Calculus : Unit 1 and 2
(Ch- 1, 2, 3, 4) Assignment

Ques 1 $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$\Rightarrow \lim_{h \to 0} \frac{2(h^2+9-6h)-18}{h}$

$\Rightarrow \lim_{h \to 0} \frac{2h^2+18-12h-18}{h}$

$\Rightarrow \lim_{h \to 0} \frac{h(2h-12)}{h}$

$\Rightarrow -12$ Ans

Ques 2. $g(x) = \begin{cases} 2x, & x < 6 \\ x-1, & x \geq 6 \end{cases}$

(a) $x = 4$

$\lim_{x \to 4^+} f(x) = 8$

$\lim_{x \to 4^-} f(x) = 8$

$f(4) = 8$

$\therefore$ Function is continuous at $x = 4$

(b) $x = 6$

$$\lim_{x \to 6^+} f(x) = 6 - 1 = 5$$

$$\lim_{x \to 6^-} f(x) = 2(6) = 12$$

$$f(6) = 5$$

$\therefore$ Func^n is discontinuous at $x = 6$.

Sol 3. $$\lim_{x \to 9} \frac{x - 9}{3 - \sqrt{x}}$$

$$\Rightarrow \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{(9-x)}$$

$$\Rightarrow \lim_{x \to 9} -(3 + \sqrt{x})$$

$$\Rightarrow -3 - \sqrt{9}$$

$$\Rightarrow -3 + 3 \text{ or } -3 - 3$$

$$\Rightarrow 0 \text{ or } -6$$

Ques 4 $$f(x) = \frac{4x + 5}{9 - 3x}$$

(a) $x = -1$

$$\lim_{x \to -1} f(x) = \frac{(-4) + 5}{9 + 3}$$

$$f(-1) = \frac{1}{12}$$

(b) $x = 0$

$$\lim_{x \to 0} f(x) = \frac{5}{9}$$

$$f(0) = \frac{5}{9}$$

c) $x = 3$

~~$f(3) = \frac{17}{0}$~~

$$\lim_{x \to 3} = \frac{4(3) + 5}{9 - 3(3)}$$

$$= \frac{17}{17}$$

$$= 0$$

Hence, funct^n is discontinuous at $x = 3$

Sol 5 $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

Note: $e^{\infty} = \infty$, $e^{-\infty} = 0$

$$\lim_{x \to \infty} = \lim_{x \to \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$= \lim_{x \to \infty} \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$$

$$= \lim_{x \to \infty} \frac{6 - 0}{8 - 0 + 0}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Ques 6 $g(y) = \begin{cases} y^2+5, & \text{if } y < -2 \\ 1-3y, & \text{if } y \geq -2 \end{cases}$

(a) $\lim_{x \to 6^+} g(y) = -17$

$\lim_{x \to 6^-} g(y) = -17$

(b) $\lim_{y \to -2^-} g(y) = 9$

$\lim_{y \to -2^+} g(y) = 7$

Sol 7 $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (8t-1)(t^3 - 8t^2 + 12) + (3t^2 - 16t)(4t^2 - t)$

$\Rightarrow 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 3t^3 - 64t^3 + 16t^2$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

Sol 8 $R(w) = \dfrac{3w + w^4}{2w^2+1}$

$= \dfrac{(3+4w^3)(2w^2+1) - (4w)(3w+w^4)}{(2w^2+1)^2}$

$= \dfrac{6w^2 + 3 + 8w^5 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$

$$= \frac{4w^5 + 4w^3 - 6w^2 + 3}{4w^4 + 4w^2 + 1}$$

Sol 9. $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log_e 8$

Sol 10. $y = z^5 - e^2 \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left[e^2 \ln(z) + \frac{e^2}{z}\right]$$

$$= 5z^4 - e^2 \ln(z) - \frac{e^2}{z}$$

Ques 11(a) $f(x) = (6x^2 + 7x)^4$

$f'(x) = 4(12x + 7)(6x^2 + 7x)^3$

(b) $f(t) = 5 + e^{4t + t^7}$

$f(t) = e^{4t+t^7}(4 + 7t^6)$

Ques 12(a) $z = \ln(7 - x^3)$

$$z' = \frac{-3x^2}{7 - x^3}$$

$$z'' = \frac{-6x(7-x^3) + 3x^2(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

(b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$Q'(v) = 2\dfrac{d}{dv}(6+2v-v^2)^{-4}$

$= \dfrac{-8\,(2-2v)}{(6+2v-v^2)^5}$

$= \dfrac{-16\,(1-v)}{(6+2v-v^2)^5}$

$Q''(v) = -16\dfrac{d}{dv}\left[\dfrac{(1-v)}{(6+2v-v^2)^5}\right]$

$= -16\left[\dfrac{-(6+2v-v^2)^5 - 5(6+2v-v^2)^4(2-2v)(1-v)}{(6+2v-v^2)^{5\times 2}}\right]$

$= -16\left[\dfrac{\cancel{(6+2v-v^2)^4}\,[-6-2v+v^2-5(2-2v)(1-v)]}{(6+2v-v^2)^{\cancel{10}\,6}}\right]$

$= -16\left[\dfrac{-6-2v+v^2-5[2-2v-2v+2v^2]}{(6+2v-v^2)^6}\right]$

$= \dfrac{-16}{(6+2v-v^2)^6}[-6-2v+v^2+20v-10v^2-10]$

$= \dfrac{-16\,[-9v^2+18v-16]}{(6+2v-v^2)^6}$

$= \dfrac{16\,(9v^2-18v+16)}{(6+2v-v^2)^6}$

(c) $f(t) = \ln(1+t^2)$

$f'(t) = \dfrac{2t}{1+t^2}$

$$f''(t) = \frac{2(1+t^2) - 4t^2}{(1+t^2)^2}$$

$$= \frac{-2t^2 + 2}{(1+t^2)^2}$$

Sol 13. $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x^2 + x - 3x - 3)$

$= 3[x(x+1) - 3(x+1)]$

$= 3(x+1)(x-3)$

$f'(x) = 0$

$x = -1$ or 3

$f''(x) = 6x - 6$

$f''(-1) = -12$

$f(x)$ is maximum at $x = -1$ $f(-1) = 17$

$f''(3) = 12$

$f(x)$ is minimum at $x = 3$ $f(3) = -15$

Sol 14 $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x^2 - x - 2x + 2)$

$= 12[x(x-1) - 2(x-1)]$

$= 12(x-1)(x-2)$

$f'(x) = 0$

$x = 1$ or 2

$f''(x) = 24x - 36$

$f''(1) = -12$

$f(x)$ is maximum at $x = 1$, $f(1) = 3$

$f''(2) = 12$

$f(x)$ is minimum at $x = 2$, $f(2) = 1$

Sol 15 $f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$\Rightarrow f'(x) = 3x^2 + 6x$

$= 3x(x+2)$

$f'(x) = 0$,

$x = 0$ or -2

$f''(x) = 6x + 6$

$f''(0) = 6$, + ve value

Thus, 0 is the point of minima

$f(0) = 0 \Rightarrow (0, 0)$

$f''(-2) = -6$, $-$ve value

Thus, -2 is the point of maxima

$f(-2) = 4 \Rightarrow (-2, 4)$

$\Rightarrow$ Inc Dec Inc

$-\infty$ $\quad -2 \quad$ 0 $\quad \infty$

$f'(-3) = 9$, + ve

$f(x)$ is increasing in $(-\infty, -2)$ & $(0, \infty)$

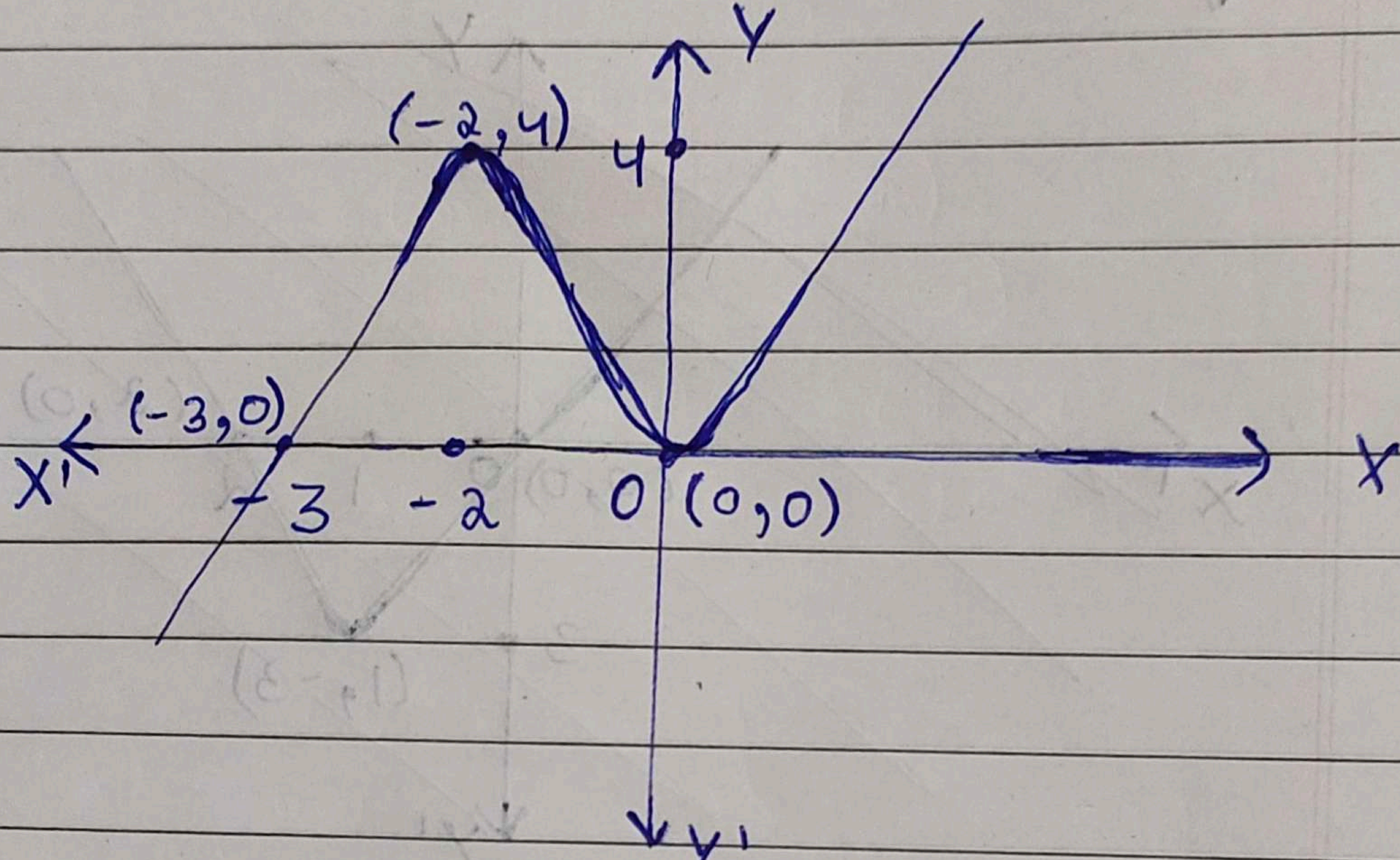
$f(x)$ is decreasing in $(-2, 0)$

⇒ y intercept

$f(0) = 0 \Rightarrow (0, 0)$

x intercept

$f(-3) = 0 \Rightarrow (-3, 0)$



Sol 16 $f(x) = 3x^2 - 6x$

⇒ $f'(x) = 6x - 6$

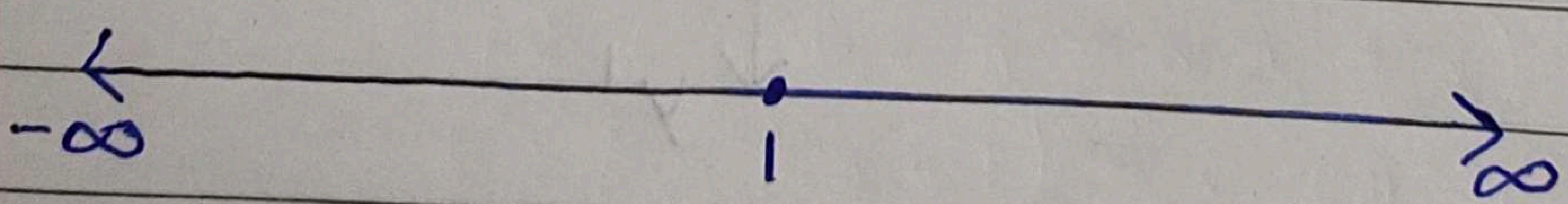
$f'(x) = 0, \quad x = 1$

$f''(x) = 6$, + ve

$f(x)$ is minimum at $x = 1$

$f(1) = -3 \Rightarrow (1, -3)$

⇒



$f'(0) = -6$

$f(x)$ is decreasing in $(-\infty, 1)$

$f'(2) = 6$

$f(x)$ is increasing in $(1, \infty)$

⇒ y intercept

$f(0) = 0 \Rightarrow (0, 0)$

x intercept

$f(2) = 0 \Rightarrow (2, 0)$