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Date: __/__/__

MON TUE WED THR FRI SAT SUN

Probability & Statistics - II

Assignments - II

Ans 1. $\dfrac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} \sim N(0,1)$

$\hat{\lambda} = 200$

$P(Z_{2.5\%} < Z < Z_{97.5\%}) = 0.95$

$P(-1.96 < Z < 1.96) = 0.95$

$P\left(-1.96 < \dfrac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} < 1.96\right) = 0.95$

$P\left(-1.96\sqrt{\dfrac{\hat{\lambda}}{n}} - \hat{\lambda} < -\lambda < 1.96\sqrt{\dfrac{\hat{\lambda}}{n}} - \hat{\lambda}\right) = 0.95$

$P\left(\hat{\lambda} - 1.96\sqrt{\dfrac{\hat{\lambda}}{n}} < \lambda < \hat{\lambda} + 1.96\sqrt{\dfrac{\hat{\lambda}}{n}}\right) = 0.95$

95% CI for $\lambda = \left(200 - 1.96\sqrt{200},\ 200 + 1.96\sqrt{200}\right)$

$= (172.28141,\ 227.7185)$

Ans 2 (i) $\bar{X} = \dfrac{\Sigma x_i}{n}$

$E(\bar{X}) = E\left(\dfrac{\Sigma x_i}{n}\right) = \dfrac{1}{n} E(\Sigma x_i) = \dfrac{1}{n} \times n \times \mu = \mu$

$V(\bar{X}) = V\left(\dfrac{\Sigma x_i}{n}\right) = \dfrac{1}{n^2} V(\Sigma x_i) = \dfrac{n\sigma^2}{n^2} = \dfrac{\sigma^2}{n}$

→ Hence, proved.

Date.: __/__/____

MON TUE WED THR FRI SAT SUN

(ii) To prove :- $E(S^2) = \sigma^2$

$$S^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$$

$$E(S^2) = E\left(\frac{1}{n-1} \Sigma (x_i - \bar{x})^2\right) = \frac{1}{n-1} E\left[\Sigma (x_i - \bar{x})^2\right)$$

$$= \frac{1}{n-1} \left[E(x_i)^2 - n\, E(\bar{x}^2)\right]$$

$$= \frac{1}{n-1} \left[\Sigma(\mu^2 + \sigma^2) + n(\mu^2 + \sigma^2/n)\right]$$

$$= \frac{1}{n-1} (\cancel{n\mu^2} + n\sigma^2 - \cancel{n\mu^2} + \sigma^2)$$

$$= \frac{\sigma^2 (n-1)}{n-1} = \underline{\underline{\sigma^2}}$$

→ Hence, proved

(iii) $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$$V\left[\frac{(n-1)S^2}{\sigma^2}\right] = V(\chi^2_{n-1})$$

$$\frac{(n-1)^2\, V(S^2)}{\sigma^4} = 2(n-1)$$

$$V(S^2) = \frac{2\sigma^4}{n-1}$$

(iv) $x_i \sim N(\mu, \sigma^2)$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$\because \sigma^2 = 100,\ n = 10$

$\Rightarrow E(S^2) = \sigma^2 = 100$

$\rightarrow$ Value of S^2 can be plus or minus 50% of $E(S)$

$= \pm 50$

$\therefore P(S^2 \le 50)$

$$= P\left[\frac{(n-1)S^2}{\sigma^2} \le \frac{50 \times 9}{100}\right] = P(\chi^2_9 < 4.5)$$

$= \underline{\underline{0.1245}}$

$$\therefore P(S^2 \le 150) = P\left[\frac{(n-1)S^2}{\sigma^2} \le \frac{150 \times 9}{100}\right]$$

$= P\left[\chi^2_9 \le 13.5\right] = \underline{\underline{0.8587}}$

$\Rightarrow P(150 \le S^2 \le 50) = 0.8587 - 0.1245$

$= \underline{\underline{0.7342}}$

Ans 3.

No. of claims	No. of policies (O_i)	E_i
0	86	88.542
1	75	68.724
2	16	19.998
3	2	2.574
4	1	0.108

$$L(p) = \left[{}^4C_0\, p^0 (1-p)^4\right]^{86} \left[{}^4C_1\, p^1 (1-p)^3\right]^{75} \left[{}^4C_2\, p^2 (1-p)^2\right]^{16}$$

$$\left[{}^4C_3\, p^3 (1-p)^1\right]^{2} \left[{}^4C_4\, p^4 (1-p)^0\right]^{1}$$

Date.: __/__/___

MON TUE WED THR FRI SAT SUN

$$= (1-p^{344})\left[4^{75}\cdot p^{75}\cdot(1-p)^{225}\right]\left[6^{16}\cdot p^{32}\cdot(1-p)^{32}\right]$$

$$\left[4^{2}\cdot p^{6}\cdot(1-p)^{2}\right](p^{4})$$

$$= 4^{77}\cdot 6^{16}\cdot p^{117}\cdot(1-p)^{603}$$

$$\ln(L(p)) = C + 117\ln(p) + 603\ln(1-p)$$

$$\frac{d\ln(L(p))}{dp} = 0 + \frac{117}{p} - \frac{603}{1-p}$$

$$0 = \frac{117}{p} - \frac{603}{1-p}$$

$$117 - 117p - 603p = 0$$

$$117 - 720p = 0$$

$$720p = 117$$

$$\underline{\underline{p = 0.1625}}$$

Ans 5 (i) $X \sim U(a, b)$

$$E(X) = \bar{X}$$

$$\frac{a+b}{2} = \bar{X} \Rightarrow a+b = 2\bar{X} \Rightarrow \hat{b} = 2\bar{X} - \hat{a}$$

$$E(X-\bar{X})^2 = S^2$$

$$\frac{1}{12}(b-a)^2 = S^2$$

$$S = \sqrt{\frac{(b-a)^2}{12}} = \frac{\hat{b}-\hat{a}}{\sqrt{12}} = \frac{2\bar{X}-\hat{a}-\hat{a}}{\sqrt{12}}$$

$$= \frac{2\bar{X}-2\hat{a}}{2\sqrt{3}} = \frac{\bar{X}-\hat{a}}{\sqrt{3}}$$

$$\Rightarrow \hat{a} = \bar{X} - \sqrt{3}S$$ (Hence, proved)

Date.: __/__/__

MON TUE WED THR FRI SAT SUN

(ii) $\hat{b} = 2\bar{X} - \hat{a}$

$= 2\bar{X} - (\bar{X} - \sqrt{3}S)$

$= \underline{\underline{\bar{X} + \sqrt{3}S}}$

Ans 7. (i) $\bar{X}_{Public} = 30$

$\bar{X}_{Private} = 35.055$

$S^2_{Public} = \frac{\Sigma(X-\bar{X})^2}{n-1} = 64.6527$

$S^2_{Private} = \frac{\Sigma(X-\bar{X})^2}{n-1} = 67.4027$

(ii) For testing equal means, the primary condition that needs to be true is that the sample size is same. This condition is satisfied in the above example.

(iii) $H_0 = \bar{X}_{Private} - \bar{X}_{Public} = 0$

$H_1 = \bar{X}_{private} - \bar{X}_{public} > 0$

$Z_{tab} = (1.96, -1.96)$

$Z_{cal} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{S^2/n}} = \frac{35.055 - 30}{\sqrt{\frac{67.4027 + 64.6527}{9}}}$

$= \underline{\underline{1.3196}}$

Date.: __/__/____

MON TUE WED THR FRI SAT SUN

$\because Z_{tab} < Z_{cal} < Z_{tab}$

$\therefore$ Do not reject H_0

$\Rightarrow$ The mean of public hospital is not higher than private hospital.

Ans 8. (i.) Unbiased estimator $\rightarrow$ $E(\hat{\theta}) = \theta$

(ii) Bais $\rightarrow$ $E(\hat{\theta}) - \theta$

(iii) $MSE(\theta) = E[(\hat{\theta} - \theta)^2]$

$= E[((\hat{\theta} - E(\hat{\theta})) + (E(\hat{\theta}) - \theta))^2]$

$= E[(\hat{\theta} - E(\hat{\theta}))^2 + 2(E(\hat{\theta}) - \theta)(\hat{\theta} - E(\hat{\theta})) + (E(\hat{\theta}) - \theta))^2]$

$= Var(\hat{\theta}) + 0 + bais^2(\hat{\theta})$

$= Var(\hat{\theta}) + bais^2(\hat{\theta})$

Ans 9 (i.) $t_k = \dfrac{N(0,1)}{\sqrt{\chi^2_k / k}}$, where

$N(0,1)$ = Standard Normal

k = degrees of freedom

χ^2_k = Chi-square with k degrees of freedom.

(ii.) $E(t_k) = 0$

$V(t_k) = \dfrac{k}{k-2}$

Date.: __/__/__

MON TUE WED THR FRI SAT SUN

(iii) $n = 10$

$\bar{X} = 50$

$S^2 = 48.667$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} = t_{n-1}$$

$$99\% \text{ of } CI = \bar{X} \pm t_{\alpha/2, n-1} \sqrt{\frac{S^2}{n}}$$

$$= 50 \pm t_{0.5\%, 9} \sqrt{\frac{48.667}{10}}$$

$$= (43.00899, 56.991)$$

Ans 10. (i) Type 1 error is the probability of rejecting H_0 when in fact ~~when~~ it is true in above case.

Type 1 error (α) is probability of rejecting $\hat{\lambda} = 3$ but in fact it is true.

(ii) Type 2 error (β) is the probability of accepting H_0 when it is actually false.

(iii) The power of a test is the probability of rejecting H_0 when it is false.

It is given by $1-\beta$.

Ans 11 $\Sigma x = 2,13,200$

$\Sigma x^2 = 4,919,860,000$

$\bar{x} = 17,767$

$S = 10,144$

$n = 12$

Doubt
→ Q4, Q6, Q12

Date: __/__/____

MON TUE WED THR FRI SAT SUN

→ new $\Sigma x = 213,200 - 11,000 - 48,000 + 27,500 + 31,500$

$= 213,200 - 59,000 + 59,000$

$= 213,200$

→ new $\bar{x} = \dfrac{\text{new } \Sigma x}{n} = \dfrac{213200}{12} = 17,767$

→ new $\Sigma x^2 = 4,919,860,000 - (11,000)^2 - (48,000)^2 + (27,500)^2 + (31,500)^2$

$= 4,243,360,000$

→ $\bar{x}^2 = 315,666,289$

→ new $S^2 = \dfrac{\text{new } \Sigma x^2 - \text{new } \bar{x}^2}{n-1}$

$= \dfrac{4,243,360,000 - 315,666,289}{12-1}$

$= 357,063,065$

→ new $S = \sqrt{S^2} = \sqrt{357,063,065}$

$= 18,896$

★ Observation:-

→ We can see that by changing the numbers, the standard deviation has increased while the mean remains the same, which therefore indicates that the variability in the data has increased.