MSE of estimator $\hat{\theta}$.

Measures the average of the squares of the errors - that is, the average squared difference between the estimated values and the actual value.

$$\theta = Var + bias^2$$

iv) $\hat{\theta}$ is more efficient than $\tilde{\theta}$, since MSE of $\hat{\theta}$ is lower than $\tilde{\theta}$.

v) when MSE more towards θ, as the n inverse, estimate would be more

vi) Method of moments

Maximum Likelihood Estimate.

Q10) $n = 1000$; $X \sim Poi(3)$

$X \sim N(3000, 3000)$

if $P(X < 3100)$ then $X \sim Poi(3)$

else X doesn't follow $Poi(3)$

(i) Type I error = P(Ho is rejected / Ho is true)

$= P(X > 3100, \lambda = 3)$ ∴ $X \sim N(3000, 3000)$

$= P\left[Z > \frac{3100 - 3000}{\sqrt{3000}}\right]$

$= P[Z > 1.826]$

$= \underline{3.39\%}$

(ii) Type 2 error = P(Ho accept / Ho is false)

(iii) Power = P(reject Ho / Ho false)

$= 1 - \left[Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right]$

Q11) $n = 12$; $\Sigma X = 213200$; $\Sigma X^2 = 4919860000$

$\overline{X} = 17767$; $S = 10144$

$X_1 = 11000$; $X_2 = 48000$; $X_1' = 27500$; $X_2' = 31500$

$\Sigma X = 213200 = A + 11000 + 48000$

A is sum of the 10 terms.

∴ $A = 154200$

$\Sigma X' = A + x_1' + x_2' = 213200$

∴ $\overline{X}' = 17767$

$\Sigma X^2 = 4919860000 = B + 11000^2 + 48000^2$

∴ $B = 2494860000$

$\Sigma X'^2 = B + x_1'^2 + x_2'^2 = 4243360000$

∴ $S = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$ $= \underline{6434.0926}$

classmate

Date
Page

Q12) $X_1, X_2, X_3, \ldots, X_n \sim U(0,\theta)$; $a=0$; $b=\theta$

(i) $E(Y) = \frac{\theta}{2}$; $V(Y) = \frac{\theta^2}{12}$

$$\therefore L = \prod_{i=1}^{n} \left(\frac{1}{\theta}\right)$$

$$= \frac{1}{\theta^n}$$

$$\ln(L) = \frac{-n}{\theta}$$

$$\frac{dn}{\theta} = \frac{-n}{\theta} \quad \text{when } \theta = \text{infinite}$$

hence CRLB won't be applicable

(ii) $$P[\text{Max } X < Y] = \prod_{i=1}^{n} \left[\int_0^y \frac{dx}{\theta}\right] = \left(\frac{y}{\theta}\right)^n$$

$$E[y^k] = \int y^k \cdot \frac{n}{\theta^n} \cdot y^{n-1} \cdot dy$$

$$= \frac{n}{n+k} \int \frac{n+k}{\theta^n} \cdot y^{n+k-1} \cdot dy$$

$$= \frac{n}{n+k} \left(\frac{\theta^{n+k} - 0}{\theta^n}\right)$$

$$= \frac{n\theta^k}{n+k}$$

(iii) $$\text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}(c) - \theta]$$

$$= E[(y-\theta)]$$

$$= cE(y] - \theta$$

$$= \frac{c \cdot n\theta}{n+1} - \theta$$

$$\therefore \text{Bias}[\hat{\theta}(c)] = \theta\left(\frac{cn}{n+1} - 1\right)$$

classmate

Date ______
Page ______

(v) $\frac{d}{dc}$ MSE $[\theta(c)] = \frac{d}{dc}\left[\frac{c^2 n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}(+\theta^2)\right]$

$$C_m = \frac{2n\theta^2}{n+1} \cdot \frac{n+2}{2n\theta} = \frac{n+2}{n+1}$$

$$\frac{d^2}{dc^2}\left[MSE\,(\hat{\theta}(c))\right] = \frac{2n\theta^2}{n-2} > 0$$

min. value

(vi) $\frac{n+2}{n+1}\, y \to y$, two estimators are same

$n \to \infty$.

—— x —— o —— x ——