1) $d^{(4)} = 8\%$ p.a.

i) $\delta = ?$

$$e^{-\delta} = \left(1 - \frac{d^{(p)}}{p}\right)^{p}$$

$$e^{-\delta} = \left(1 - \frac{0.08}{4}\right)^{4}$$

$$e^{-\delta} = (1 - 0.02)^{4}$$

$$e^{-\delta} = (0.98)^{4}$$

$$e^{-\delta} = 0.922368160$$

$$-\delta =$$

Taking log on both sides

$$-\delta \ln e = \ln(0.922368160)$$

$$-\delta = -0.080810829$$

$$\delta = 0.080810829$$

$$\delta = 8.08108\%$$

ii) $i = ?$

~~2)~~

$$\left(1 - \frac{d^{(p)}}{p}\right)^{p} = \frac{1}{1+i}$$

$$1 + i = \left(1 - \frac{d^{(p)}}{p}\right)^{-p} \cancel{-4}$$

$$i = \cancel{1} \left(1 - \frac{d^{(p)}}{p}\right)^{-p} - 1$$

$$i = \cancel{1} (1 - 0.02)^{-4} - 1$$

$$i = \cancel{1} (0.98)^{-4} - 1$$

$$i = \cancel{1}\ 1.084165785 - 1$$

$$i = 0.084165785$$

$$i = 8.417\%$$

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iii) $d^{(12)} = ?$

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = (1-d)$$

$$\left(\left(1-\frac{d^{(4)}}{4}\right)^4\right)^{1/4} = \left(1-\frac{d^{(12)}}{12}\right)^{12/4}$$

$$\left(1-\frac{d^{(12)}}{12}\right)^3 = 1-\frac{d^{(4)}}{4}$$

$$1-\frac{d^{(12)}}{12} = \left(1-\frac{d^{(4)}}{4}\right)^{1/3}$$

$$\frac{d^{(12)}}{12} = 1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}$$

$$d^{(12)} = \left[1-(1-0.02)^{1/3}\right]\times 12$$

$$d^{(12)} = \left[1-(0.98)^{1/3}\right]\times 12$$

$$d^{(12)} = (1-0.993288388)\,12$$

$$d^{(12)} = 12\times(0.006711612)$$

$$d^{(12)} = 0.080539344$$

$$d^{(12)} = 8.0539\%$$

2) $\delta(t) = 0.004t + 0.0002t^2$

i) $AV_{10} = 1000$, $PV_0 = ?$

$$PV_0 = AV_{10} \times e^{-\int_0^t \delta(t)\,dt}$$

$$= AV_{10} \times e^{-\int_0^{10}(0.004t+0.0002t^2)\,dt}$$

$$= 1000 \times e^{-\left[\frac{0.004}{2}t^2 + \frac{0.0002}{3}t^3\right]_0^{10}}$$

$$= 1000 \times e^{-\left[0.002t^2 + 0.0000667t^3\right]_0^{10}}$$

$$= 1000 \times e^{-\left[0.002(10)^2 + 0.0000667(10)^3\right]}$$

$$= 1000 \times e^{-(0.2+0.0667)}$$

$$= 1000 \times e^{\cancel{0.2667}} \quad e^{-0.2667}$$

$$= 1000 \times \cancel{0.875202492}\ 0.765902808$$

~~$PV_0 = Rs\ 875.202$~~ $PV_0 = Rs.\ 765.903$

ii) $PV_0 = 875.202$, $AV_{10} = 1000$, $i = ?$

$AV_{10} = PV_0 \times \cancel{A} A(0,10)$

$1000 = \cancel{PV}\ \cancel{(875.202)\times} (1+i)^{10} \times (765.903)$

$(1+i)^{10} = 1000 \div \cancel{875.202}\ 765.903$

$(1+i)^{10} = \cancel{1.142593367}\ 1.305648365$

$1+i = \cancel{(1.142593367)^{1/10}}\ (1.305648365)^{1/10}$

$1+i = \cancel{1.013419297}\ 1.027028802$

$i = \cancel{1.013419297-1}\ 1.027028802-1$

$i = \cancel{0.013419297}\ 0.027028802$

$i = \cancel{1.3419\%}\ 2.703\%$

3) i) $d = iv$

We know that

$$\frac{1}{1+i} = 1-d$$

$1 = (1-d)(1+i)$

$1 = 1+i-d-di$

$1 = 1+i-d(1+i)$

~~$1 = (1+i)$~~

$d(1+i) = 1+i-1$

$$d = \frac{i}{1+i} \rightarrow (1)$$

We also that, discounting factor is-

$$v = \frac{1}{1+i} \rightarrow (2)$$

So, in eq. (1)

$$d = \frac{i}{1+i}$$

$$d = i \times \frac{1}{1+i}$$

Substituting eq. ② ~~is~~ in d

$$d = i \times v \quad \left[\text{where } v = \frac{1}{1+i}\right]$$

Hence, $d = iv$

ii) $d^{(2)} = ?$

a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = v$$

$$1 - \frac{d^{(2)}}{2} = v^{1/2}$$

$$1 - \frac{d^{(2)}}{2} = \left[\left(1 - \frac{d^{(4)}}{4}\right)^4\right]^{1/2}$$

$$1 - \frac{d^{(2)}}{2} = \left(1 - \frac{d^{(4)}}{4}\right)^2$$

$$\frac{d^{(2)}}{2} = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^2$$

$$d^{(2)} = 2\left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^2\right]$$

$$d^{(2)} = 2\left[1 - (1 - 0.0225)^2\right]$$

$$d^{(2)} = 2\left[1 - (0.9775)^2\right]$$

$$d^{(2)} = 2\left[1 - 0.955506250\right]$$

$$d^{(2)} = 2(0.044493750)$$

$$d^{(2)} = 8.8988\%$$

b) $d^{(1/2)} = 5\%$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = (1-d)$$

$$\left(1-\frac{d^{(2)}}{2}\right) = \left[\left(1-\frac{d^{(1/2)}}{1/2}\right)^{1/2}\right]^{1/2}$$

$$1-\frac{d^{(2)}}{2} = \left(1-2d^{(1/2)}\right)^{1/4}$$

$$\frac{d^{(2)}}{2} = 1-\left(1-2d^{(1/2)}\right)^{1/4}$$

$$d^{(2)} = 2\left[1-\left(1-2d^{(1/2)}\right)^{1/4}\right]$$

$$d^{(2)} = 2\left[1-(1-2(0.05))^{1/4}\right]$$

$$d^{(2)} = 2\left[1-(1-0.1)^{1/4}\right]$$

$$d^{(2)} = 2\left[1-(0.9)^{1/4}\right]$$

$$d^{(2)} = 2(1-0.974003746)$$

$$d^{(2)} = 2(0.025996254)$$

$$d^{(2)} = 0.051992508$$

$$d^{(2)} = 5.199\%$$

iii) $t = 91$ days, $i = 5\%$, d (S.D.) = ?

Let us assume that the bill gives a return of 100 after 91 days. So, its PV at time 0, will be,

$$AV_t = PV_0 (1+i)^t$$

$$100 = PV_0 (1+0.05)^{\frac{91}{365}}$$

$$PV_0 = 100 \div (1.05)^{91/365}$$

$$PV_0 = 100 \div 1.012238407$$

$$PV_0 = 98.79096$$

So, with the PV at time 0 of 98.7910, the annual simple discount rate will be,

$$PV_0 = AV_t \times \cancel{(1-d)^t}\,(1-td)$$

$$98.7910 = 100\left(1-\frac{91}{365}d\right)$$

$$1-\frac{91}{365}d = \frac{98.7910}{100}$$

$$\frac{91}{365}d = 1-0.987910$$

$$d = \frac{0.01209 \times 365}{91}$$

$$d = \frac{4.41285}{91}$$

$$d = 0.048492857$$

$$d = 4.849\,\%$$

4) i) $d = iv$

We Know that

$$\frac{1}{1+i} = 1-d$$

$$\frac{1}{1+i} = v \rightarrow (1)$$

$$\frac{1}{1+i} = 1-d$$

$$1 = (1-d)(1+i)$$

$$1 = 1-d-di+i$$

$$1 = 1+i\cancel{+1}-d(1+i)$$

$$d(1+i) = 1+i-1$$

$$d = \frac{i}{1+i}$$

$$d = i \times \frac{1}{1+i}$$

Substituting ① in d

$$d = i \times \frac{1}{1+i}$$

$$d = i \times v$$

Hence $d = iv$

ii) $i = 0.08$

a) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$$

$$1 - \frac{d^{(12)}}{12} = (1+i)^{-1/12}$$

$$\frac{d^{(12)}}{12} = 1 - (1+i)^{-1/12}$$

$$d^{(12)} = 12\left[1 - (1+i)^{-1/12}\right]$$

$$d^{(12)} = 12\left[1 - (1+0.08)^{-1/12}\right]$$

$$d^{(12)} = 12\left[1 - (1.08)^{-1/12}\right]$$

$$d^{(12)} = 12\,(1 - 0.993607102)$$

$$d^{(12)} = 12\,(0.006392898)$$

$$d^{(12)} = 0.076714776$$

$$d^{(12)} = 7.6714776\ \%$$

$$d^{(12)} = 7.671\ \%$$

b) $i^{(365)} = ?$

$$\left(1 + \frac{i^{(p)}}{p}\right)^{p} = 1+i$$

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$$\left(1+\frac{i^{(365)}}{365}\right)^{365} = 1+i$$

$$1+\frac{i^{(365)}}{365} = (1+i)^{1/365}$$

$$\frac{i^{(365)}}{365} = (1+i)^{1/365} - 1$$

$$i^{(365)} = 365\left[(1+i)^{1/365} - 1\right]$$

$$i^{(365)} = 365\left[(1+0.08)^{1/365} - 1\right]$$

$$i^{(365)} = 365\left[(1.08)^{1/365} - 1\right]$$

$$i^{(365)} = 365\left[1.000210874 - 1\right]$$

$$i^{(365)} = 365\,(0.000210874)$$

$$i^{(365)} = 0.07696 9010$$

$$i^{(365)} = 7.697\ \%$$

c) $\delta = ?$

$$\delta = \ln(1+i)$$

$$\delta = \ln(1+0.08)$$

$$\delta = \ln(1.08)$$

$$\delta = 0.076961041$$

$$\delta = 7.696\%$$

d) $i^{(0.5)} = ?$

$$\left(1+\frac{i^{(0.5)}}{0.5}\right)^{0.5} = 1+i$$

$$1+2i^{(0.5)} = (1+i)^2$$

$$2i^{(0.5)} = (1+i)^2 - 1$$

$$i^{(0.5)} = \frac{(1+i)^2 - 1}{2}$$

$$i^{(0.5)} = \frac{(1+0.08)^2 - 1}{2}$$

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$$i^{(0.5)} = \frac{(1.08)^2 - 1}{2}$$

$$i^{(0.5)} = \frac{1.1664 - 1}{2}$$

$$i^{(0.5)} = \frac{0.1664}{2}$$

$$i^{(0.5)} = 0.0832$$

$$i^{(0.5)} = 8.32\%$$

5) i) $d^{(4)} = 6\%$

a) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^{-2} = v$$

$$\left(1 + \frac{i^{(2)}}{2}\right) = v^{-1/2}$$

$$1 + \frac{i^{(2)}}{2} = \left(1 - \frac{d^{(4)}}{4}\right)^{4 \times \frac{-1}{2}}$$

$$\frac{i^{(2)}}{2} = \left(1 - \frac{d^{(4)}}{4}\right)^{-2} - 1$$

$$i^{(2)} = 2\left[\left(1 - \frac{d^{(4)}}{4}\right)^{-2} - 1\right]$$

$$i^{(2)} = 2\left[\left(1 - \frac{0.06}{4}\right)^{-2} - 1\right]$$

$$i^{(2)} = 2\left[(1 - 0.015)^{-2} - 1\right]$$

$$i^{(2)} = 2\left[(0.985)^{-2} - 1\right]$$

$$i^{(2)} = 2(1.030688758 - 1)$$

$$i^{(2)} = 2(0.030688758)$$

$$i^{(2)} = 0.061377516$$

$$i^{(2)} = 6.138\%$$

b) $d^{(12)} = ?$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^{4}$$

$$1-\frac{d^{(12)}}{12} = \left(1-\frac{d^{(4)}}{4}\right)^{1/3}$$

$$\frac{d^{(12)}}{12} = 1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}$$

$$d^{(12)} = 12\left[1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}\right]$$

$$d^{(12)} = 12\left[1-(1-0.015)^{1/3}\right]$$

$$d^{(12)} = 12\left[1-(0.985)^{1/3}\right]$$

$$d^{(12)} = 12(1-0.994974790)$$

$$d^{(12)} = 12(0.005025210)$$

$$d^{(12)} = 0.060302520$$

$$d^{(12)} = 6.0303\%$$

(i) $C = 2{,}00{,}000$, $AV_1 = 2{,}20{,}000$

a)

2,00,000	x	2,20,000
15/04/2005	17/07/2005	15/04/2006

Let us assume that he repaid the loan on 15/04/2006, then the rate of interest would be,

[The interest is reducing proportionately, hence, it must be coumpound interest]

$$AV = C \cdot A(t)$$

$$212000 = (2{,}00{,}000)(1+i)^{365}$$

$$(1+i)^{365} = \frac{2{,}20{,}000}{2{,}00{,}000}$$

$$1+i = \cancel{(1+1)^{365}} \quad (1.1)^{1/365}$$

$\cancel{i = 0.1}$ $\qquad i = 1.000261158 - 1$

$\cancel{i = 10\%}$ $\qquad i = 0.000261158$

$$i = 0.026\%$$

Amit repaid the loan after 93 days, instead of 1 year, So, the total amount paid by him will be,

$$AV = C.A(t)$$
$$AV = (2,00,000)(1+0.00026)^{93/365}$$
$$AV = (2,00,000)(1.00026)^{93/365}$$
$$AV = (2,00,000)(1.000066240)$$
$$AV = Rs.\ 200013.248$$

6) i) a) The retailers who make cash payment only have one payment. Hence, $d^{(1)} = d = 30\%$.

b) The retailers who make credit purchase, for six months, so, there are two payment periods. Hence $d^{(2)} = 25\%$.

So, $d = 1 - \left(1 - \frac{d^{(2)}}{2}\right)^2$

$$d = 1 - (1 - 0.125)^2$$
$$d = 1 - 0.765625$$
$$d = 0.234375$$
$$d = 23.44\%$$

ii) a) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - d$$

$$1 - \frac{d^{(12)}}{12} = (1-d)^{1/12}$$

$$\frac{d^{(12)}}{12} = 1 - (1-d)^{1/12}$$

$$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$
$$d^{(12)} = 12\left[1 - (1 - 0.2344)^{1/12}\right]$$
$$d^{(12)} = 12\left[1 - (0.7656)^{1/12}\right]$$

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$d^{(12)} = 12[1 - 0.977987927]$

$d^{(12)} = 12(0.022012073)$

$d^{(12)} = 0.264144876$

$d^{(12)} = 26.41\%$

b) $i^{(2)} = ?$

$\left(1 + \frac{i^{(2)}}{2}\right)^{-1} = 1 - d$

$1 + \frac{i^{(2)}}{2} = (1-d)^{-1}$

$\frac{i^{(2)}}{2} = (1-d)^{-1} - 1$

$i^{(2)} = 2[(1-d)^{-1} - 1]$

$i^{(2)} = 2[(1-0.2344)^{-1} - 1]$

$i^{(2)} = 2[(0.7656)^{-1} - 1]$

$i^{(2)} = 2[1.306165099 - 1]$

$i^{(2)} = 2(0.306165099)$

$i^{(2)} = 0.612330198$

$i^{(2)} = 61.23\%$

c) $\delta = ?$

$\delta = \ln(1-d)^{-1}$

$\delta = -\ln(1-d)$

$\delta = -\ln(1-0.2344)$

$\delta = -\ln(0.7656)$

$\delta = -(-0.267095439)$

$\delta = 26.71\%$

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7) $C = 20{,}000$, $AV_{17} = 26{,}000$

i) $i^{(4)} = ?$

$AV_t = C\,A(t)$

$26{,}000 = 20{,}000\,(1+i)^{\frac{17}{12}}$

$(1+i)^{\frac{17}{12}} = 26{,}000 \div 20{,}000$

$1+i = (1.3)^{12/17}$

$i = 1.203457064 - 1$

$i = 0.203457067$

$i = 20.3457$

$1+i = \left(1 + \frac{i^{(4)}}{4}\right)^4$

$1 + \frac{i^{(4)}}{4} = (1+i)^{1/4}$

$\frac{i^{(4)}}{4} = (1+i)^{1/4} - 1$

$i^{(4)} = 4\left[(1+i)^{4} - 1\right]$

$= 4\left[(1+0.20346)^4 - 1\right]$

$= 4\left[(1.20346)^4 - 1\right]$

$= 4\,(2.097619154 - 1)$

$= 4\,(1.097619154)$

$= 4.390476616$

$= 4.39.$

$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$i^{(4)} = 4\left[(1+0.20346)^{1/4} - 1\right]$

$i^{(4)} = 4\left[(1.20346)^{1/4} - 1\right]$

$i^{(4)} = 4\,(1.047388775 - 1)$

$i^{(4)} = 4\,(0.047388775)$

$i^{(4)} = 0.1895551$

$i^{(4)} = 18.96\ \%$

ii) $d^{(2)} = ?$

$PV_0 = AV_{19} \times V(17)$

$20{,}000 = 26{,}000\,(1-d)^{17/12}$

$(1-d)^{17/12} = 20{,}000 \div 26{,}000$

$1-d = (0.769230769)^{12/17}$ [$\Rightarrow d = 0.16906 = 16.91\%$]

$\left(1-\frac{d^{(2)}}{2}\right)^2 = 0.830939488$

$1-\frac{d^{(2)}}{2} = (0.830939488)^{1/2}$

$\frac{d^{(2)}}{2} = 1 - 0.911558823$

$d^{(2)} = 2(0.088441177)$

$d^{(2)} = 0.176882354$

$d^{(2)} = 17.69\%$

iii) S.I. $= ?$

$AV_{19} = C \times A(17)$

$26{,}000 = 20{,}000 \times \left(1 + \frac{17}{12}i\right)$

$1 + \frac{17}{12}i = \frac{26{,}000}{20{,}000}$

$1 + \frac{17}{12}i = 1.3$

$\frac{17}{12}i = 1.3 - 1$

$i = \frac{0.3 \times 12}{17}$

(S.I.) $i = 0.211764706$

(S.I.) $i = 21.18\%$

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iv) a) When we compare the compound interest rate with the simple interest rate, we see that Simple Interest is higher.

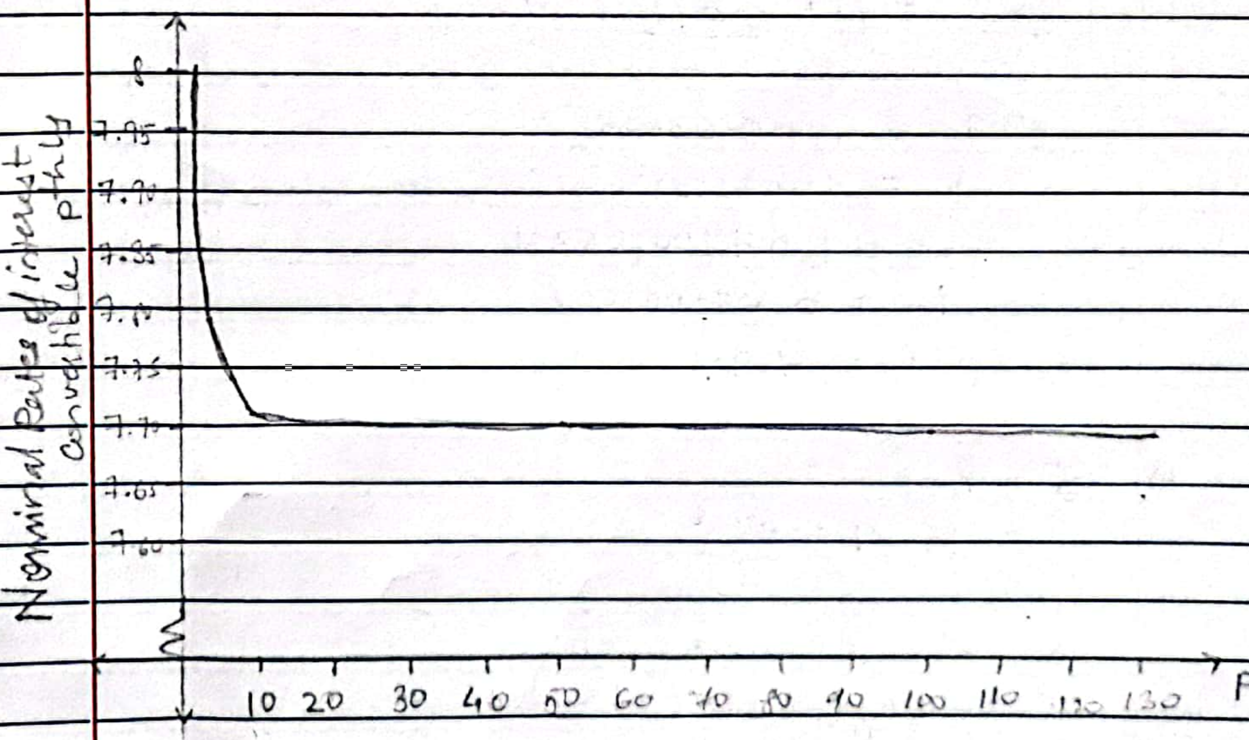
~~b) When com~~ $20.346\%\,(C.I.) < 21.18\%\,(S.I.)$

b) ~~When~~ We ~~can~~ also have the following observation,

$16.91\% < 17.69\% < 18.96\% < 20.346\%$, i.e.,

$$d < d^{(2)} < i^{(4)} < i$$

8) $i = 8\%$ p.a.



As we can see from the graph, as the value of p increases, the value of the nominal interest rate ~~also~~ decreases, and eventually it limits itself. Hence, equivalent nominal rates of interest convertible pthly are inversely related to p.

9) i) Force of interest (δ) is the nominal rate of interest per unit time convertible continuously. This is also referred to as the rate continuously compounded.

ii) $i^{(2)} = 0.0775$

a) $i = ?$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i$$

$$i = \left(1+\frac{i^{(2)}}{2}\right)^2 - 1$$

$$i = \left(1+\frac{0.0775}{2}\right)^2 - 1$$

$$i = (1+0.03875)^2 - 1$$

$$i = (1.03875)^2 - 1$$

$$i = 1.079001563 - 1$$

$$i = 0.079001563$$

$$i = 7.9\%$$

b) $\delta = ?$

$$\delta = \ln\left(1+\frac{i^{(2)}}{2}\right)^2$$

$$\delta = 2\ln(1+0.03875)$$

$$\delta = 2\ln(1.03875)$$

$$\delta = 2(0.038018067)$$

$$\delta = 0.076036134$$

$$\delta = 7.6\%$$

c) $d^{(12)} = ?$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1+\frac{i^{(2)}}{2}\right)^{-2}$$

$$1-\frac{d^{(12)}}{12} = \left(1+\frac{i^{(2)}}{2}\right)^{-2/12}$$

$$\frac{d^{(12)}}{12} = 1-\left(1+\frac{i^{(2)}}{2}\right)^{-1/6}$$

$$d^{(12)} = 12\left[1-\left(1+\frac{i^{(2)}}{2}\right)^{-1/6}\right]$$

$$d^{(12)} = 12\left[1-(1.03875)^{-1/6}\right]$$

$$d^{(12)} = 12\left[1-0.9936836888\right]$$

$$d^{(12)} = 12\,(0.006316312)$$

$$d^{(12)} = 0.075795744$$

$$d^{(12)} = 7.58\%$$

d) $i^{(1/2)} = ?$

$$\left(1+\frac{i^{(2)}}{2}\right)^{2} \quad {}^{2}\left(1+\frac{i^{(1/2)}}{1/2}\right)^{1/2}$$

$$1+2i^{(1/2)} = \left(1+\frac{i^{(2)}}{2}\right)^{4}$$

$$2i^{(1/2)} = \left(1+\frac{i^{(2)}}{2}\right)^{4}-1$$

$$i^{(1/2)} = \frac{1}{2}\left[\left(1+\frac{i^{(2)}}{2}\right)^{4}-1\right]$$

$$i^{(1/2)} = \frac{1}{2}\left[(1.03875)^{4}-1\right]$$

$$i^{(1/2)} = 1/2\,(1.164244372-1)$$

$$i^{(1/2)} = 1/2\,(0.164244372)$$

$$i^{(1/2)} = 0.082122186$$

$$i^{(1/2)} = 8.21\%$$

10)

$$\delta(t) = \begin{cases} 0.08, & 0 \leq t \leq 5 \\ 0.06, & 5 \leq t \leq 10 \\ 0.04, & t \geq 10 \end{cases}$$

$$v(t) = e^{-\int_{t_1}^{t_2} \delta(t)\,dt}$$

$$v(5) = e^{-\int_0^5 0.08\,dt}$$

$$v(5) = e^{-[0.08t]_0^5}$$

$$v(5) = e^{-0.08(5)}$$

$$v(5) = e^{-0.4}$$

$v(t)$, $t \leq 5$

$$v(t) = e^{-\int_0^t 0.08\,ds}$$

$$v(t) = e^{-[0.08s]_0^t}$$

$$v(t) = e^{-0.08t}$$

$v(t)$, $5 \leq t \leq 10$

$$v(t) = e^{-\int_5^t 0.06\,ds} \times e^{-0.08t} \quad v(0,5)$$

$$v(t) = e^{-[0.06s]_5^t} \times e^{-0.08t}$$

$$v(t) = e^{-0.06t+0.06(5)} \times e^{-0.08t}$$

$$v(t) = e^{-0.14t+0.3}$$

$v(t)$, $t \geq 10$

$$v(t) = e^{-\int_{10}^t 0.04\,ds} \times v(0,10)$$

$$v(t) = e^{-[0.04s]_{10}^t} \times e^{-0.14t+0.3}$$

$$v(t) = e^{+0.4-0.04t} \times e^{-0.14t+0.3}$$

$$v(t) = e^{-0.18t+0.7}$$

$$v(t) = \begin{cases} e^{-0.08t}, & 0 \leq t \leq 5 \\ e^{-0.14t+0.3}, & 5 \leq t \leq 10 \\ e^{-0.18t+0.7}, & t \geq 10 \end{cases}$$

11) a) $t = 9$ month , $AV_9 = 50,000$, $d^{(12)} = 18\%$

$PV = ?$

$PV_0 = AV_t \cdot v(t)$

$PV_0 = (50,000) \times (1-0.18)^{9/12}$

$PV_0 = (50,000)(0.82)^{3/4}$

$PV_0 = (50,000)(0.861708525)$

$PV_0 = 43,085.42625$

b) i) $\delta = 6\%$, $d^{(12)} = ?$

$e^{-\delta} = 1-d$

$e^{-\delta} = \left(1-\frac{d^{(12)}}{12}\right)^{12}$

$e^{-\delta/12} = 1-\frac{d^{(12)}}{12}$

$\frac{d^{(12)}}{12} = 1-e^{-\delta/12}$

$d^{(12)} = 12\left[1-e^{-\delta/12}\right]$

$d^{(12)} = 12\left[1-e^{-\frac{0.06}{12}}\right]$

$d^{(12)} = 12\left[1-e^{-0.005}\right]$

$d^{(12)} = 12(1-0.995012479)$

$d^{(12)} = 12(0.004987521)$

$d^{(12)} = 0.059850252$

$d^{(12)} = 5.99\%$

ii) $4d = 6\%$, $4i^{(12)} = ?$

$d = 0.015$

$(1-d)^{-4} = \left(1+\frac{i^{(12)}}{12}\right)^{12}$

$1+\frac{i^{(12)}}{12} = (1-d)^{-1/12}$

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$$\frac{i^{(12)}}{12} = (1-d)^{-1/12} - 1$$

$$i^{(12)} = 12\left[(1-d)^{-1/12} - 1\right]$$

$$4\,i^{(12)} = 48\left[(1-d)^{-1/12} - 1\right]$$

$$4\,i^{(12)} = 48\left[(1-0.015)^{-1/2} - 1\right]$$

$$4\,i^{(12)} = 48\left[(0.985)^{-1/2} - 1\right]$$

$$4\,i^{(12)} = 48(1.007585444 - 1)$$

$$4\,i^{(12)} = 48(0.007585444)$$

$$4i^{(12)} = 0.36410312$$

$$4i^{(12)} = 36.41\%.$$

c) $d, \delta, i^{(4)}, i$

We know that $\lim_{p\to\infty} i^{(p)} = \delta$

and $\lim_{p\to\infty} d^{(p)} = \delta$

$d^{(p)}$ tends to this limit from below whereas $i^{(p)}$ tends to this limit from above.

Hence, $d < \delta < i^{(4)} < i$.

d) $d = iv$

We know that

$$1-d = \frac{1}{1+i}$$

$$\frac{1}{1+i} = v \rightarrow (1)$$

$$1-d = \frac{1}{1+i}$$

$$(1-d)(1+i) = 1$$

$$1+i-d-di = 1$$

$$\therefore d(1+i) = 1+i-1$$

$$d = \frac{i}{1+i}$$

d

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$$d = i \times \frac{1}{1+i}$$

Substituting eq-(1) in 'd'

$$d = i \times v$$

Hence, $d = iv$

12) 7049

$0 \quad i^{(12)} = 6\% \quad 2 \quad \frac{i^{(4)}}{4} = 1.5\% \quad 4.5 \quad d^{(12)} = 6\% \quad 6$

$PV_0 = 7049$

$AV_6 = PV_0 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$

$\cancel{A(0,2) = (1+i)^2}$

$\cancel{= (1+0.06)^2}$

$\cancel{= (1.06)^2}$

$\cancel{A(0,2) = 1.1236}$

$A(2,4.5) = (1+2.5\,i)$

$= 1+(2.5)(0.015)(4)$

$= 1+0.15$

$A(2,4.5) = 1.15$

$$A(4.5,6) = \left(1-\frac{d^{(12)}}{12}\right)^{-12}$$

$$= \left(1-\frac{0.06}{12}\right)^{-12}$$

$$= (1-0.005)^{-12}$$

$$= (0.995)^{-12}$$

$A(4.5,6) = 1.061996367$

$$A(0,2) = \left(1+\frac{i^{(12)}}{12}\right)^{12}$$

$$= (1 + 0.005)^{12}$$

$$= (1.005)^{12}$$

$$A(0,2) = 1.061677812$$

$$AV_6 = (7049)(1.061677812)(1.15)(1.061996367)$$

$$AV_6 = 9139.893244$$

13) $PV_0 = x$, $AV_t = 2x$

i) $i = 10\%$

$$AV_t = PV_0 \cdot (1+i)^t$$

$$2x = x(1+0.1)^t$$

$$(1.1)^t = \cancel{x}\, 2$$

Taking log on both sides

$$\ln (1.1)^t = \ln \cancel{x}\, 2$$

$$t \ln(1.1) = \ln \cancel{x} 2$$

$$t = \frac{\ln \cancel{x} 2}{\ln(1.1)}$$

$$t = \frac{0.693147181}{0.09531018}$$

$$t = 7.272540887$$

$$t \approx 7.27 \text{ years}$$

$$t \approx 7 \text{ years } 4 \text{ month}$$

ii) $d^{(12)} = 10\%$

$$PV_0 = AV_t (1-d)^t$$

$$PV_0 = AV_t \left(1 - \frac{d^{(12)}}{12}\right)^{12t}$$

$$x = 2x\left(\frac{12 - 0.1}{12}\right)^{12t}$$

$$\left(\frac{11.9}{12}\right)^{12t} = \frac{1}{2}$$

Taking log on bothsides

$$12t \ln\left(\frac{11.9}{12}\right) = \ln\frac{1}{2}$$

$$12t\,(\ln 11.9 - \ln 12) = \ln 1 - \ln 2$$

$$12t = \frac{0 - 0.693147181}{2.4765384 - 2.48490650}$$

$$12t = \frac{-0.693147181}{-0.00836825}$$

$$t = \frac{82.83060150}{12}$$

$$t = 6.902550125$$

$$t \approx 6.9 \text{ years}$$

$$t \approx 7 \text{ years}$$