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Probability and Statistics-1
Assignment-2 (Unit 3 and 4)

Ques 10: $f(x,y) = k x^{-\alpha} e^{-y/\beta}$

$1 < x < \infty$; $1 < y < \infty$

$\alpha > 1$; $\beta > 0$

$$\Rightarrow k \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/\beta} \, dx\, dy = 1.$$

$$\Rightarrow k \int_1^\infty e^{-y/\beta} \left[\frac{x^{-(\alpha-1)}}{-(\alpha-1)} \right]_1^\infty dy = 1$$

$$\Rightarrow \frac{k}{1-\alpha} \int_1^\infty e^{-y/\beta} (-1)\, dy = 1$$

$$\Rightarrow -\frac{k}{1-\alpha} \int_1^\infty e^{-y/\beta} \, dy = 1$$

$$\Rightarrow -\frac{k}{1-\alpha} \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^\infty = 1$$

$$= \frac{-k\beta}{1-\alpha} \left(e^{-\infty} - e^{-y/\beta} \right) = 1$$

$$\frac{-k\beta}{1-\alpha} (-0 + e^{-y/\beta}) = 1$$

$$\boxed{k = \frac{\alpha - 1}{\beta (e^{-y/\beta})}}$$

~~Hence proved~~.

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Que 9: X $\quad E(X) = 3 \quad V(X) = 4.$

$E(Y) = 4 \quad V(Y) = 1$

$$\rho = -0.3 = \frac{\text{cov}(X,Y)}{\sqrt{V(X) \cdot V(Y)}} = \frac{\text{cov}(X,Y)}{\sqrt{(4)(1)}}$$

$Z = \cancel{X} + Y.$ $\quad \boxed{\text{cov}(X,Y) = -0.6}$

① $\cancel{\text{cov}(X,Z) = \text{Var}(X,Z,X,Z)}$

$\cancel{= \text{Var}(X,X) + \text{Var}(X,Z)}$

$\text{cov}(X,Z) = \text{cov}(X, X+Y)$

$= \text{cov}(X,X) + \text{cov}(X,Y)$

$= V(X) + \text{cov}(X,Y).$

$= 4 + (-0.6)$

$= 4 - 0.6$

$\boxed{\text{cov}(X,Z) = 3.4}$

② $\text{Var}(Z) = \text{Var}(X+Y) = \text{cov}(X+Y, X+Y)$

$= \text{cov}(X,X) + \text{cov}(X,Y) + \cancel{\text{cov}}(Y,Y) + \text{cov}(Y,X)$

$= V(X) + V(Y) + 2\,\text{cov}(X,Y)$

$= 4 + 1 + (-0.6)$

$= 5 - 0.6$

$\boxed{V(Z) = 4.4}$

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Ans 8: $X \sim Gamma(3, 2)$; $E(X) = 3/2$ $V(X) = 3/4$.

$E(Y/X = x) = 3x + 1$ $Var(Y/X = x) = 2x^2 + 5$

$E(Y) = E[E(Y|X=x)]$

$= E[3x + 1] = E(3x) + E(1)$

$= 3E(X) + 1$

$= \cancel{3}\left(\frac{3}{\cancel{2}}\right) + 1 = 3 + 1$

$\boxed{E(Y) = 4}$

EVE VE:

$V(Y) = E(V(X/Y)) + V(E(X/Y))$

$= E(2x^2 + 5) + V(3x + 1)$

$= E(2X^2) + E(5) + V(3X) + V(1)$

$= 2E(X^2) + 5 + 9V(X) + 0$.

$V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = V(X) + (E(X))^2$

$= \frac{3}{4} + \left(\frac{3}{2}\right)^2 = \frac{3}{4} + \frac{9}{4} = \frac{\cancel{12}\,3}{\cancel{4}\,1}$

$\boxed{E(X^2) = \cancel{12/4}\,3}$

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$V(y) = \cancel{2}\left(\frac{13}{4}\right) + 5 + 9\left(\frac{3}{4}\right)$

$= \frac{26 + 20 + 27}{4}$

$V(y) = \frac{73}{4} \quad 18.25$

$V(y) = 2(3) + 5 + 9(3/4)$

$= 6 + 5 + 27/4 = 11 + 27/4$

$= \frac{44 + 27}{4}$

$\boxed{V(y) = \frac{71}{4}}$ or $\boxed{V(y) = 17.75}$

$S.D\ y = \sqrt{71/4} = \frac{\sqrt{71}}{2}$

$\boxed{S.D.\ y = \frac{\sqrt{71}}{2}}$

Ques 7:

		X			
		1	2	3	4
	1	0.2	0	0.05	0.15
Y	2	0	0.3	0.1	0.2

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(c) $V(X/Y=2) = E(X^2/Y=2) - [E(X/Y=2)]^2$

$$E[X^2/Y=2] = \sum_{x=0}^{4} x^2 \frac{P(X=x; y=2)}{P(y=2)}$$

$$= \frac{(1)^2(0)}{0.6} + \frac{(2)^2(0.3)}{0.6} + \frac{(3)^2(0.1)}{0.6} + \frac{(4)^2(0.2)}{0.6}$$

$$= 0 + 2 + \frac{3}{2} + \frac{16}{3}$$

$$\boxed{E(X^2/Y=2) = 8.8333}$$

$$E[X/Y=2] = \sum_{i=1}^{4} x \frac{P(X=x, y=2)}{P(y=2)}$$

$$= \frac{1(0)}{0.6} + \frac{(2)(0.3)}{0.6} + \frac{(3)(0.1)}{0.6} + \frac{(4)(0.2)}{(0.6)}$$

0.2

$$= 0 + 1 + \frac{1}{2} + \frac{4}{3}$$

$$\boxed{E[X/Y=2] = 2.8333}$$

$$V(X/Y=2) = 8.83333 - (2.8333)^2$$

$$\boxed{V(X/Y=2) = 0.807}$$

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(ii) $f(U,V) = \frac{48}{67}(2uV - U^2)$ $\quad 0 < U < 1$, $\frac{U}{2} < V < 2$

$E(U/V=v) = ?$

$\frac{f(U,V)}{f(V)} = f(U/V=v)$

$$f(v) = \frac{48}{67}\int_0^1 (2uv - v^2)\,du = \frac{48}{67}\left[2U^2V - \frac{U^3}{3}\right]_0^1$$

$$= \frac{48}{67}\left[V - \frac{1}{3}\right] = \frac{16}{67}(3v-1)$$

$$\boxed{f(V) = \frac{16}{67}(3v-1)}$$

$$\frac{f(U,V)}{f(V)} = \frac{\cancel{48}^{3}}{\cancel{67}}\frac{(2uv^2 - v^2)}{\cancel{16}_{1}(3v-1)} \times \cancel{67}$$

$$= \frac{3}{3v-1}(2uV^2 - V^2) = 2U^2V^2 - U^3.$$

Now $E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - U^2)\,du$

$$= \frac{3}{3v-1}\left[\frac{2U^3v^2}{3} - \frac{v^4}{4}\right]_0^1 = \frac{3}{3v-1}\left[\frac{2}{3}v^2 - \frac{1}{4}\right]$$

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$$= \frac{\cancel{3}}{3v-1}\,\frac{(8v^2-3)}{\cancel{12}\,4} = \frac{8v^2-3}{4(3v-1)}$$

$$\boxed{\text{Ans}\quad E(U/V=v) = \frac{8v^2-3}{4(3v-1)}}$$

Ques6:- **Arthur statement.**

$X \sim Poi(\lambda) \qquad Y \sim Poi(\mu)$

To: Prove $Z = X - Y$. ; $Z \sim Poi(\lambda-\mu)$.

Sol^n

$X \sim Poi(\lambda)$

$M_x(t) = e^{\lambda(e^t-1)} \qquad M_y(t) = e^{\mu(e^t-1)}$

$Z = X - Y$

$$M_z(t) = E[e^{(tz)}]$$

$$= E[e^{t(x-y)}]$$

$$= E[e^{tx-ty}]$$

$$= E[[e^{tx}] - [e^{ty}]]$$

$$= E[e^{tx}] - E[e^{ty}]$$

$$= e^{\lambda(e^t-1)} - e^{\mu(e^t-1)}$$

$$= e^{\lambda(e^t-1) - \mu(e^t-1)}$$

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$$M_Z(t) = e^{(\lambda-\mu)(e^t-1)}$$

$$\boxed{\text{Hence } Z \sim Poi(\lambda-\mu)}$$

Christine statement

Let $Z = X + Y$.

$$M_Z(t) = E(e^{tZ}) = E[e^{t(X+Y)}] = E[e^{tX+tY}]$$

$$= E[e^{tX}] + E[e^{tY}]$$

$$= e^{\lambda(e^t-1)} + e^{\mu(e^t-1)}$$

$$= e^{\lambda(e^t-1)+\mu(e^t-1)}$$

$$\boxed{M_Z(t) = e^{(\lambda+\mu)(e^t-1)}}$$

Hence $Z \sim Poi(\lambda+\mu)$

Ques:

$$f(x,y) = ax(x+y) \qquad 0<x<5 \;;\; 10<y<20.$$

(i) $P(Y > \frac{1}{3}X + 10)$

$$= a\int_{10}^{20}\int_0^5 (x^2+xy)\,dx\,dy = 1.$$

$$\int_{10}^{20}\left[\frac{x^3}{3} + \frac{x^2y}{2}\right]_0^5 dy = \frac{1}{a}$$

$$\int_{10}^{20}\left(\frac{5^3}{3} + \frac{25y}{2}\right)dy = \frac{1}{a} \Rightarrow \int_{10}^{20}\left(\frac{125}{3} + \frac{25y}{2}\right)dy$$

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$$= \left[\frac{125}{3} y + \frac{25}{4} y^2\right]_{10}^{20} = \frac{1}{a}.$$

$$a\left(\frac{2500}{3} + 2500 - \frac{1250}{3} - 625\right) = 1$$

$$a\left(\frac{6875}{3}\right) = 1 \quad ; \quad a = \frac{3}{6875}.$$

$$\boxed{\cancel{a = \frac{3}{6875}}}$$

$$8.33.33 + 2500 - (416.667+625) = \frac{1}{a}$$

$$\boxed{a = 0.00043636}.$$

$$\cancel{\frac{1}{a} = 6875}$$

(iii) $E(Y/X)$

$$f(y/x) = \frac{f(x,y)}{f(x)}$$

$$f(x) = 0.00043636 \int_{10}^{20} (x^2 + xy)\, dy.$$

$$= 0.00043636 \left[x^2 y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$= 0.00043636 \left[20x^2 + \frac{400x}{2} - 10x^2 - \frac{100x}{2}\right]$$

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$$= 0.00043636\left[10x^2 + 150x\right]$$

$$= 0.0043636\left(x^2 + 15x\right).$$

$$f\,y/x) = \frac{3}{\cancel{6875}}\,\frac{x(x+y)}{30x} \times \cancel{6875}\,(x+15)$$
$$10$$

$$\boxed{f\,y/x = \frac{x+y}{10(x+15)}}$$

$$E(y/x) = \int_{10}^{20} y\cdot\frac{(x+y)}{10(x+15)}\,dy = \frac{1}{10(x+15)}\int_{10}^{20}(xy + y^2)\,dy$$

$$= \frac{1}{10(x+15)}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}.$$

$$= \frac{1}{10(x+15)}\left[200x + \frac{8000}{3} - 50x - \frac{1000}{3}\right].$$

$$= \frac{1}{10(x+15)}\left[150x + \frac{7000}{3}\right].$$

$$\boxed{E(y/x) = \frac{1}{x+5}\left(15x + \frac{700}{3}\right)}$$ Ans.

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Ques4: $G_V(t) = \left(\frac{p}{1-qt}\right)^k$

$p+q=1.$

$G_V(t) = p^k(1-qt)^{-k}.$

$G_V(t) = \sum_{0}^{4} t^v \, P(V=v)$

$P(V=v) = \frac{\lceil k+4}{\lceil 5 \lceil k} p^k q^k$

$= \frac{\lceil k+4}{4! \lceil k} p^k q^k$

$$\boxed{G_V(t) = \frac{(k+3)!}{(k-1)!\,4!} p^k (1-p)^k.}$$

Ques3: $f(x) = \lambda e^{-\lambda(x-\alpha)}$

$MGF = E(e^{tx}) = \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$

$M_x(t) = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda\alpha} dx$

$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{-x(t+\lambda)} dx. = \frac{e^{\alpha\lambda}\lambda}{t-\lambda} \left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty}$

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$$= e^{t\alpha} \frac{\lambda}{\lambda - t}.$$

$E(X) = M'_X(t) = \lambda e^{t\alpha} (\lambda - t)^{-1}$

$= \lambda [\alpha e^{t\alpha}(\lambda - t) + e^{t\alpha}(-1)].$

$= \lambda [e$

$$M_X(t) = \frac{\lambda e^{t\alpha}}{\lambda - t} = \lambda e^{t\alpha} (\lambda - t)^{-1}$$

$$M'_X(t) = \lambda [e^{t\alpha} (\lambda - t)^{-2}(1) + (\lambda - t)^{-1} e^{t\alpha}(\alpha)].$$

$$= \lambda [(\lambda - t)^{-1} e^{t\alpha} \alpha + e^{t\alpha} (\lambda - t)^{-2}]$$

$$= \lambda e^{t\alpha} (\lambda - t)^{-2} [\alpha(\lambda - t) + 1]$$

$t = 0$ $$= \lambda (\lambda)^{-2} [\alpha(\lambda + 1]$$ $(\alpha + 1/\lambda)$

$$= \frac{1}{\lambda} [\lambda\alpha + 1] = \frac{\lambda\alpha + 1}{t}.$$

$$M''_X(t) = \lambda [\alpha e^{t\alpha} (\lambda - t)^{-2} + \alpha^2 (\lambda - t)^{-1} e^{t\alpha} + e^{t\alpha}(2) (\lambda - t)^{-3}\alpha + (\lambda - t)^{-2} e^{2}]$$

Put $t = 0$

$$M''_X(t) = \lambda [\lambda (\lambda)^{-2} + \alpha^2 (\lambda)^{-1} + 2(\lambda)^{-3} + \alpha (\lambda)^{-2}]$$

$$= \lambda \left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3} \right]$$

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$$E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} \quad (4)$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + 1/\lambda\right]^2$$

$$= \cancel{\frac{2\alpha}{\lambda}} + \cancel{\alpha^2} + \frac{2}{\lambda^2} - \cancel{\alpha^2} - 1/\lambda^2 - \cancel{\frac{2\alpha}{\lambda}}$$

$$\boxed{V(x) = 1/\lambda^2}$$

Ques: Let X_i be the size of on home insurance and Y_i be the size on car insurance.

ATQ

$X \sim N(800, 100^2)$

$Y \sim N(1200, 300^2)$

$$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$$

$$\Rightarrow P((Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$$

$$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_7) \sim N(3 \times 1200 - 4 \times 800, 300^2 \times 3 + 4 \times 100^2)$$

$$\sim N(400, 310000)$$

$$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$$

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$P(Z > 0.71842)$
$= 1 - P(Z < 0.71842)$
$= 1 - P(Z < 0.72)$
$= 1 - 0.76424$

$\boxed{Ans = 0.23576}$

Ques: $N \sim Ng.\ Binomial(K, P)$

$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$
$= {}^{n+3-1}C_{n-1} (0.9)^3 (0.1)^n$

so $k = 3$; $p = 0.7$

$M_y t = M_N(\log M_x t)$

$X \sim Gamma(6, 2)$

$Y \to$ Total claim amount.

(i) MGF of Y

$M_y(t) = E[E(e^{yt}/N=n)]$

$E(e^{yt}/N=n) = E[e^{x_1 t + x_2 t + x_3 t \cdots x_n t}]$

$= E(e^{tx_1}) \cdot E(t^{x_2}) \cdots E(e^{tx_n})$

$= \left(1 - \frac{t}{2}\right)^{-6n}$

So $E(E(e^{yt}/N=n)) = E\left[\left(1-\frac{t}{2}\right)^{-6n}\right]$.

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$$= E\left(e^{N\log(1-t/2)^{-6}}\right)$$

$$= M_N\left(\log\left(1-t/2\right)^{-6}\right)$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k = \left[\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right]^k$$

$$= \left[\frac{p\left(1-t/2\right)^{-6}}{1-q\left(1-t/2\right)^{-6}}\right]^3$$

$$= \left(\frac{(0.9)\left(1-t/2\right)^{-6}}{1-0.1\left(1-t/2\right)^{-6}}\right)^3$$

$$V(Y) = E(N)\,V(X) + \left(E(X)\right)^2 V(N)$$

$$= \left(\frac{\cancel{3}^{10}}{\cancel{0.9}_{0.3}} \times \frac{6^2}{4}\right) + \left(\frac{\cancel{6}^3}{2}\right)\left(\frac{3\,(0.1)}{(0.9)^2}\right)$$

$$= 5 + \frac{9 \times 0.1 \times 10 \times 10}{0.9 \times 0.9 \times 10}$$

$$= 5 + 0.111$$

$$\boxed{S.D = 2.887}$$

$$= 5.1111\ 2$$

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