Uddhav Jadhav

Financial Mathematics

Roll number 91

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Q1.i) Solution :-

$$d^{(4)} = 0.08 \qquad \delta = ?$$

$$\delta = \ln(1+i)$$

$$\delta = -\ln\left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\delta = -\ln\left(1 - \frac{0.08}{4}\right)^4$$

$$\delta = -\ln(0.92236816)$$

$$\delta = 0.0808108293$$

$$\delta = 8.08108293\ \%$$

Q1.ii) $d^{(4)} = 0.08 \qquad i = ?$

$$\left(1 - \frac{d^{(p)}}{p}\right)^{-p} = 1+i$$

$$\left(1 - \frac{d^{(p)}}{p}\right)^{-p} - 1 = i$$

$$i = \left(1 - \frac{d^{(4)}}{4}\right)^{-4} - 1$$

$$\cancel{i = \left(1 - 0.04\right.} \cancel{\left(1 - \frac{0.08}{4}\right.} \left(\right.$$

$$i = \left(1 - \frac{0.08}{4}\right)^{-4} - 1$$

$$i = 1.0841657847 - 1$$

$$i = 0.0841657847$$

$$i = 8.41657847\ \%$$

Q1.iii) $d^{(4)} = 0.08 \quad d^{(12)} = ?$

$$\therefore \left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{0.08}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.92236816$$

$$\left(1 - \frac{d^{(12)}}{12}\right) = (0.92236816)^{\frac{1}{12}}$$

$$\left(1 - \frac{d^{(12)}}{12}\right) = 0.9932883884$$

$$\frac{d^{(12)}}{12} = 1 - 0.9932883884$$

$$d^{(12)} = 12 \times 0.0067116116$$

$$d^{(12)} = 0.0805393392$$

$$d^{(12)} = 8.05393392\,\%$$

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Q2.i) $PV = C \times V(0,10)$

$= 1000 \times e^{-\delta}$

$= 1000 \times e^{-\int_0^{10} \delta(t)\, dt}$

$= 1000 \times e^{-\int_0^{10} 0.04t + 0.000t^2\, dt}$

$= 1000 \times e \times \left[-\int_0^{10} (0.004t + 0.0002t^2)\, dt \right]$

$= \cancel{1000 \times e}$

$= 1000 \times e \times \left[-\left(\frac{0.004t^2}{2} + \frac{0.0002t^3}{3} \right]_0^{10}$

$= 1000 \times e \times \left[-\left(\frac{0.004 \times 100}{2} + \frac{0.002 \times 1000}{3} \right] \right.$

$= 1000 \times e \times (\cancel{0.0} - 0.2 \cancel{0.666} - 0.0666666667)$

$= 1000 \times e \times (-0.2666666667)$

$= \cancel{7248.755152}\ \cancel{1000 \times}\ 765.926$

Present value payable at the end of 10 years is 765.926

Q2.ii $(1+i)^{-10} = \cancel{0.765926}\ \cancel{7248.755152}$

$(1+i) = (7248.755152)^{\frac{1}{10}}$

$(1+i) = 0.4111248031$

$i = 1 - 0.4111248031$

$i = 0.5888751969$

~~Q2.iii~~

~~Q2.ii~~

ii) $(1+i)^{-10} = 0.765926$

$(1+i)^{10} = 1.305609$

$i = 1 - (1.305609)^{\frac{1}{10}}$

$i = 2.7025705\ \%$

Q3. i) d is the interest payable at beginning of year on a loan of 1 unit repayable at the end of year. The corresponding interest ~~rate~~ payable at the end of ~~year~~ year on 1 unit of loan is i. Thus, d must equal the present value at ~~beginning~~ at beginning of year of repayment i payable at the end of the year i.e $d = iv$

~~Q3.iia)~~ ~~$\frac{d^{(4)}}{4} = 0.0225 \quad d^{(2)} = ?$~~

~~$\therefore \left(1 - \frac{d^{(2)}}{2}\right)^{-2} = \left(1 + \frac{i^{(4)}}{4}\right)^{4}$~~

~~$\left(1 - \frac{d^{(2)}}{2}\right)^{-2} = (1.0225)^{4}$~~

~~$\left(1 - \frac{d^{(2)}}{2}\right)^{-2} = 1.0930833188$~~

~~$\left(1 - \frac{d^{(2)}}{2}\right) = (1.0930833188)^{-\frac{1}{2}}$~~

~~$\frac{d^{(2)}}{2} = 1 - 0.9564744352$~~

~~$d^{(2)} = 2 \times 0.0435255648$~~

~~$d^{(2)} = \cancel{0.087} \; 8.70511296\,\%$~~

Q3.iia) $\frac{d^{(4)}}{4} = 0.0225 \quad d^{(2)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{4} = \left(1 - \frac{d^{(2)}}{2}\right)^{2}$$

$$(1 - 0.0225)^{4} = \left(1 - \frac{d^{(2)}}{2}\right)^{2}$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^{2} = (0.9775)^{4}$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^{2} = 0.9129921938$$

$$d^{(2)} = 2 \times \left[1 - (0.9129921938)^{\frac{1}{2}}\right]$$

$$d^{(2)} = 2 \times (1 - 0.95550625)$$

$$d^{(2)} = 2 \times 0.04449375$$

$$d^{(2)} = \cancel{0.889875\,\%} \; \cancel{0.8} \; 8.89875\,\%$$

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Q3 iib) $1-d = (1 - 2 \times 0.05) = 0.9$

$d = \cancel{0.9}\ 0.1$

$\left(1 - \frac{d^{(2)}}{2}\right)^4 = 0.9$

$d^{(2)} = 2 \times \left[1 - (0.9)^{\frac{1}{4}}\right]$

$d^{(2)} = 5.19925072\ \%$

Q3-iii) $i = 0.05$

$(1+i)^{\frac{-91}{365}} = (1-nd)$

$(1.05)^{-\frac{91}{365}} = \left(1 - \frac{91}{365} \times d\right)$

$1 - 0.9879095608 = \frac{91}{365} \times d$

$d = \frac{365}{91} \times 0.0120904392$

$d = 0.0484946188$

$d = 4.84946188\ \%$

Q4.i) d is the interest ~~rate~~ payable at the beginning of the year on loan of 1 unit repayable at the end of the year.
The corresponding interest payable at the end of year on 1 unit of loan is i.
Thus, d must equal the present value at the beginning of the year i.e $d = iv$

Q4.ii c) $i = 0.08 \quad \delta = ?$

$\delta = \ln(1+i)$

$\delta = \ln(1.08)$

$\delta = 7.69610411\%$

Q4.ii b) $i^{(365)} = ? \quad i = 0.08$

$i^{(p)} = p[(1+i)^{\frac{1}{p}} - 1]$

$i^{(365)} = 365[(1.08)^{\frac{1}{365}} - 1]$

$i^{(365)} = 365 \times 0.0002108744$

$i^{(365)} = 7.6969156\%$

Q4.ii d) $i^{(\frac{1}{2})} = ? \quad i = 0.08$

$i^{(p)} = p[(1+i)^{\frac{1}{p}} - 1]$

$i^{(\frac{1}{2})} = \frac{1}{2}[(1.08)^{2} - 1]$

$i^{(\frac{1}{2})} = \frac{1}{2} \times \cancel{(1.66} \cancel{(16} (1.1664 - 1)$

$i^{(\frac{1}{2})} = \frac{1}{2} \times 0.1664$

$i^{(\frac{1}{2})} = 8.32\%$

Q4.ii a) $i = 0.08 \quad d^{(12)} = ?$

$d^{(p)} = p[1 - (1+i)^{-\frac{1}{p}}]$

$d^{(12)} = 12[1 - (1.08)^{-\frac{1}{12}}]$

$d^{(12)} = \cancel{12} \; 12 \times (1 - 0.99360710 2)$

$d^{(12)} = 12 \times 0.006392898$

$d^{(12)} = 0.076714776$

$d^{(12)} = 7.6714776\%$

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Q5.i a) $d^{(4)} = 0.06$ ~~$i^{+} = ?$~~ $i^{(2)} = ?$

~~$i^{(p)} = p\left[(1+i)^{-\frac{1}{p}} - 1\right]$~~

$$\left(1 - \frac{d^{(p)}}{p}\right)^{-p} = \left(1 + \frac{i^{(p)}}{p}\right)^{p}$$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(p)}}{p}\right)^{p}$$

$$\left(1 - \frac{0.06}{4}\right)^{-4} = \left(1 + \frac{i^{(p)}}{p}\right)^{p}$$

$$1.0623193154 = \left(1 + \frac{i^{(2)}}{2}\right)^{2}$$

$$(1.0623193154)^{\frac{1}{2}} = \left(1 + \frac{i^{(2)}}{2}\right)$$

$$1.0306887578 = 1 + \frac{i^{2}}{2}$$

$$\frac{i^{2}}{2} = 1.0306887578 - 1$$

$$i^{2} = 2 \times 0.0306887578$$

$$i^{2} = 0.0613775156$$

$$i^{2} = 6.13775156\,\%$$

Q5.i b) $d^{(4)} = 0.06$ $d^{(12)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{4} = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{0.06}{4}\right)^{4} = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = ~~1-~~\ 0.9413365506$$

$$d^{(12)} = 12\left[1 - (0.9413365506)^{\frac{1}{12}}\right] = 0.0603025248$$

$$d^{(12)} = 6.03025248\%$$

Q5. ii) ⓐ Original term of loan = 365 days
No deferment - settlement date is ~~actually~~ actual date of repayment 17^{th} July 2005
Outstanding term of loan = 272 days
Rebate allowed = $\frac{272}{365} \times 20000 = 14904.10959$

Sum paid to settle the loan.
$= 220000 - 14904.10959$
$= 205095.89041$

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Q6.i) Let retail price be 100

Cash price is 70

Credit price in six months is 75

∴ ~~Discount for retailers who pay using cash is 30 %~~

Discount for retailers who will pay using credit in six months

$$PV = AV \cancel{(1+} (1-d)^n$$

$$70 = 75 (1-d)^{\frac{1}{2}}$$

$$\left(\frac{70}{75}\right)^2 = (1-d)$$

$$d = 1 - 0.871111111$$

$$\therefore d = 12.8888889\ \%$$

Discount for retailers who pay using cash

$$PV = AV \cancel{(1+d)} (1-d)^n$$

$$70 = 100 (1-d)^1$$

$$\frac{70}{100} = (1-d)$$

$$d = 1 - 0.7 = 0.3$$

$$\therefore d = 30\%$$

Q6.ii) ~~d = 12.8888889~~ $d = 0.128888889\ \%$ $d^{(12)} = ?$

$$d^{(p)} = p\left[1 - (1-d)^{\frac{1}{p}}\right]$$

$$d^{(12)} = 12\left[1 - (1 - 0.128888889)^{\frac{1}{12}}\right]$$

$$d^{(12)} = 12 \times 0.0114329533$$

$$d^{(12)} = \cancel{0.0}\ 13.71954396\ \%$$

Q6.ii) $i^{(2)} = ?$ $d = 0.128888889$

$i^{(p)} = p[(1-d)^{-\frac{1}{p}} - 1]$

$i^{(2)} = 2[(1-0.128888889)^{\frac{-1}{p}} - 1]$

$i^{(2)} = 2[(0.871111111)^{-\frac{1}{2}} - 1]$

$i^{(2)} = 2(1.0714285715 - 1)$

$i^{(2)} = 2 \times 0.0714285715$

$i^{(2)} = \cancel{14.28 \ldots} \; 14.2857143\%$

Q6.ii) $d = 0.128888889$ $\delta = ?$

$\delta = \ln(1+i)$

$\delta = -\ln(1-d)$

$\delta = -\ln(1-0.128888889)$

$\delta = -\ln(0.871111111)$

$\delta = 13.79857431\%$

~~Q7.i) AV = PV x C x (1+i)^n~~

$$\cancel{26000 = 20000 \times \left(1 + \frac{i^{(4)}}{4}\right)}$$

~~Q7.i) AV = C x (~~

Q7.i) $AV = C \times \left(1 + \frac{i^{(4)}}{4}\right)^{4 \times \frac{17}{12}}$

$$26000 = 20000 \times \left(1 + \frac{i^{(4)}}{4}\right)^{4 \times \frac{17}{12}}$$

$$\left(\frac{26}{20}\right)^{\frac{3}{17}} = \left(1 + \frac{i^{(4)}}{4}\right)$$

$$i^{(4)} = 4 \times (1.0473881364 - 1)$$

$$i^{(4)} = 4 \times (0.0473881364)$$

$$i^{(4)} = 18.955254 56\,\%$$

Q7.ii) $PV = C \times \left(1 - \frac{d^{(2)}}{2}\right)^{2 \times \frac{17}{12}}$

$$\left(\frac{20000}{26000}\right)^{\frac{6}{17}} = \left(1 - \frac{d^{(2)}}{2}\right)$$

$$\left(1 - \frac{d^{(2)}}{2}\right) = \cancel{0.7811945375}\ 0.9115588234$$

$$d^{(2)} = \cancel{2 \times (1 - 0.7811945375)}\ 2 \times (1 - 0.9115588234)$$

$$d^{(2)} = \cancel{2 \times 0.}\ 2 \times 0.0884411766$$

$$d^{(2)} = 17.68823532\,\%$$

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Q7.iii) $AV = C \times (1+ni)$

$26000 = 20000 \times \left(1 + \frac{17}{12} i\right)$

$\frac{26\cancel{000}}{20\cancel{000}} = \left(1 + \frac{17}{12} \times i\right)$

$\frac{26}{20} - 1 = \frac{17}{12} \times i$

$\frac{6}{20} = \frac{17}{12} \times i$

$i = \frac{6}{20} \times \frac{12}{17}$

$i = 21.17647059\%$

compound interest = 20.34570672

Q7.iv) If using simple interest, the rate of interest needs to be higher when compared to compound interest in order to achieve the same overall amount.
Interest rate will get lower with higher frequency of compounding.

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Q8) ~~Rela~~ The relationship between i and $i^{(p)}$ can be represented in a formula as follows:-

$$\left(1+\frac{i^{(p)}}{p}\right) = (1+i)$$

Also when $p \to \infty$, the nominal rate of interest will ~~be~~ become convertible continuously and is known as force of interest.

& ~~Force~~ Calculating force of interest:

$e^{\delta} = (1+i) = 1.08$

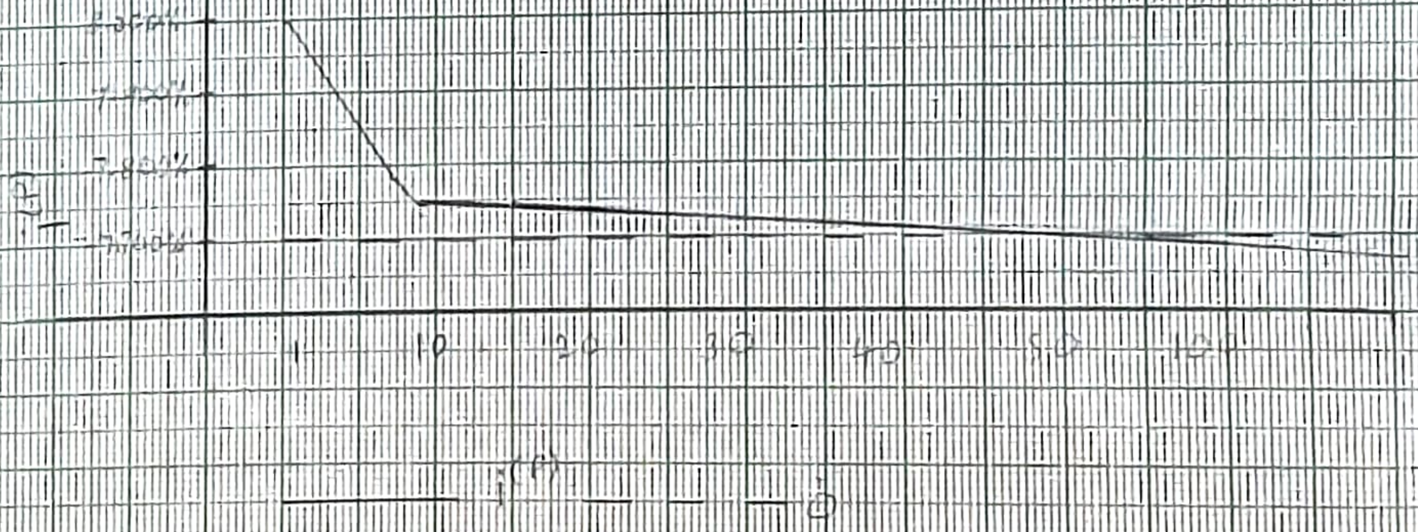
$\delta = \ln(1.08)$

$\delta = \cancel{7.967}\ 7.69610411\%$

The force of interest will be the lowest value which will be attained by $i^{(p)}$

The graph depicting relationship between $i^{(p)}$ and p will be :

Q8)



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Q9.i) The force of interest can be defined as the nominal rate of interest per unit time at time t convertible momently i.e for each value of t there is a number $\delta(t)$ such that limit $i_{h}t = \delta(t)$ where $\delta(t)$ is called force of interest per unit time at time t. $h \to 0+$

We may also define $\delta(t)$ directly in terms of accumulation factor as

$$\delta(t) = \lim_{h \to 0+} \left[\frac{A(t, t+h) - 1}{h} \right]$$

Q9.ii) $i^{(2)} = 0.0775$ $i = ?$

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^{p} - 1$$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^{2} - 1$$

$$i = \left(1 + \frac{0.0775}{2}\right)^{2} - 1$$

$$i = 7.90015625\%$$

Q9.iii) $i^{(\frac{1}{2})} = ?$ ~~i=0.7~~ ~~i=0.0~~ i 0.0790015625

$$i^{(\frac{1}{2})} = \frac{1}{2}\left[(1+i)^{2} - 1\right]$$

$$i^{(\frac{1}{2})} = \frac{1}{2}\left[(1.0790015625)^{2} - 1\right]$$

~~$i^{(\frac{1}{2})} = \frac{1}{2} \times 0.164244$~~$3719$ $i^{(\frac{1}{2})} = \frac{1}{2}(1.164243719 - 1)$

~~$i^{(\frac{1}{2})} = 8.2122186\%$~~ $i^{(\frac{1}{2})} = \frac{1}{2} \times 0.164243719$

$i^{(\frac{1}{2})} = 0.082122186$

$$i^{(\frac{1}{2})} = 8.2122186\%$$

Q9. ii) $d^{(12)} = ?$ $\quad i^{(2)} = 0.0775$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 + \frac{i^{(2)}}{2}\right)^{-2}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.0775}{2}\right)^{-2}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.9267827172$$

$$d^{(12)} = 12 \times \left[1 - (0.9267827172)^{\frac{1}{12}}\right]$$

$$d^{(12)} = 7.57957464\ \%$$

ii)

$$\cancel{i^{(2)} = 0.0775 \qquad i^{(\frac{1}{2})} = i^{0.5} = ?}$$

$$\cancel{i^{(\frac{1}{2})} = \left(\frac{1}{2}\right) \times \left[(1+i)^2 - 1\right]}$$

$$\cancel{i^{(\frac{1}{2})} = \frac{1}{2}\left[(1.0775)^2 - 1\right]}$$

$$\cancel{i^{(\frac{1}{2})} = 8.0503125}$$

Q11a)

$$PV = C \times (1 - nd)$$

$$PV = 50000 \times \left(1 - \frac{9}{12} \times 0.18\right)$$

$$PV = 50000 \times 0.865$$

$$PV = 43250$$

Amount lent ~~by~~ to the borrower is 43250

Q10 ans) $v(t) = e\left[-\int_0^t \delta(s)\,ds\right]$

$$\int_0^t \delta(s)\,ds = \begin{cases} 0.08t & \text{for } 0 \le t \le 5 \\ 0.1 + 0.06t & \text{for } 5 \le t \le 10 \\ \cancel{0.3}\ 0.4 & \text{for } t \ge 10 \end{cases}$$

$$\cancel{v(v t)}\ v(t) = \begin{cases} e(-0.08) & \text{for } 0 \le t \le 5 \\ e(-0.1 - 0.06t & \text{for } 5 \le t \le 10 \\ e(-0.4) & \text{for } t \ge 10 \end{cases}$$

Q11 bii)

~~$d^{(4)}$~~ $d^{(4)} = 0.06 \quad i^{(12)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$\left(1 - \frac{0.06}{4}\right)^{-4} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1.0623193154$$

$$i^{(12)} = 12\left[(1.0623193154)^{\frac{1}{12}} - 1\right]$$

$$i^{(12)} = 6.06070884\ \%$$

Q11 bii)

$d^{(4)} = 0.06 \quad i^{(4)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(4)}}{4}\right)^{4}$$

$$\left(1 - \frac{0.06}{4}\right)^{-4} = \left(1 + \frac{i^{(4)}}{4}\right)^{4}$$

$$1.0623193154 = \left(1 + \frac{i^{(4)}}{4}\right)^{4}$$

$$\left(1 + \frac{i^{(4)}}{4}\right) = (1.0623193154)^{\frac{1}{4}}$$

$$\left(1 + \frac{i^{(4)}}{4}\right) = 1.0152284264$$

$$\frac{i^{(4)}}{4} = \cancel{1.0152282 4} \; 1.0152284264 - 1$$

$$i^{(4)} = 4 \times 0.0152284264$$

$$i^{(4)} = 6.09137056\ \%$$

Q11bi) $(1+i) = e^{\delta} = 1.0618365465$

$\cancel{v = 0.9418}$ $v = 0.9417645336$

$d^{(12)} = 12(1 - v^{\frac{1}{12}})$

$d^{(12)} = 12[1 - (0.9417645336)^{\frac{1}{12}}]$

$d^{(12)} = 12 \times 0.0049875208$

$d^{(12)} = 5.98502496\%$

Q11 c) The order is determined based on when the interest is paid, so, for example d corresponds to interest paid immediately which requires a smaller payment amount. Similarly, i corresponds to interest paid at the end of time and δ corresponds to payment throughout.

$d < \delta < i^{(4)} < i$

Q11d) With 1 unit of capital, d is the interest payable at the beginning of the year. Similarly, with the same capital, i is the interest payable at the end of the year. So d must be equal to the present value at the beginning of year of payment i. ~~payable~~ payable at the end of the year i.e

$d = iv$

Q12 ans) $i^{(12)} = 0.06$, thus monthly effect is given by.

$$i = \frac{i^{(12)}}{12} = \frac{0.06}{12} = \cancel{0.5\%}\ 0.005 = 0.5\%$$

Accumulation factor for 2 years (working in months)

$= \cancel{(1+0.05)^{24}}\ (1+0.005)^{24} = 1.1271597762$

Amount accumulated after years $= 7945.3492624338$

Accumulation factor for next 2.5 years (working in quarters) $= (1+10\times0.005) = 1.15$

Accumulated amount after 4 and a half years is

9137.1516517989

$$V = 1-d = \left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.9416228069$$

$i = v^{-1} - 1 = 0.061996367$

Accumulation factor for next 1.5 years (working in years) $= (1+0.061996367)^{1.5} = 1.0944213246$

~~Accumulated amoun~~ ~~A~~ ~~Amount accum~~ Amount accumulated after 6 years =

$7049 \times 1.1271597762 \times 1.15 \times 1.0944213246$

$= 9999.8936138328$

≈ 10000

Q13) Let $C = 1$

$\therefore AV = 2$ .... {investment doubles in n years}

i) $AV = C \times (1+i)^n$

$2 = 1 \times (1 + 0.1)^{\cancel{n/365}}$

$2 = (1.1)^{\cancel{n/365}}$

$\ln 2 = \cancel{\frac{n}{365}} \ln(1.1)$

$n = \cancel{\frac{\ln 2}{\ln(1.1)}} \quad \frac{\ln 2}{\ln(1.1)} \times \cancel{365}$

$n = \cancel{7} \; \cancel{2654.4774275145} \; 7.272540897\cancel{3} \; \cancel{5}$ years

Q13-ii) $AV = C \times \left(1 - \frac{d^{(p)}}{p}\right)^{-p}$

$2 = 1 \times \left(1 - \frac{d^{(12)}}{12}\right)^{-12 \times n}$

$2 = \cancel{\left(1 - d\right.} \left(\left(1 - \frac{0.1}{12}\right)^{-12 \times n}\right.$

$\cancel{\tan} \ln 2 = -12 \times n \times \ln\left(1 - \frac{0.1}{12}\right)$

$\cancel{n = 6.902}$ $\cancel{n} = \ln 2 = \cancel{-120 n} \times n \times 0.100418996$

$0.6931471806 = n \times 0.100418996$

$n = 6.9025503959$ years