

HOMEWORK-1

Page :

Date :

①

Q1]

x	7	8	9	10
$f(x)$	3	1	1	9

$f(9.5) = 9$

$$\therefore f(9.5) = \frac{[9.5-8] \cdot (9.5-9) \cdot (9.5-10)(3)}{(7-8)(7-9)(7-10)}$$

$$+ \frac{(9.5-7)(9.5-9) \cdot (9.5-10)}{(1)(-1)(-2)} + \frac{(9.5-7)(9.5-8)(9.5-10)}{(2)(1)(-1)}$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(3)(2)(1)}$$

$$= \frac{3}{16} - \frac{5}{16} + \frac{9}{16} + \frac{45}{16}$$

$$= 3.625$$

Ans) 3.625

Q2)

x	3	7	9	10
$f(x)$	108	120	72	63

$f(6) = ?$

$$\therefore f(6) = \frac{(6-7)(6-9)(6-10)(10 \cdot 8)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)}$$

$$+ \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)}$$

$$= 12 + 180 + 27 - 72$$

$$= 147$$

Q3)

x	1	2	7	8
$f(x)$	4	5	5	4

$$f(6) = ?$$

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)}$$

$$+ \frac{(6-1)(6-2)(6-7)(4)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)}$$

$$= \frac{-16}{21} + \frac{5}{3} - \frac{40}{21} + \frac{20}{3}$$

$$= \underline{\underline{5.6667}}$$

Q4)

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)}$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)}$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}$$

$$\Rightarrow \frac{-2}{3} + \frac{32}{5} - \frac{24}{3} + \frac{128}{3} + \frac{128}{15}$$

$$\Rightarrow 32.9334$$

Q5)

x	10	20	14	16
$f(x)$	0.25	0.20	0.15	0.10

$$f(15) = \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)(0.20)}{(20-10)(20-14)(20-16)}$$

$$+ \frac{(15-10)(15-20)(15-16)(0.15)}{(24-10)(14-20)(14-16)} + \frac{(15-10)(15-20)(15-16)(0.10)}{(16-10)(16-20)(16-14)}$$

$$\Rightarrow \frac{-1}{12} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96} = 0.1208$$

Q2)
①

$$x^3 + x - 1 = 0 \quad \text{blw } 0.5 \text{ and } 1$$

$$\therefore y = x^3 + x - 1$$

x	y
0.5	-0.315
0.6	-0.184
0.7	0.043

$$\begin{array}{ll} x_1 = 0.6 & x_2 = 0.7 \\ y_1 = -0.184 & y_2 = 0.043 \end{array}$$

using,

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$x_n = \frac{(0.043)(0.6) + (0.7)(-0.184)}{0.043 + 0.184}$$

$$= 0.6810$$

$$\therefore n = -0.003042$$

$$\text{Now, } x_1 = 0.6810 \quad x_2 = 0.7$$

$$y_1 = -0.003042 \quad y_2 = 0.043$$

$$x_n = \frac{(0.6810)(0.043) + (0.7)(-0.003042)}{0.043 + 0.003042}$$

$$= \cancel{0.68} \quad 0.6823$$

$$y_n = -0.000045$$

$$\text{Root } y'x^3 + x - 1 = 0$$

Q2) $x^3 + 2x + 0.5 = 0$

blw 0 & 1

$$y = x^3 + 2x + 0.5$$

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073

$x_1 = 0.2$

$x_2 = 0.3$

$y_1 = 0.108$

$y_2 = -0.073$

$$x_n = \frac{(-0.073)(0.2) - (0.3)(0.108)}{-0.073 - 0.108}$$

$$= 0.259669$$

$y_n = -0.001824$

$x_1 = 0.2$

$x_2 = 0.259669$

$y_1 = 0.108$

$y_2 = -0.001829$

$$x_n = \frac{(-0.001829) - (0.259669)(0.108)}{-0.001829 - 0.108}$$

$$= 0.255675$$

$y_n = -0.000041$

Root of $x^3 + 2x + 0.5 = 0.258625$

Q4) $3x - 105x - 1 = 0$ blw 0 & 1

$y = 3x - 105x - 1$

x	y
0	-2
0.1	-1.69998
0.2	-1.39994
0.3	-1.09990
0.4	-0.79986
0.5	-0.49982
0.6	-0.19978
0.7	0.10002

Now,

$x_1 = 0.6$

$x_2 = 0.7$

$y_1 = 0.1999$

$y_2 = 0.10002$

$$x_n = \frac{(0.10007)(0.6) + (0.7)(0.1999)}{0.10007 + 0.1999}$$

$$= 0.6666$$

$$y_n = -0.0000003$$

$$\therefore \text{Root} = -0.666$$

Q5)

$$x^3 - 3x - 5 = 0 \quad \text{blw } 0 \text{ and } 2.5$$

$$\therefore y = x^3 - 3x - 5$$

x	y
2	-3
2.1	-2.039
2.2	-0.952
2.3	0.267
2.4	1.679

Now,

$$x_1 = 2.2 \quad x_2 = 0.3$$

$$y_1 = -0.952 \quad y_2 = 0.267$$

$$x_n = \frac{(0.267)(0.2) + (0.952)(2.3)}{0.267 + 0.952} = 2.27809$$

$$y_n = -0.01159$$

$$x_1 = 2.27809 \quad x_2 = 2.3$$

$$y_1 = -0.01159 \quad y_2 = 0.261$$

$$x'_n = \frac{(0.267)(\cancel{2.27809}) + (2.27809)(0.01159)}{(0.207) + (0.01159)}$$

$$= 2.27901$$

$$y'_n = 0.0001$$

$$\text{Root} = 2.27901$$

Q7) $x^3 - 4x + 1 = 0$

$$y = x^3 - 4x + 1$$

x	y
0	1
0.1	0.60
0.2	0.108
0.3	-0.173

Now,

$$x_1 = 0.2$$

$$y_1 = 0.208$$

$$x_2 = 0.3$$

$$y_2 = -0.173$$

$$x_n = \frac{(-0.173)(0.3) - (0.3)(0.208)}{-0.173 - 0.208}$$

$$= 0.254666$$

$$y_n = -0.00187$$

$$x_1 = 0.2 \quad y_1 = 0.208$$

$$x_2 = 0.2546 \quad y_2 = -0.00187$$

$$x'_n = \frac{(-0.00187)(0.2) - (0.208)(0.2546)}{-0.00187 - 0.208}$$

$$= 0.2541$$

$$y''x = -0.0002$$

$$\text{Root} = \underline{\underline{0.254}}$$

Q8) $x^3 + 2x^2 - 8 = 0$

$$\therefore y = x^3 + 2x^2 - 8$$

x	y
0	-8
1	-5
2	8

$$x_1 = 1 \quad x_2 = 2$$

$$y_1 = -5 \quad y_2 = 8$$

$$x_n = \frac{8(1) + 5(2)}{8 + 5}$$

$$= 1.3846$$

$$y_n = -1.512$$

Now,

$$x_1 = 1.3846 \quad y_1 = -1.5112$$

$$x_2 = 2 \quad y_2 = 8$$

$$x'_n = \frac{8(1.3846) + 2(-1.5112)}{8 + (-1.5112)}$$

$$= 1.4824$$

$$y'_1(x) = -0.3475$$

$$x_1 = 1.4824 \quad y_1 = -0.3475$$

$$x_2 = 2 \quad y_2 = 8$$

$$x''_n = \frac{8(1.4824) + 2(-0.3475)}{8 + (-0.3475)}$$

$$= 1.5039$$

$$x'''_n = \frac{8(1.5039) + 2(0.0747)(2)}{8 + 0.747} = 1.5085$$

$$y'''_n = -0.0158$$

$$x_1 = 1.5085 \quad y_1 = 0.1158$$

$$x_2 = 2 \quad y_2 = 2$$

$$x''''_n = \frac{8(1.5085) + (0.958)(2)}{8 + 0.0158} = 1.5095$$

$$y_n = -0.0033$$

$$x_n = \frac{8(1.5095) + (0.0033)(2)}{8 + 0.0033} = 1.5097$$

Root therefore becomes 1.5097.

Q9)

$$x^3 - 9x + 1 = 0$$

$$y = x^3 - 9x + 1$$

x	y
0	1
0.1	0.101
0.2	0.792

$$x_1 = 0.1 \quad x_2 = 0.2$$

$$y_1 = 0.101 \quad y_2 = -0.792$$

$$x_n = \frac{(-0.792)(0.1) - (0.2)(0.101)}{-0.792 - 0.101}$$

$$= 0.1113$$

$$y_n = 0.00041$$

$\therefore$ Root is 0.1113

Q10)

$$x^3 - x - 4 = 0$$

$$\therefore y = x^3 - x - 4 = 0$$

x	y
0	-4
1	-4
2	2

$$x_1 = 1$$

$$y_1 = -4$$

$$x_2 = 2$$

$$y_2 = 2$$

$$x_n = \frac{2(1) + 4(2)}{2 + 4} = 1.334$$

$$y_n = -2.9629$$

$$x_1 = 1.334$$

$$x_2 = 2$$

$$y_1 = -2.9629$$

$$y_2 = 2$$

$$x_n = \frac{2(1.334) + 2(2.9624)}{2 + 2.9624}$$

$$= 1.7319$$

$$y_n = -0.5415$$

$$\text{Now, } x_1 = 1.7313$$

$$x_2 = 2$$

$$y_1 = -0.5415$$

$$y_2 = 2$$

$$x_n = \frac{2(1.7313) + 2(0.5415)}{2 + 0.5415} = 1.2885$$

$$y_n = 0.6680$$

$$x_1 = 1.7885 \quad x_2 = 2$$

$$y_1 = 0.6680 \quad y_2 = 2$$

$$x_n = \frac{2(1.7885) + 2(0.6680)}{2 + 0.6680} = 1.7954$$

$$y_n = -0.0078$$

Now Again,

$$x_1 = 1.7954$$

$$x_2 = 2$$

$$y_1 = 0.0078$$

$$y_2 = 2$$

$$x_n = \frac{2(1.7954) + 2(0.0078)}{2 + 0.0078} = 1.7912$$

$$y_n = 0.0009$$

$$\text{Root} = 1.7962$$