Calculus Assignment [Unit 1 & 2]

1. $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$= \lim_{h \to 0} \frac{2(h^2 + 9 - 6h) - 18}{h}$

$= \lim_{h \to 0} \frac{2h^2 + 18 - 12h - 18}{h}$

$= \lim_{h \to 0} \frac{h(2h - 12)}{h}$

$= -12$

2. $f(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$

a) $x = 4$

$\lim_{x \to 4^+} f(x) = 8$

$\lim_{x \to 4^-} f(x) = 8$

$f(4) = 8$

$\therefore$ Function is continuous at $x = 4$

b) $x = 6$

$\lim_{x \to 6^+} f(x) = 6 - 1 = 5$

$\lim_{x \to 6^-} f(x) = 2(6) = 12$

$f(6) = 5$

$\therefore$ Function is discontinuous at $x = 6$

3. $\lim\limits_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$= \lim\limits_{x \to 9} \dfrac{x-9}{3-\sqrt{x}} \times \dfrac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \lim\limits_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim\limits_{x \to 9} \dfrac{-(9-x)(3+\sqrt{x})}{(9-x)}$

$= \lim\limits_{x \to 9} -(3+\sqrt{x})$

$= -(3+\sqrt{9})$

$= -3+3 \quad OR \quad -3-3$

$= 0 \quad OR \quad -6$

4. $f(x) = \dfrac{4x+5}{9-3x}$

a) $x=-1$

$\lim\limits_{x \to (-1)} f(x) = \dfrac{-4+5}{9+3}$

$= \dfrac{1}{12}$

$f(-1) = \dfrac{1}{12}$ $\therefore$ function is continuous.

b) $x=0$

$\lim\limits_{x \to 0} f(x) = \dfrac{5}{9}$

~~lim~~ $f(0) = \dfrac{5}{9}$

$\therefore$ function is continuous

c) $x = 3$

$f(3) = \frac{17}{0}$; not a possible value, indeterminant value

$\lim_{x \to 3^+} f(x) = -\infty$

$\lim_{x \to 3^-} f(x) = \infty$

Hence function is discontinuous at $x = 3$. Infinite discontinuity

$$\boxed{\begin{array}{c} e^{\infty} = \infty \\ \hline e^{-\infty} = 0 \end{array}}$$

5. $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim_{x \to \infty} \cdot = \lim_{x \to \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$

$= \lim_{x \to \infty} \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}}$

$= \lim_{x \to \infty} \frac{6 - 0}{8 - 0 + 0}$

$= \frac{6}{8}$

$= 3/4$

6. $f(y) = \begin{cases} y^2 + 5 & y < -2 \\ 1 - 3y & y \geq -2 \end{cases}$

a) $\lim_{y \to 6^+} f(y) = -17$

$\lim_{y \to 6^-} f(y) = -17$

b) $\lim_{y \to -2^-} f(y) = 9$

$\lim_{y \to -2^+} f(y) = 7$

$\therefore$ It is discontinuous.

7. $f(t) = (4t^2 - t)(t^2 - 8t^2 + 12)$

$f'(t) = (8t - 1)(t^3 - 8t^2 + 12) + (3t^2 - 16t)(4t^2 - t)$
$= 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 3t^3 - 64t^3 + 16t^2$
$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$= \dfrac{(3 + 4w^3)(2w^2 + 1) - (4w)(3w + w^4)}{(2w^2 + 1)^2}$

$= \dfrac{6w^2 + 3 + 8w^5 + 4w^3 - 12w^2 - 4w^5}{4w^4 + 4w^2 + 1}$

$= \dfrac{4w^5 + 4w^3 - 6w^2 + 3}{4w^4 + 4w^2 + 1}$

9. $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log_e 8$

10. $y = z^5 - e^2 \ln(z)$

$\dfrac{dy}{dz} = 5z^4 - \left[e^2 \ln(z) + \dfrac{e^2}{2}\right]$

$= 5z^4 - e^2 \ln(z) - \dfrac{e^2}{2}$

11. a) $f(x) = (6x^2+7x)^4$

$= \cancel{4(6}$

$f'(x) = 4(6x^2+7x)^3 \cdot 12x+7$

$= 4(6x^2+7x)^3(12x+7)$

~~12.~~ b) $f(t) = 5+e^{4t+t^7}$

$f'(t) = e^{4t+t^7}(4+7t^6)$

12. a) $z = \ln(7-x^3)$

$z' = \frac{1}{7-x^3} \cdot -3x^2$

$z'' = \frac{(7-x^3)-6x + 3x^2(-3x^2)}{(7-x^3)^2}$

$= \frac{-42x+6x^4-9x^4}{(7-x^3)^2}$

$= \frac{-42x-3x^4}{(7-x^3)^2}$

b) $Q(v) = \frac{2}{(6+2v-v^2)^4}$

$Q'(v) = 2 \cdot \frac{d}{dv}(6+2v-v^2)^{-4}$

$= \frac{-8(2-2v)}{(6+2v-v^2)^5}$

$= \frac{-16(1-v)}{(6+2v-v^2)^5}$

$$g''(v) = -16 \frac{d}{dv}\left(\frac{1-v}{(6+2v-v^2)^5}\right)$$

$$= -16\left[\frac{-(6+2v-v^2)^5 - 5(6+2v-v^2)^4(2-2v)(1-v)}{(6+2v-v^2)^{5\times 2}}\right]$$

$$= -16\left[\frac{(6+2v-v^2)^4[-6-2v+v^2-5(2-2v)(1-v)]}{(6+2v-v^2)^{10}}\right]$$

$$= -16\left[\frac{-6-2v+v^2-5(2-2v-2v+2v^2)}{(6+2v-v^2)^6}\right]$$

$$= -16\left[\frac{-6-2v+v^2+20v-10v^2-10}{(6+2v-v^2)^6}\right]$$

$$= \frac{-16[-9v^2+18v-16]}{(6+2v-v^2)^6}$$

$$= \frac{16(9v^2-18v+16)}{(6+2v-v^2)^6}$$

c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{2t}{1+t^2}$$

$$f''(t) = \frac{2(1+t^2)-4t^2}{(1+t^2)^2}$$

$$= \frac{-2t^2+2}{(1+t^2)^2}$$

13. $f(x) = x^3 - 3x^2 - 9x + 12$

$$f'(x) = 3x^2 - 6x - 9$$

$$= 3(x^2-2x-3)$$

$$= 3(x^2+x-3x-3)$$

$$= 3[x(x+1)-3(x+1)]$$

$$= 3(x+1)(x-3)$$

$f'(x) = 0$

$x = -1$ or 3

$f''(x) = 6x - 6$

$\cancel{f''(x)}$ $f''(-1) = -12$

$f(x)$ is maximum at $x = -1$; $f(-1) = 17$

$f''(3) = 12$

$f(x)$ is minimum at $x = 3$; $f(3) = -15$

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x^2 - x - 2x + 2)$

$= 12[x(x-1) - 2(x-1)]$

$= 12(x-1)(x-2)$

$f'(x) = 0$

$x = 1$ or 2

$f''(x) = 24x - 36$

$f''(1) = -12$

$f(x)$ is maximum at $x = 1$; $f(1) = 3$

$f''(2) = 12$

$f(x)$ is minimum at $x = 2$; $f(2) = 1$

15. $f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

$= 3x(x+2)$

$f'(x) = 0$

$x = 0$ or -2

$f''(x) = 6x + 6$

$f''(0) = 6$; +ve value

Thus, 0 is the point of minima

$f(0) = 0 \Rightarrow (0, 0)$

$f''(-2) = -6$; -ve value

Thus, -2 is the point of maxima

$f(-2) = 4 \Rightarrow (-2, 4)$

Increase — Decrease — Increase

$-\infty$ — -2 — 0 — ∞

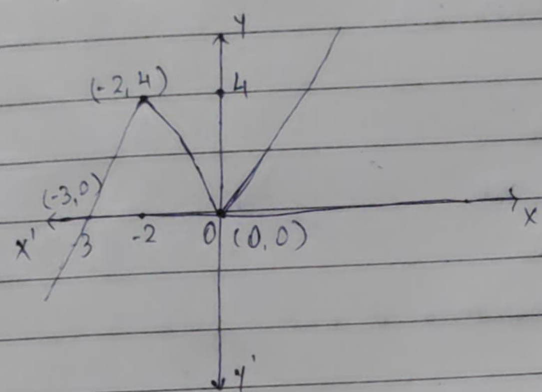
$f'(-3) = 9$; +ve

$f(x)$ is increasing in $(-\infty, -2)$ & $(0, \infty)$

$f(x)$ is decreasing in $(-2, 0)$

$\Rightarrow$ y intercept : $f(0) = 0 \Rightarrow (0, 0)$

x intercept : $f(-3) = 0 \Rightarrow (-3, 0)$



16. $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

$f'(x) = 0$

$x = 1$

$f''(x) = 6$; +ve

$f(x)$ is maximum at $x = 1$

$f(1) = -3 \Rightarrow (1, -3)$

$-\infty$ — 1 — ∞

$f'(0) = -6$

$f(x)$ is decreasing in $(-\infty, 1)$

$f'(2) = 6$

$f(x)$ is increasing in $(1, \infty)$

y intercept : $f(0) = 0 \Rightarrow (0,0)$

x intercept : $f(2) = 0 \Rightarrow (2,0)$