Prob & Stats → Assignment 2 [43]

Q1

$X \sim N(800, 100^2)$ [Home Insurance claim]

$Y \sim N(1200, 300^2)$ [Car Insurance claim]

$\therefore P((Y_1+Y_2+Y_3) > (X_1+X_2+X_3+X_4)+800)$

$P((Y_1+Y_2+Y_3)-(X_1+X_2+X_3+X_4) > 800)$

$\therefore$

$(Y_1+Y_2+Y_3)-(X_1+X_2+X_3+X_4) \sim$
$N(3\times1200 - 4\times800, 3\times300^2 + 4\times100^2)$

$\sim N(400, 310\,000)$

$\therefore P\left(Z > \frac{800-400}{\sqrt{310,000}}\right)$

$= P(Z > 0.718)$

$= 1 - \Phi(0.718)$

$= 0.236$

$\therefore \cancel{M''_Y(t)} = Y = X_1 + X_2 + X_3 + \dots + X_N$

Q2] $M_Y(t) = M_N(\ln M_X(t))$

$= M_N(\ln(1 - t/\lambda)^{-a})$

$= \cancel{M_N(1 - t/2)} = M_N(\ln(1 - t/2)^{-b})$

$= M_N(-b\ln(1 - t/2))$

$\therefore \left(\frac{0.9e^t}{1 - 0.1e^t}\right)^k$

$\left(\frac{0.9\, e^{\ln(1-t/2)^{-b}}}{1 - 0.1\, e^{\ln(1-t/2)^{-b}}}\right)^k$

$\left(\frac{0.9 \times (1 - t/2)^{-b}}{1 - 0.1(1 - t/2)^{-b}}\right)^3$

$M_Y(t) \left(\frac{0.9(1 - 0.5t)^{-b}}{1 - 0.1(1 - 0.5(t))^{-b}}\right)^3 \leftarrow$ [Not sure]

$\cancel{M_Y(t)} = \boxed{\begin{array}{l} M_X(t) = \ln(1 - t/\lambda)^{-a} \\ M_N(t) = \left(\frac{pe^t}{1 - qe^t}\right)^k \end{array}}$ Ignore

ii) $E[Y] = M'_Y(0)$ $\quad V[Y] = M''_Y(0) - [M'_Y(0)]^2$

~~Q3] i) ∴ MGF parameter by uniqueness property of MGF~~

~~$X \sim Exp($~~

$E[X] = M'_X(0)$

$= \cancel{-\lambda \cdot \frac{d}{dx}}$

Q3] i)

$$M_X(t) = E[e^{tX}]$$

$$= \int_a^\infty e^{tx} \cdot \lambda e^{-\lambda(x-a)}\, dx$$

$$= \lambda \int_a^\infty e^{-\lambda x + \lambda a + tx}$$

$$\left(\frac{\lambda - t}{\lambda}\right)^a$$

$$= \lambda \int_a^\infty e^{x(t-\lambda)} \cdot e^{\lambda a}\, dx$$

$$\cancel{\therefore M_X(t) = \left(1 - \frac{t}{}\right.}$$

$$= \lambda e^{\lambda a} \int_a^\infty e^{x(t-\lambda)}\, dt$$

$$= \lambda e^{-\lambda a} \left(\frac{e^{x(t-\lambda)}}{(t-\lambda)}\right)_a^\infty$$

$$\cancel{= \lambda e^{-\lambda a}\left(0 - \frac{1}{a(t-\lambda)}\right)} = \lambda e^{-\lambda a}\left(-\frac{e^{a(t-\lambda)}}{(t-\lambda)}\right)$$

$$MGF = \cancel{\frac{\lambda e^{\lambda a}}{\lambda - t}} \quad \cancel{\frac{-\lambda e^{\lambda a}}{a-\lambda}} \quad \frac{-\lambda e^{-\lambda a + \lambda a - ta}}{t+\lambda}$$

$$\boxed{MGF_X(t) = \frac{-\lambda e^{-ta}}{(t-\lambda)}}$$

Qu) $G_v(t) = p^k(1-qt)^{-k}$

$\therefore G_v(t) = t^0 P(X=0) + t^1 P(X=1) + \frac{t^2}{2!} P(X=2) + \frac{t^3}{3!} P(X=3) + \frac{t^4}{4!} P(X=4)$

$\therefore \cancel{G_v'(t) = p^k\left[-k(1-qt)^{-k-1}\cdot(-q)\right]}$

$= \cancel{p^k}$

$G_v(t) = p^k(1-(1-p)t)^{-k}$

$= p^k(1-t+pt)^{-k}$

$\cancel{= p^k \cdot t p^k}$

$G_v'(t) = p^k\left[-k(1-qt)^{-k-1} \cdot -q\right]$

$= p^k qk\left[1-qt^{-k-1}\right]$

$\vdots$

$G_v''''(t) = p^k q^4 k(k+1)(k+2)(k+3)(1-qt)^{-(k+4)}$

$\therefore P(X=4) = \frac{t^4}{4!} \times p^k q^4 k(k+1)(k+2)(k+3)$

$= \frac{t^4}{4!} p^k (1-p)^4 k(k+1)(k+2)(k+3)$

Q5

$\cancel{P(Y > 1/3X + 10)}$

$\cancel{P(3Y - X \ldots)}$

$\cancel{P(3Y - X > 10)}$ ← Don't know

$\Rightarrow$

$$E[Y|X] = \int_{10}^{20} y \cdot \frac{a\cancel{x}(x+y)}{10a\cancel{x}(x+15)}$$

$$f_X(x) = \int_{10}^{20} ax^2 + axy = 10ax(x+15)$$

$$\frac{1}{10} \int_{10}^{20} \frac{yx + y^2}{(x+15)} dy$$

$$= \frac{1}{10(x+15)} \left(\frac{300x}{2} + \frac{7000}{3} \right)$$

$\cancel{P(Y > 1/3X + 10)}$

$$\frac{10 + x}{3}$$

$$\cancel{= \int_{10}^{1/3x+10} \int_0^5 f_{X,Y}(x,y) \, dx \, dy}$$

$$\cancel{\int_{10}^{1/3x+10} \left[2ax^2 + axy \right]_0^5 dy}$$

$P(Y > X/3 + 10)$

$\cancel{P(X \ldots < 10)}$ $-3Y$

$$\cancel{\int_{10}^{1/3x+10} 10a + ay - ay}$$

$$\int_0^5 \int_{y=10}^{10+x/3} ax^2 + axy \, dy \, dx$$

$$\cancel{\int_0^5 \left[ax^2 \left(10 + \frac{x}{3}\right) + \right.}$$

$$\int_0^5 \left[ax^2 y + \frac{axy^2}{2} \right]_{10}^{10+\frac{x}{3}}$$

$$\int_{10}^{10+x/3} \int_{0}^{5} ax^2 + axy \, dx \, dy$$

$$\int_{10}^{10+x/3} \left[\frac{ax^3}{3} + \frac{ax^2 y}{2} \right]_0^5 dy$$

$$\int_{10}^{10+x/3} \frac{125a}{3} + \frac{25a}{2} y \, dy$$

$$\cancel{12a}\ \frac{125}{3} a \left[\cancel{10} + \frac{x}{3} - \cancel{10} \right] + \frac{25a}{2} \left[\frac{(10 + \frac{x}{3})^2}{2} - 50 \right]$$

$$\frac{125}{9} ax + \frac{25a}{2} \left[\cancel{100}\ 50 + a/3x + x^2/18 - \cancel{50} \right]$$

$$\frac{125}{9} ax + \frac{25a^2 x}{6} + \frac{25}{36} ax^2$$

Q6] $X \sim Poi(\lambda)$ $\qquad Y \sim Poi(\mu)$

$X - Y = Z$

$$\therefore M_Z(t) = E[e^{tZ}]$$
$$= E[e^{t(X-Y)}]$$
$$= E[e^{tX - tY}]$$
$$= E[e^{tX} \cdot e^{-tY}]$$
$$= M_X(t) \cdot M_Y(-t)$$
$$\therefore = e^{\lambda(e^t - 1)} \cdot e^{\mu(e^{-t} - 1)}$$
$$= e^{\lambda(e^t - 1) + \mu(e^{t} - 1)}$$

Hence Arthur Statement is wrong as the MGF is not of Poisson

$Z = X + Y$

$$M_Z(t) = E[e^{t(X+Y)}]$$
$$= M_X(t) \cdot M_Y(t)$$
$$= e^{\lambda(e^t - 1)} \cdot e^{\mu(e^t - 1)}$$
$$= e^{(\lambda + \mu)(e^t - 1)}$$

On comparing and by uniqueness property
$X + Y \sim Poi(\lambda + \mu)$

Date :

Q7] ii) $V(X|Y=2)$

$= E[X^2|Y=2] - (E[X|Y=2])^2$

$= [(1^2 \times 0) + (2^2 \times 0.3) + (3^2 \times 0.1) + (4^2 \times 0.2)]$

$- [(1\times0) + (2\times0.3) + (3\times0.1) + (4\times0.2)]$

$= [0 + 1.2 + 0.9 + 3.2] - [0 + 0.6 + 0.3 + 0.8]$

$= 5.3 - 1.7 = \boxed{3.6}$

iii) $E(U|V=v)$

$f_V(u) = \int_0^1 \frac{48}{67}(2uv - u^2)\, du$

$\frac{48}{67\times3}[3v-1]$

$= \int_0^1 u \cdot f(U|V=v)$

$\int_0^1 u \cdot \frac{f(u,v)}{f_V(v)}\, du$

$\int_0^1 u \cdot \frac{\cancel{48}/\cancel{67}(2uv - u^2)}{\cancel{48}/3\times\cancel{67}(3v-1)}\, du$

$= \frac{8v-3}{12v-4}$

Q8 $X \sim \text{Gamma}(3, 2)$

$E[Y|X=x) = 3x+1 \qquad V(Y|X=x) = 2x^2+5$

$E[Y] = E[E[Y|X=x]]$

$= E[3X+1]$

$= 3E[X]+1$

$= 3\times(1.5)+1$

$= 5.5$

$V[Y] = E[V(Y|X=x)] + V[E(Y|X=x)]$

$= E[2X^2+5] + V[3X+1]$

$= 2E[X^2]+5 + 9V[X]$

$= 2\left[3/4+(3/2)^2\right] + 5 + 9[3/4]$

$= 2\left[\frac{\cancel{12}^{3}}{\cancel{4}}\right] + 5 + 27/4$

$= 11 + \frac{27}{4} = \boxed{\frac{71}{4}}$

Q9] $E[X] = 3$ $V[X] = 2^2$

$E[Y] = 4$ $V[Y] = 1$

$E[Z] = E[X+Y]$
$= E[X] + E[Y]$
$= 3 + 4$
$= 7$

$corr(X,Y) = -0.3$

$Z = X + Y$

i) $Cov(X,Z) = E[XZ] - E[X]E[Z]$

$= 24.4 - [3 \times 7]$

$E[XZ] =$
$= E[X(X+Y)]$

$= 24.4 - 21 = \underline{\underline{3.4}}$

$= E[X^2 + XY]$

$= E[X^2] + E[XY]$

$= 13 + 11.4 = 24.4$

ii) $Var(Z) =$
$= Cov(X+Y, X+Y)$
$= V(X) + V(Y) + 2Cov(X,Y)$
$= 4 + 1 + (2 \times -0.6)$

$= 5 + (-1.2) = \underline{\underline{3.8}}$

$V[X] = E[X^2] - E[X]^2$

$4 = E[X^2] - 9$

$E[X^2] = 13$

$Cov(X,Y) = E[X,Y] - E[X]E[Y]$
$(-0.3 \times 2) = E[X,Y] - [12]$

$E[XY] = 11.4$

$cora = \frac{Cov(X,Y)}{\sqrt{Var(X)V(Y)}} =$

$-0.3 \times \sqrt{4} = Cov(X,Y)$

Date :

Q10 $f(x,y) = kx^{-a} e^{-y/\beta}$

$$\therefore \int_1^\infty \int_1^\infty k x^{-a} e^{-y/\beta} \, dy \, dx = 1$$

$$k \int_1^\infty x^{-a} \int_1^\infty e^{-y/\beta} \, dy \, dx = 1$$

$$k \int_1^\infty x^{-a} \cdot \left[\frac{e^{-y/b}}{-y/\beta} \right]_1^\infty$$

$$k \int_1^\infty x^{-a} \left[0 - \left[\frac{e^{-1/\beta}}{-1/\beta} \right] \right]$$

$$k \frac{1}{\beta} e^{-1/\beta} \cancel{a} \int_1^\infty x^{-a}$$

$$k \frac{1}{\beta} e^{-1/\beta} \left[\frac{x^{-a+1}}{a+1} \right]_1^\infty$$

$$k \frac{1}{\beta} e^{-1/\beta} \left[0 - \left[1/(a+1) \right] \right] = 0$$

$$k \cdot \frac{e^{-1/\beta}}{\beta} \cdot \frac{1}{(a+1)} = 1$$

$$k = (a+1)\beta \,/\, \exp\left[-1/\beta\right]$$