

Financial Engineering - 2 - Assignment 1.

Q1.

Ans. 7 6m rate = 5%.

9m rate = 6%.

Rate locked using FRA = 7%.

$$\text{Implied forward rate} = \frac{(6\% \times 7.5\%) - (5\% \times 50\%)}{25\%}$$

$$= 8\%$$

The agreed FRA is 7%. Therefore, arbitrage profit can be made by borrowing for 6m at 5%, lending for 9m at 6% & entering into the FRA for 7% between 6-9m.

Q2.

Ans. Case A : The company is better off using a forward contract. If the company uses futures contract, it will have to settle the losses daily which may cause liquidity crunch.

* Case B : Using future contract would be a better hedging instrument. This is because the company will realise gains on a daily basis rather than at the end.

* Case C : For this case, the company is better off using a 'futures contract' as they will realise gains initially. Although they are being offset later, the initial gain paints a better picture of the hedging strategy.

← * Case D : The company should use forward contract as it will help them in avoiding the initial -ve cash flow settlement that would be if future contract was used.

Q3.

Ans. 7 * The selection of the delivery date will be one that is most preferable & beneficial to the company. This might not be beneficial to the bank & can cause losses to them. Thus, the bank has to consider multiple factors while pricing the product.

* If the bank assumes when the company will take delivery correctly, it will be able to price the product accordingly.

* The bank will also make profit if the company selects a suboptimal delivery date.

2.

Qu.

Ans.] To find the hedging strategy, first:

$$\text{Contract value} = 10,000 [100 - 0.25 (100 - 92)]$$

$$= 9,80,000.$$

$$\text{No. of contracts} = \frac{\text{portfolio forward value}}{\text{future contract price}} - \frac{\text{portfolio duration}}{\text{future duration}}$$

$$= \frac{48,20,000}{9,80,000} \times 2$$

$$= 9.84.$$

∴ An effective hedge would be to short 10 contracts.

Q.5.

Ans.] No. of short futures contract req:

$$= \frac{\text{portfolio forward price}}{\text{futures contract price}} \times \frac{\text{portfolio duration}}{\text{futures duration}}$$

$$\frac{100,000,000}{122,000} \times \frac{4}{9}$$

$$= 364.3$$

∴ 364 short contracts are required.

$$(a.) \text{ No. of contracts shorted} = \frac{100,000,000}{122,000} \times \frac{4}{7}$$

$$= 468.4$$

∴ 468 contracts will be required

(b) In this case, the bond portfolio could probably be more than the profit on shorter futures position.

Q6.

Ans. Per annum differences :

1.5% → yen

0.4% → dollars

$$\text{Total gain from swap} = (1.5 - 0.4)\%$$

$$= 1.1\% \text{ p.a.}$$

Bank profit : 0.5% per annum

$$\therefore 0.3\% \text{ for } X \text{ \& } Y.$$

$$X \text{ borrows dollar at } : 9.6 - 0.3 = 9.3\%$$

$$Y \text{ borrows yen at } : 6.5 - 0.3 = 6.2\%$$

3.

Q7.

Ans.] The swap rate is the fixed rate that a party receives in return for paying a variable rate.

At the same time, the swap rate for a particular maturity is the swap par yield for the maturity.

Q8.

Ans.] The bank must enter into int. rate swaps with other institutions. Here, the bank can pay fixed rate & receive floating rate.

Q9.

Ans.] Disc. rate:

12% p.a. with quarterly compounding

11.8% p.a. with continuous compounding.

$$\text{Next floating payment} = 11.8\% \times 100 \times 0.25 = 2.95$$

$$\text{Value of floating-rate bond} = 102.95 \cdot e^{-0.1182 \times 2/12} = 100.94$$

$$\begin{aligned} \text{Value of fixed-rate bond} &= 2.5e^{-11.8\% \times 1/6} + 2.5e^{-11.8\% \times 5/12} + \\ &+ 2.5e^{-11.8\% \times 8/12} + 2.5e^{-11.8\% \times 11/12} + 2.5e^{-11.8\% \times 14/12} \\ &= 98.678 \end{aligned}$$

$$\begin{aligned} \text{Value of swap} &= 100 - 94 - 98.678 \\ &= \$2.26 \text{ million} \end{aligned}$$

Q 10
 New 1 Forward rate : 8%
 Strike rate : 7.5%
 TTM : 4 yr.
 Rf rate : 8%
 Volatility : 25%

Pay-off from swaption = $\max [0.076 - F, 0]$

Value of annuity at the end of year 5, 6, 7, 8 & 9 = $\sum_{i=5}^9 e^{-0.078i}$

$$= 2.972$$

$$d_1 = \frac{\ln(0.08 / 0.076) + 0.25^2 \times 2}{0.25 \sqrt{4}} = 0.3526$$

$$d_2 = -0.1474$$

$$\text{Value of swaption} = 2.914 \left[0.076 \cdot \Phi(-d_2) - 0.08 \cdot \Phi(d_1) \right]$$

$$= 0.0401$$

$$= \$40,100$$

4

$$S_0 = 1/1.48 = 0.6757$$

$$K = 1/1.50 = 0.6667$$

$$r = 0.08$$

$$\sigma = 0.12$$

$$q = 0.04$$

$$T = 1$$

$$\text{Value of derivative} = 10,000 \cdot \Phi(-d_2) e^{-0.08 \times 1}$$

$$d_2 = \frac{\ln(0.6757 / 0.6667) + (0.08 - 0.04 - 0.12^2 / 2)}{0.12 \sqrt{1}} = 0.3852$$

$$\Phi(-d_2) = 0.3501$$

$$\text{Value of derivative} = 10,000 \times 0.3501 \times e^{-0.08} = 3231$$

$$\text{In dollars, } 3231 \times 1.48 = \$4782$$

Q.10.

Ans.] In an asian optⁿ the payoff becomes more certain as the time passes and the delta always approaches zero as the maturity date is approached. This makes delta hedging easy.

Q.13

Ans.] Value = $100 \Phi(-d_2) \cdot e^{-0.08 \times 0.5}$

$$d_2 = \frac{\ln(960/1000) + (0.08 - 0.03 + 0.2^2/2) \times 0.05}{0.2 \cdot \sqrt{0.5}}$$

$$= 0.1826$$

$$\Phi(-d_2) = 0.4276$$

$$\text{Value} = 1000 \times 0.4276 \times e^{-0.08 \times 0.5}$$

$$= \$ 4108$$