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Subject : Calculus
Chapter : 5, 6, 7 (Unit 3 and 4)
Category : Assignment 2

1. $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

let $I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

let $y = 2/w$
differentiating on both sides,
$\frac{dy}{dw} = \frac{-2}{w^2}$

$dw = -\frac{w^2}{2} dy$

~~Substit~~ if $w = 2$, $y = 1$; if $w = 1/50$, $y = 100$

$\therefore I = \int_{100}^{1} \frac{e^y}{\cancel{w^2}} \times -\frac{\cancel{w^2}}{2} dy$

$= -\int_{100}^{1} \frac{e^y}{2} dy$

since $-\int_a^b f(x) = \int_b^a f(x)$

$\therefore I = \int_{1}^{100} \frac{e^y}{2} dy$

$$\therefore I = \left[\frac{e^y}{2}\right]_1^{100}$$

$$= \frac{e^{100}}{2} - \frac{e^1}{2}$$

$$= 1.344058 \times 10^{43}$$

Ans : $\dfrac{e^{100} - e}{2} = 1.344058 \times 10^{43}$

2. $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

let $I = \displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

let $y = t^4 + 2t$

differentiate y w.r.t t

$$\frac{dy}{dt} = 4t^3 + 2$$

$$\frac{dy}{dt} = 2(2t^3+1)$$

$$\therefore \frac{1}{2(2t^3+1)}\, dy = dt$$

$$\therefore I = \int \frac{2t^3+1}{\cancel{(t}\ y^3} \times \frac{1}{2(2t^3+1)}\, dy$$

$$= \int \frac{y^{-3}}{2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^{-3+1}}{-3+1}\right] + c \qquad = -\frac{1}{4} y^{-2} + c$$

Ans: $-\frac{1}{4} y^{-2} + c = -\dfrac{1}{4y^2} + c$

3. $f'(x) = x^4 + 3x - 9$

On Integrating $f'(x)$ we get $f(x)$

$\therefore$ Integrating on both sides,

$\int f'(x) = \int x^4 + 3x - 9 \, dx$

$f(x) = \int x^4 + 3x - 9 \, dx$

$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

Ans: $\frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

4. $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

Let $I = \int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$\left[\because \int_a^c f(x)\, dx = \int_a^b f(x)\, dx + \int_b^c f(x)\, dx \right]$

$\therefore I = \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$

$= \left[\frac{3x^3}{3} \right]_{-2}^{1} + \left[6x \right]_1^3$

$= \left[x^3 \right]_{-2}^{1} + \left[6x \right]_1^3$

~~let~~ $\therefore I = [x^3]_{-2}^{1} + [6x]_{1}^{3}$

$= [1^3 - (-2)^3] + [6(3) - 6(1)]$

$= [1 - (-8)] + 6 \times 2$

$= (1 + 8) + 12$

$= 21$

$\therefore \int_{-2}^{3} f(x) = 21$

5. $\int 3(8y-1)\, e^{4y^2-y}\, dy$

let $x = e^{(4y^2-y)}$

differentiate w.r.t y

$\frac{\cancel{d}}{\cancel{d}}\frac{dx}{dy} = 8y - 1$

$dy = \frac{1}{(8y-1)} dx$

let $I = \int 3(8y-1)\, e^{4y^2-y}\, dy$

$= \int 3\cancel{(8y-1)} \cdot e^{x}\, dx \times \frac{1}{\cancel{(8y-1)}}$

$= \int 3e^{x}\, dx$

$= 3e^{x} + c$

$\therefore \int 3(8y-1)\, e^{4y^2-y}\, dy = 3e^{x} + c$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$ } given
$f(1) = -\frac{5}{4}$, $f(4) = 404$

To get $f'(x)$: Integrate $f''(x)$

$$\therefore f'(x) = \int 15\sqrt{x} + 5x^3 + 6 \; dx$$

$$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + c_1$$

$$= 10x^{3/2} + \frac{5x^4}{4} + 6x + c_1$$

To get $f(x)$: Integrate $f'(x)$

$$\therefore f(x) = \int \cancel{15}\; 10x^{3/2} + \frac{5x^4}{4} + 6x + c_1$$

$$= \frac{10x^{5/2}}{5/2} + \frac{5}{4} \times \frac{x^5}{5} + \frac{6x^2}{2} + c_1x + c_2$$

$$= 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 + c_1x + c_2$$

given : $f(1) = -\frac{5}{4}$

$$f(1) = 4(1)^{5/2} + \frac{1}{4}(1)^5 + 3(1)^2 + c_1(1) + c_2$$

$$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + c_1 + c_2$$

$$-\frac{5}{4} = \frac{49}{4} + c_1 + c_2$$

$$-\frac{54}{4} = c_1 + c_2 \qquad \therefore 4c_1 + 4c_2 = -54$$

— (1)

$$f(4) = 404$$

$$4(4)^{5/2} + \frac{1}{4}(4)^5 + 3(4)^1 + 4c_1 + c_2 = 404$$

$$4^{7/2} + 4^4 + 12 + 4c_1 + c_2 = 404$$

$$4c_1 + c_2 = 8 \quad — ②$$

Subtract ② from ①

$$\begin{array}{rcl} \cancel{4c_1} + 4c_2 & = & -54 \\ - \; \cancel{4c_1} + c_2 & = & 8 \\ (-) \quad & & (-) \\ \hline 3c_2 & = & 62 \end{array}$$

$$c_2 = \frac{62}{3}$$

Substituting, $4c_1 + c_2 = 8$

$$4c_1 + \frac{62}{3} = \frac{24}{3}$$

$$4c_1 = -\frac{38}{3}$$

$$c_1 = -\frac{9.5}{3}$$

$$\therefore f(x) = 4x^{5/2} + \frac{1}{4}x^5 + 3x \cancel{+4} \cancel{-38} - \frac{9.5}{3}x + \frac{62}{3}$$

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta} \qquad r(1) = 2$

Integrate on both sides on ~~reag~~ rearranging

$$\cancel{\int \frac{dr}{d\theta}} \qquad \frac{1}{r^2}\, dr = \frac{1}{\theta}\, d\theta$$

$$\int \frac{1}{r^2}\, dr = \int \frac{1}{\theta}\, d\theta$$

$$\frac{r^{-2+1}}{-2+1} = \cancel{\theta} \log|\theta| + c$$

$$-\frac{1}{r} = \log \theta + c$$

$$-\log \theta = \frac{1}{r} + c$$

when $r = 2, \quad \theta = 1$

$$\therefore -\log 1 = \frac{1}{2} + c$$

$$-0 = \frac{1}{2} + c$$

$$c = -2$$

$\therefore$ equation is : $-\log \theta = \frac{1}{r} \cancel{+} -2$

$$\frac{1}{r} + \log \theta = 2$$

9. $\dfrac{dv}{dt} = 9.8 - 0.196v$

$\dfrac{dv}{dt} + 0.196v = 9.8$

On comparing with $\dfrac{dy}{dx} + Py = Q$

$P = 0.196$

$Q = 9.8$

$e^{\int P dx} = e^{\int 0.196 dv}$

$= e^{0.196v}$

$\therefore \cancel{y}v . e^{\int P dv} = \int Q \cdot (e^{\int P dv}) dv + c$

$v . e^{0.196v} = \int 9.8 \, e^{0.196v} dv + c$

$\cancel{v . e^{0.196v}} = \cancel{9.8} . \dfrac{e^{0.196v}}{\cancel{0.196}} + c$

$\cancel{v = \dfrac{9.8}{}}$

$v . e^{0.196v} = 50 \, e^{0.196v} + c$ is the general solution

12. <u>Taylor Series</u>

$f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f'(x) = 3x^2 + (-20)x$

$f''(x) = 6x - 20$

$f'''(x) = 6$

$$f(x) = f(3) + \frac{(x-3)^1}{1!} f'(3) + \frac{(x-3)^2}{2!} f''(3) + \frac{(x-3)^3}{3!} f'''(3)$$

$$= \left[3^3 - 10(3)^2 + 6\right] + (x-3)\left[3(3)^2 - 20(3)\right] + \frac{6(3)-20}{2}(x-3)^2 + \frac{(x-3)^3}{3!} \times 6$$

$$= -57 + (x-3)(-33) + (-1)(x-3)^2 + (x-3)^3$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

13. <u>Maclaurin</u> <u>Series</u> $(1+x)^{\mu}$

let $y = (1+x)^{\mu}$

$$f(x) = (1+x)^{\mu}$$
$$f'(x) = \mu(1+x)^{\mu-1}$$
$$f''(x) = \mu \times \mu-1\ (1+x)^{\mu-2}$$
$$f'''(x) = \mu \times \mu-1 \times \mu-2\ (1+x)^{\mu-3}$$

$(1+x)^{\mu}$ at $x=0 \Rightarrow 1^{\mu} = 1$

$$f'(0) = \mu(1+0)^{\mu-1} = \mu$$
$$f''(0) = \mu \times \mu-1\ (1+0)^{\mu-2} = \mu(\mu-1)$$
$$f'''(0) = \mu \times \mu-1 \times \mu-2\ (1+0)^{\mu-3} = \mu(\mu-1)(\mu-2)$$

$$(1+x)^{\mu} = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{3!}(f'''(0))$$

$$(1+x)^{\mu} = 1 + \mu x + \frac{x^2}{2}(\mu(\mu-1)) + \frac{x^3}{6}\mu(\mu-1)(\mu-2)$$

14. $$f(x) = \sqrt{1+x}$$

$$f(0) = \sqrt{1+0} = 1$$

$$f'(x) = \frac{1}{2\sqrt{1+x}} \quad \therefore f'(0) = \frac{1}{2\sqrt{1+0}} = \frac{1}{2}$$

$$f''(x) = \frac{1}{2}\frac{d}{dx}(1+x)^{-1/2}$$

$$= \frac{1}{2} \times -\frac{1}{2}(1+x)^{-3/2}$$

$$= -\frac{1}{4}(1+x)^{-3/2} \quad \therefore f''(0) = -\frac{1}{4}$$

$$f'''(x) = -\frac{1}{4} \times -\frac{3}{2}(1+x)^{-5/2}$$

$$= \frac{3}{8}(1+x)^{-5/2} \quad f'''(0) = \frac{3}{8}$$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$$= 1 + \left(x \times \frac{1}{2}\right) + \frac{x^2}{2}\left(-\frac{1}{4}\right) + \frac{x^3}{2\cancel{6}}\left(\frac{\cancel{3}}{8}\right)$$

$$f(x) = 1 + \frac{x}{2} \cancel{+} - \frac{x^2}{8} + \frac{x^3}{16}$$

$$\therefore \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$$

15. Taylor Series

$f(x) = e^{-6x}$ about $x=-4$

$$f(x-\cancel{4}) = f(-4) + (x+4) f'(-4) + \frac{(x+4)^2}{2} f''(-4) + \frac{(x+4)^3}{3!} f'''(4)$$

$$f'(x) = -6e^{-6x} \qquad f(-4) = e^{24}$$
$$f''(x) = 36e^{-6x} \qquad f'(-4) = -6e^{24}$$
$$f'''(x) = -216e^{-6x} \qquad f''(-4) = 36e^{24}$$
$$f'''(-4) = -216e^{24}$$

$$f(x) = e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2}(36e^{24}) + \frac{(x+4)^3}{3!}(-216e^{24})$$

$$= e^{24} - 6(x+4)e^{24} + \frac{36}{2}(x+4)^2(e^{24}) - \frac{216}{6}(x+4)^3 e^{24}$$

$$= e^{24} - 6(x+4)e^{24} + 18(x+4)^2 e^{24} - 36(x+4)^3 e^{24}$$

$\therefore$ Taylor Series $= e^{24} - 6(x+4)e^{24} + 18(x+4)^2 e^{24} - 36(x+4)^3 e^{24}$

16. $f(x) = \ln(3+4x) \quad x=0$

Taylor Series

$$f(x) = f(a) + \frac{(x-a)^1}{1!} f'(x) + \frac{(x-a)^2}{2} f''(x)$$

$$+ \frac{(x-a)^3}{3!} f'''(x)$$

Since $x=0 \quad \therefore a=0$

$$\therefore f(x) = f(0) + (x) f'(x) + \frac{x^2}{2} f''(x) +$$

$$+ \frac{x^3}{3!} f'''(x)$$

$$f(x) = \ln(3+4x)$$

$$f'(x) = \frac{1}{3+4x} \cdot 4 = \frac{4}{3+4x} = 4(3+4x)^{-1}$$

$$f''(x) = 4(-1)(3+4x)^{-2} \cdot 4 = -16(3+4x)^{-2}$$

$$f'''(x) = 32(3+4x)^{-3} \cdot 4$$

$$= 128(3+4x)^{-3}$$

Ans : $f(x) = \ln 3 + x \times \frac{4}{3} + \frac{x^2}{\cancel{2}}\left(-\frac{\overset{8}{\cancel{16}}}{9}\right) + \frac{x^3}{\cancel{6}3}\left(\frac{\overset{64}{\cancel{128}}}{27}\right)$

$$= \ln 3 + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64x^3}{81}$$