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Calculus Assignment 1
Chapter 1, 2, 3, 4 (Unit 1 and 2)

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Question 1

1. $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$

Solution $= \lim_{h\to 0} \frac{2(9-6h+h^2)-18}{h}$

$= \lim_{h\to 0} \frac{\cancel{18}-12h+2h^2-\cancel{18}}{h}$

$= \lim_{h\to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}}$

$= 2(0-6)$

$= \boxed{-12}$

2. Question 2

Soln (a) $x = 4$

$$g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$$

Step (i) : $g(4) = 2(4) = 8$

step (ii) :

$$L \cdot H \cdot L = \lim_{x \to 4^-} 2x = 2(4) = 8$$

$$R \cdot H \cdot L = \lim_{x \to 4^+} 2x = 2(4) = 8$$

$$L \cdot H \cdot L = R \cdot H \cdot L$$

∴ The limit exists

step (iii) :

$$\lim_{x \to 4^-} 2x = \lim_{x \to 4^+} 2x = g(4)$$

Hence the function is continuous

(b) $x = 6$

$$g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$$

step (i) : $g(6) = (6) - 1 = 5$

step (ii) :

$$L \cdot H \cdot L = \lim_{x \to 6^-} 2x = 2(6) = 12$$

$$R.H.L = \lim_{x \to 6^+} x - 1 = 6 - 1 = 5$$

$$L.H.L \neq R.H.L$$

Therefore the limit doesnot exists

Hence the function is discontinuous at 6.

Question 3

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

Solution :

$$= \lim_{x \to 9} -\left(\frac{x-9}{\sqrt{x}-3}\right)$$

$$= \lim_{x \to 9} -\left(\frac{(\sqrt{x}+3)(\sqrt{x}-3)}{(\sqrt{x}-3)}\right)$$

$$= \lim_{x \to 9} -(\sqrt{x}+3)$$

$$= -(\sqrt{9}+3)$$

$$= -(3+3)$$

$$= \boxed{-6}$$

Question 4

$$f(x) = \frac{4x+5}{9-3x}$$

Solution

(a) $x = -1$

step (i) $f(-1) = \dfrac{4(-1)+5}{9-3(-1)} = \dfrac{-4+5}{9+3}$

$= \dfrac{1}{12}$

step (ii)

$$L.H.L = \lim_{x \to -1^-} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3(-1)}$$

$$= \frac{1}{12}$$

$$R.H.L = \lim_{x \to -1^+} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3x}$$

$$= \frac{1}{12}$$

$L.H.L = R.H.L$

$\therefore$ limit exists

step (iii)

$f(-1) = L.H.L = R.H.L$

$\boxed{\text{Hence the function is continuous at } x = -1.}$

(b) $x=0$

step (i)

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

step (ii)

$$L.H.L = \lim_{x \to 0^-} \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$$R.H.L = \lim_{x \to 0^+} \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$L.H.L = R.H.L$

$\therefore$ The limit exists

step (iii)

$f(0) = L.H.L = R.H.L$

Hence the function is continuous at $x=0$.

(c) $x = 3$

step (i)

$$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \infty$$

$\therefore$ $f(3)$ doesnot exist

Hence the function is discontinuous at $x=3$.

Question 5

$$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

Solution

$$= \lim_{x\to\infty} \frac{6e^{4x} - \frac{1}{e^{2x}}}{8e^{4x} - e^{2x} + \frac{3}{e^{x}}}$$

$$= \lim_{x\to\infty} \frac{\cancel{6e^{4x}}\ \frac{6e^{6x} - 1}{e^{2x}}}{\frac{8e^{5x} - e^{3x} + 3}{e^{x}}}$$

$$= \lim_{x\to\infty} \frac{6e^{6x} - 1}{e^{x}(8e^{5x} - e^{3x} + 3)}$$

~~divide throughout by e^{x}~~

$$= \lim_{x\to\infty} \frac{6e^{5x} - 1/e^{x}}{(8e^{5x} - e^{3x} + 3)}$$

divide throughout by e^{5x}

$$= \lim_{x\to\infty} \frac{6 - 1/e^{6x}}{8 - 1/e^{2x} + 3/e^{5x}}$$

$$= \frac{6}{8}$$

$$= \boxed{\frac{3}{4}}$$

Question 6

$$g(y) = \begin{cases} y^2+5 & y<-2 \\ 1-3y & y \geq -2 \end{cases}$$

Solution

(a) $\lim\limits_{y \to 6} g(y)$

$$= \lim_{y\to 6} \cancel{g}\; 1-3y$$
$$= 1-3(6)$$
$$= 1-18$$
$$= \boxed{-17}$$

(b) $\lim\limits_{y \to -2} g(y)$

$$\lim_{y\to -2^-} g(y) = \lim_{y\to -2^-} y^2+5$$
$$= (-2)^2+5$$
$$= 9$$

$$\lim_{y\to -2^+} g(y) = \lim_{y\to -2^+} 1-3y$$
$$= 1-3(-2)$$
$$= 1+6$$
$$= 7$$

L.H.L $\neq$ R.H.L $\left[\lim\limits_{y\to -2^-} \neq \lim\limits_{y\to -2^+}\right]$

$\boxed{\therefore \text{The limit does not exist.}}$

Question 7

$$f(t) = (4t^2 - 4)(t^3 - 8t^2 + 12)$$

Solution

$$f'(t) = (4t^2 - 4)\frac{d}{dt}(t^3 - 8t^2 + 12) +$$
$$(t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - 4)$$

$$= (4t^2 - 4)(3t^2 - 16t \cancel{4}) +$$
$$(t^3 - 8t^2 + 12)(8t)$$

$$= 12t^4 - 64t^3 - 12t^2 + 64t +$$
$$8t^4 - 64t^3 + 96t$$

$$= \boxed{20t^4 - 128t^3 - 12t^2 + 160t}$$

Question 8

$$R(w) = \frac{3w + w^4}{2w^2 + 1}$$

Solution

$$R(w) = \frac{3w + w^4}{2w^2 + 1}$$

$$R'(w) = \frac{(2w^2+1)\frac{d}{dw}(3w+w^4) - (3w+w^4)\frac{d}{dw}(2w^2+1)}{(2w^2+1)}$$

$$= \frac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)}$$

$$= \frac{6w^2 + 8w^5 + 3 + 4w^3 - [12w^2 + 4w^5]}{(2w^2+1)}$$

$$= \frac{4w^5 + 4w^3 - \cancel{1}6w^2 + 3}{(2w^2+1)}$$

$$= 4w^3(w^2+1) - 3(2w^2-1)$$

Question 9

$f(x) = 2e^{x} - 8^{x}$

Solution

$f'(x) = \cancel{2e^{x}} \; 2\frac{d}{dx}e^{x} - \frac{d}{dx}8^{x}$

$= \boxed{2e^{x} - 8^{x} \cdot \log 8}$

Question 10

$\cancel{f(x) =} y = z^{5} - e^{z}\ln(z)$

Solution

$\frac{dy}{dz} = \frac{d}{dz}z^{5} - \left[e^{z}\frac{d}{dz}\ln(z) + \ln(z) \cdot \frac{d}{dz}e^{z}\right]$

$= 5z^{4} - \left[e^{z} \cdot \frac{1}{z} + \ln(z) \cdot e^{z}\right]$

$= \boxed{5z^{4} - \frac{e^{z}}{z} - e^{z}\ln(z)}$

Question 11

a) $f(x) = (6x^2 + 7x)^4$

Solution

$$f'(x) = 4(6x^2 + 7x)^3 \cdot \frac{d}{dx}(6x^2 + 7x)$$

$$= 4(6x^2 + 7x)^3 \cdot (12x + 7)$$

$$= \boxed{4(12x + 7)(6x^2 + 7x)}$$

b) $f(t) = 5 + e^{4t + t^7}$

Solution

$$f'(t) = \frac{d(5)}{dt} + \frac{d}{dt} e^{4t + t^7}$$

$$= 0 + e^{4t + t^7} \cdot \frac{d}{dt}(4t + t^7)$$

$$= e^{4t + t^7} \cdot (4 + 7t^6)$$

$$= \boxed{(4 + 7t^6) \cdot e^{4t + t^7}}$$

Question 12

a) $z = \ln(7-x^3)$

Solution

$$\frac{dz}{dx} = \frac{d}{dx}\ln(7-x^3)$$

$$= \frac{1}{(7-x^3)} \cdot \frac{d}{dx}(7-x^3)$$

$$= \frac{1}{7-x^3} \cdot (0-3x^2)$$

$$= \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx^2} = \frac{d}{dx}\left(\frac{-3x^2}{7-x^3}\right)$$

$$= \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\left(\cancel{\frac{d}{dx}}\right.\ \frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)(-6x) + 3x^2(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \boxed{\frac{-3x^4 - 42x}{(7-x^3)^2}}$$

b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

<u>Solution:</u>

$Q(v) = 2(6+2v-v^2)^{-4}$

$$Q'(v) = 2\times(-4)\times(6+2v-v^2)^{-5} \cdot \frac{d}{dx}(2v-v^2)$$

$$= 2(-4)\times(6+2v-v^2)^{-5} \cdot (2-2v)$$

$$= -16(1-v)(6+2v-v^2)^{-5}$$

$$Q''(v) = -16\left[(1-v)\cdot\frac{d}{dv}(6+2v-v^2)^{-5} + (6+2v-v^2)^{-5}\frac{d}{dv}(1-v)\right]$$

$$= -16\left[(1-v)\cdot(-5)(6+2v-v^2)^{-6} \cdot (2-2v) + (6+2v-v^2)^{-5}(-1)\right]$$

$$= 16\left[5(1-v)^2(6+2v-v^2)^{-6}\times 2 + (6+2v-v^2)^{-5}\right]$$

$$= \boxed{16(6+2v-v^2)^{-5}\left[\frac{10(1-v)^2}{(6+2v-v^2)} + 1\right]}$$

c) $f(t) = \ln(1+t^2)$

Solution

$$f'(t) = \frac{d}{dt} \ln(1+t^2)$$

$$= \frac{1}{(1+t^2)} \frac{d}{dt} (1+t^2)$$

$$= \frac{1}{(1+t^2)} \cdot (0+2t)$$

$$= \frac{2t}{(1+t^2)}$$

$$f''(t) = \frac{(1+t^2)\frac{d}{dt}2t - 2t\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{2(1+t^2) - 2t.2t}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

$$= \boxed{\frac{2(1-t^2)}{(1+t^2)^2}}$$

Question 13

$f(x) = x^3 - 3x^2 - 9x + 12$

Solution

$f(x) = x^3 - 3x^2 - 9x + 12$

differentiate w.r.t x

$f'(x) = 3x^2 - 6x - 9$ 27

$f'(x) = 0$ 9 3

$3x^2 - 6x - 9 = 0$

$3x^2 - \cancel{3}9x + 3x - 9 = 0$

$3x(x-3) + 3(x-3) = 0$

$3(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

differentiate w.r.t x : $f'(x)$

$\therefore f''(x) = 6x - 6$

$f''(3) = 6(3) - 6 = 18 - 6 = 12$

$f''(-1) = 6(-1) - 6 = -6 - 6 = -12$

Since $f''(3) > 0$ $\therefore$ function is minium at $x = 3$

Since $f''(-1) < 0$ $\therefore$ function is maximum at $x = -1$

$f(3) = 3^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -13$ $\therefore$ Minima $= \boxed{-13}$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$ $\therefore$ Maxima $= \boxed{17}$

Question 14

$f(x) = 4x^3 - 18x^2 + 24x - 7$

Solution

$f(x) = 4x^3 - 18x^2 + 24x - 7$

differentiate w.r.t x

$f'(x) = 12x^2 - 36x + 24$

Equating $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$12(x^2 - 3x + 2) = 0$

$12(x^2 - 2x - x + 2) = 0$

$12(x(x-2) - (x-2)) = 0$

$12(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

differentiating $f'(x)$ w.r.t x

$f''(x) = 24x - 36$

$f''(1) = 24(1) - 36 = -12 < 0$

$\therefore$ function is maximum at $x = 1$

$f''(2) = 24(2) - 36 = 48 - 36 = 12 > 0$

$\therefore$ function is minimium at $x = 2$

Minima :

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7 = \boxed{1}$

Maxima :

$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7 = \boxed{3}$

Question 15

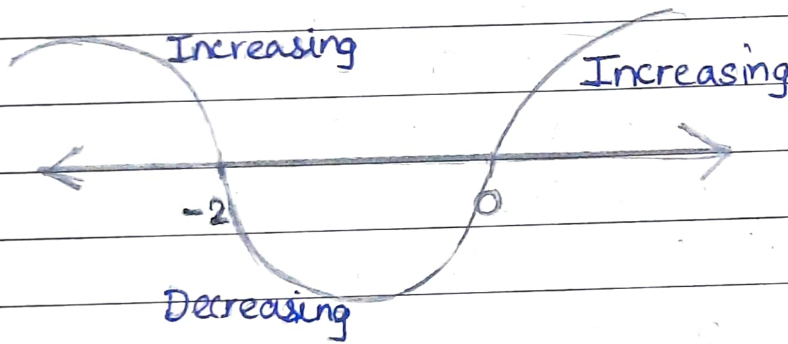
$f(x) = x^2(x+3)$

Solution

$$f(x) = x^2(x+3)$$
$$= x^3 + 3x^2$$

differentiate w.r.t x

$$f'(x) = 3x^2 + 6x$$
$$= 3x(x+2)$$



Question 16

$f(x) = 3x^2 - 6x$

Solution

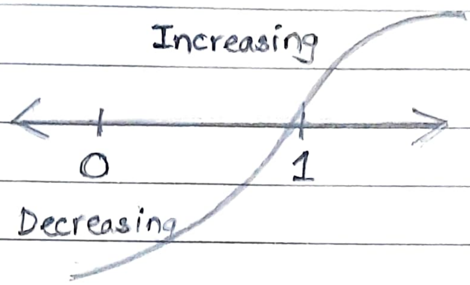
$f(x) = 3x^2 - 6x$

differentiate w.r.t x

$f'(x) = 6x - 6$

$= 6(x-1)$

Curve:



Equating $f'(x)$ to 0

$f'(x) = 0$

$6(x-1) = 0$

$x = 1$

differentiating $f'(x)$ w.r.t x

$f''(x) = 6 \cancel{>0}$ $\quad f''(1) = 6 > 0$

$\therefore$ The function is minimium at $x = 1$

Minima:

$f(1) = 3(1)^2 - 6(1) = 3 - 6 = \boxed{-3}$

Since $f''(x)$ cannot be less than 0 hence there is no maxima for this function.

Detailed graph :