

Date: _____

Numerical Method And Algebra.

$$1) \sqrt{7\sqrt{7\sqrt{7}\dots}}$$

$$= 7^{1/2} \cdot 7^{1/4} \cdot 7^{1/8} \cdot 7^{1/16} \dots$$

$$= 7^{1/2 + 1/4 + 1/8 + 1/16 \dots}$$

$1/2 + 1/4 + 1/8 + 1/16 \dots$ is in form of infinite G.P.

$$\begin{aligned}\text{Sum of infinite G.P} &= \frac{a}{1-r} \\ &= \frac{1/2}{1-1/2} \\ &= 1\end{aligned}$$

$$\therefore 7^{1/2 + 1/4 + 1/8 \dots}$$

$$= 7^1$$

$$= 7$$

$$2) 7x^2 - 6y^2 = -11x^2 - 4y^2$$

$$18x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{2}{18}$$

$$\frac{x}{y} = \sqrt{\frac{2}{18}}$$

$$3) \quad x + y = 4 \quad \text{---(1)}$$

$$x^2 + y^2 - 4y = 12 \quad \text{---(2)}$$

$$x = 4 - y \quad \text{--- from (1) in (2)}$$

$$(4 - y)^2 + y^2 - 4y = 12$$

$$16 + y^2 - 8y + y^2 - 4y = 12$$

$$16 + 2y^2 - 12y = 12$$

$$y^2 - 6y + 8 = 6$$

$$y^2 - 6y + 2 = 0$$

$$y = \frac{6 \pm \sqrt{36 - 8}}{2}$$

$$= \frac{6 \pm \sqrt{28}}{2}$$

$$= 3 \pm \sqrt{7}$$

$$\begin{aligned} x_1 &= 4 - y_1 \\ &= 4 - 3 + \sqrt{7} \\ &= 1 + \sqrt{7} \end{aligned}$$

$$\begin{aligned} x_2 &= 4 - y_2 \\ &= 4 - 3 - \sqrt{7} \\ &= 1 - \sqrt{7} \end{aligned}$$

The ordered pairs are

$$(1 + \sqrt{7}, 3 - \sqrt{7}) \text{ and } (1 - \sqrt{7}, 3 + \sqrt{7})$$

$$u) d = 2r^2 - 16r + 34 \Rightarrow \text{density.}$$

$$i) r = 3.5$$

$$d = 2.5 \text{ mill particles/8t}^3$$

$$ii) d - 2r^2 - 16r + 34 = 0$$

differentiation.

$$4r - 16 = 0 \rightarrow (1)$$

$$r = 4 \quad (400 \text{ RPM})$$

differentiation of (1)

4

minima.

$$d = 2(4)^2 - 16(4) + 34$$

$$d = 2$$

$\therefore$ The no. of revolution for minimum smoke = 4.

$$iii) d = 100$$

$$100 = 2r^2 - 16r + 34$$

$$50 = r^2 - 8r + 17$$

$$0 = r^2 - 8r + (-33)$$

$$r = (u-11)(u+3)$$

$$r = 11$$

The speed must be 11 revolution per minute.

$$5) \begin{aligned} C_1 &= 21500 + 25 \text{ (km)} \\ C_2 &= 2100 + 22 \text{ (km)} \end{aligned}$$

i) Cost to rent car from Co. 1.
 $C_1 = 1500 + 25n$

ii) Cost to rent car from Co. 2
 $C_2 = 2100 + 22n$

iii) $C_2 < C_1$
 $1500 + 25n > 2100 + 22n$

iv) $3n > 600$
 $n > 200$

It is less expensive to take C_2 when the no. of km is more than 200.

6) $x^3 - 7x^2 + 14x - 6 = 0$
a) $a = 0, b = 1$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f(\frac{a+b}{2})$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.628	0.9844	0.2508

$$x = 0.625$$

b) $a=1$ $b=3.2$ $f(x) = x^3 - 7x^2 + 16x - 6$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f(a+b/2)$
1	3.2	2.1	2	-0.42	1.791
2.1	3.2	2.65	1.791	-0.112	0.5521
2.65	3.2	2.925	0.5521	-0.112	0.08522

$x = 2.925$

c) $[3.2, 4]$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f(a+b/2)$
3.2	4	3.6	-0.112	2	0.336
3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	-0.016	0.0461	0.0130

$x = 3.425$

$$7) \quad f(u) = 230u^4 + 18u^3 + 9u^2 - 221u - 9$$

$$f'(u) = 920u^3 + 54u^2 + 18u - 221$$

a)	i	u_i	$f(u)$	$f'(u)$	$u_{i+1} = u_i - \frac{f(u)}{f'(u)}$
	1	-1	156.1398	-1105	-0.60814
	2	-0.6081	43.994	-418.898	-0.2354
	3	-0.2354	1.537	-234.206	-0.04754
	4	-0.04754	0.0004	-221.704	-0.04666
	5	-0.04066	≈ 0	-221.70443	-0.04066
		-0.04066	≈ 0	-221.70443	-0.04066

$$u = -0.04060$$

b)	i	u_i	$f(u)$	$f'(u_i)$	$u_{i+1} = u_i - f(u_i)$
	1	0.9	-36.585	509.62	0.97179
	2	0.97178	6.37668	691.80	0.86259
	3	0.9625	0.11523	666.87	0.9624
		0.9624	0.3267	666.411	0.9624
		0.9624			

$$u = 0.9624$$

$$8) \quad a) \text{ Average} = \frac{70 + 86 + 81 + 83 + u}{5}$$

$$= \frac{320 + u}{5}$$

$$b) \text{ Average} = B$$

$$80 < \frac{320 + u}{5} < 90$$

$$or \quad 400 < 320 + u < 450$$

$$80 < u < 130$$

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$$a) \quad u_1 = 0.04 \quad u_2 = 0.2 \quad u = 0 \\ f(u_1) = 8.54 \quad f(u_2) = -11.81 \quad f(u) = ?$$

$$\frac{u - u_1}{u_2 - u_1} = \frac{f(u) - f(u_1)}{f(u_2) - f(u_1)}$$

$$\frac{-0.04}{0.16} = \frac{f(u) - 8.54}{-20.35}$$

$$f(u) = 13.6275$$

$$10) \quad R(x) = 32u - 0.21u^2 \\ C(x) = 195 + 12u$$

for profit $R(u) > C(u)$

$$\therefore 32u - 0.21u^2 > 195 + 12u$$

$$20u - 195 - 0.21u^2 > 0$$

$$0.21u^2 - 20u + 195 < 0$$

$$0.21u^2 - 20u + 195 = 0$$

$$u = \frac{20 \pm \sqrt{252 - 8}}{0.42}$$

$$u = 82.35, 12.88$$

$$(u - 82.38)(u - 12.88) < 0 \\ u \in (12.88, 82.38)$$