classmate

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# CALCULUS - I

1. Evaluate the following limit

$$\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$$

$$\lim_{h\to 0} \frac{2(h^2+9-6h)-18}{h}$$

$$\lim_{h\to 0} \frac{2h^2+18-6h-18}{h}$$

$$\lim_{h\to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}}$$

$$\lim_{h\to 0} 2(0-6)$$

$$= \boxed{-12}$$

2. $g(x) = \begin{cases} 2x & ; x < 6 \\ x-1 & ; x \geq 6 \end{cases}$

continuous or discontinuous at

(a) $x = 4$

(b) $x = 6$

At $x = 4$,

$\lim_{x\to 4} 2x = \boxed{8}$ is continuous at $x=4$

(b) At $x = 6$

$\lim_{x\to 6^-} 2x$

$\lim_{h\to 6^-} 2x$

$\lim_{h\to 0} 2(h-0)$

$= 12$

$$\lim_{x \to 6^+} x-1$$

$$\lim_{h \to 6^+} (6+h)-1$$

$$\lim_{h \to 0} 6-1 = \boxed{5}$$

~~At~~

As LHL ≠ RHL, it is discontinuous at $x=6$

3. Solve the limit

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x \to 9} \frac{x-9}{-(\sqrt{x}-3)}$$

$$\lim_{x \to 9} \frac{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)}{-\cancel{(\sqrt{x}-3)}} = -(\sqrt{x}+3)$$

$$= -(\sqrt{9}+3)$$

$$= -3-3 = \boxed{-6}$$

4. continuous or discontinuous

$$f(x) = \frac{4x+5}{9-3x}$$

(a) $x = -1$

at $x=1$

$$\lim_{x \to -1} \frac{4(x)+5}{9-3(x)}$$

$$\lim_{x \to -1} \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9-3(-1)} = f(-1)$$

= As $\displaystyle\lim_{x \to -1} f(x) = f(-1)$, the function is continuous at $x=-1$

(b) at $x=0$,

$$\lim_{x \to 0} \frac{4x+5}{9-3x}$$

$$= \frac{4(0)+5}{9-3(0)} = f(0)$$

As $\lim_{x \to 0} f(x) = f(0)$, the function is

continuous at $x=0$

(c) $\lim_{x \to 3^-} f(x)$ at $x=3$

$\lim_{x \to 3^-} = \infty$ & $\lim_{x \to 3^+} = -\infty$

the limit does not exist so the function is discontinuous.

5. Evaluate the limit

$$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \frac{\cancel{e^{4x}}\left(6 - e^{-6x}\right)}{\cancel{e^{4x}}\left(8 - e^{-2x} + 3e^{-5x}\right)}$$

$$= \frac{[6 - 1/e^{6(\infty)}]}{[8 - 1/e^{2(\infty)} + 3/e^{5(\infty)}]}$$

$$= \frac{(6-0)}{(8-0+0)} = 6/8 = 3/4$$

6. $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

(a) $\lim_{y \to 6} g(y)$

$\lim_{y \to 6} 1-3y$

$\Rightarrow \lim_{y \to 6} 1-3(6)$

$\Rightarrow \lim_{y \to 6} 1-18 = \boxed{-17}$

(b) $\lim_{y \to -2} g(y)$

LHL

$\lim_{y \to -2^-} g(y)$

$\lim_{h \to -2^-} y^2+5$

$\lim_{h \to -2^-} (-2)^2+5$

$\lim_{h \to -2^-} 4+5 = 9$

$= \boxed{9}$

RHL

$\lim_{y \to -2^+} g(y)$

$\lim_{h \to -2^+} 1-3y$

$\lim_{h \to -2^+} 1-3(-2)$

$\lim_{h \to -2^+} 1+6 = 7$

$= \boxed{7}$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (4t^2-t).(3t^2-16t) + (t^3-8t^2+12).(8t-1)$

$= (12t^4 - 64t^3 - 3t^3 + 16t^2) +$

$[8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12)$

$= [12t^4 - 67t^3 + 16t^2] + [8t^4 - 65t^3 + 8t^2 + 96t - 12]$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

$f'(t) = 5t^4 - 33t^3 + 6t^2 + 24t - 3$

8. $R(w) = \dfrac{3w + w^4}{2w^2+1}$

$R'(w) = \dfrac{(3w+w^4)(4w) - (2w^2+1)(3+4w^3)}{(2w^2+1)^2}$

$= \dfrac{(12w^2 + 4w^5) - (6w^2 + 8w^5 + 3 + 12w^3)}{4w^4 + 1 + 4w^2}$

$= \dfrac{12w^2 + 4w^5 - 6w^2 - 8w^5 - 3 - 12w^3}{4w^4 + 1 + 4w^2}$

$= \cancel{\dfrac{4w^2}{4w^2}} [$

$=$

9. Differentiate

$$f(x) = 2e^x - 8^x$$
$$= 2e^x - 8^x \log(8)$$

10. $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left[e^z \cdot \frac{1}{z} + \log(z) \cdot e^z\right]$$
$$= 5z^4 - \left[\frac{e^z}{z} + \log z e^z\right]$$

11. (a) $f(x) = (6x^2 + 7x)^4$

$$f'(x) = 4(6x^2+7x)^3 \cdot (12x+7)$$

(b) $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = 0 + [e^{4t+t^7}] \cdot [4 + 7t^6]$$
$$= (7t^6 + 4t) \cdot (e^{4t+t^7})$$

12. (a)' $z = \log(7-x)^3$

$$z' = \frac{1}{(7-x)^3} \cdot 3(7-x)^2(-1)$$
$$= -\frac{3(7-x)^2}{(7-x)^3}$$

12. (a) $z = \log(7 - x^3)$

$$z' = \frac{1}{(7-x^3)} \cdot (0 - 3x^2)$$

$$z' = \frac{-3x^2}{(7-x^3)}$$

$$z'' = \frac{(7-x^3) \cdot (-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

$$= \frac{-3x(14 - x^3)}{(7-x^3)^2}$$

(b) $(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$v' = \frac{[6+2v-v^2]^4 \cdot [0] - 2[4(6+2v-v^2)^3(0+2-2v)]}{(6+2v-v^2)^6}$$

$$v' = \frac{2[24+8v-4v^2]^3[2-2v]}{\cancel{-4[24}}$$

$$v' = \frac{-\cancel{2.2}[2.8(6+2v-v^2)^3(1-v)]}{(6+2v-v^2)^6}$$

$$v'' = 16(6+2v-v^2)^{-5} + 40(6+2v-v^2)^{-6}(2-2v)^2$$

12) $f(t) = \log(1+t^2)$

$f'(t) = \frac{1}{(1+t^2)} \cdot 2t$

$f'(t) = \frac{2t}{(1+t^2)}$

$f''(t) = \frac{2 \cdot (1+t^2) - 2t \cdot 2t}{(1+t^2)^2}$

$= \frac{2+2t^2 - 4t^2}{(1+t^2)^2}$

$= \frac{2(1+t^2-2t^2)}{(1+2t^2+t^4)}$

$= \frac{2(1-t^2)}{(1+t^2)^2}$

13. $y = x^3 - 3x^2 - 9x + 12$

$y' = 3x^2 - 6x - 9$

Equating to 0

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$x(\cancel{x-2})$

$x^2 + x - 3x - 3 = 0$

$x(x+1) - 3(x+1)$

$x = -1$ or $x = +3$

$y'' = 6x - 6$

$y''(3) = 6(3) - 6$

$= 12 \leftarrow$ minima

$y''(-1) = 6(-1) - 6$

$= -12 \leftarrow$ maxima

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

equating to 0

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$x^2 - x - 2x + 2 = 0$

$x(x-1) - 2(x-1) = 0$

so $x = 1$ & $x = 2$

$f''(x) = 24x - 36$

$f''(1) = 24(1) - 36$

$= -12 \rightarrow$ maxima

$f''(2) = 24(2) - 36$

$= 12 \rightarrow$ minima

15. $f(x) = x^2(x+3)$

$f'(x) = x^2 \cdot 1 + 2x(x+3)$

$= x^2 + 2x^2 + 6x$

$= 3x^2 + 6x$

$= 3x(x+2)$

$= x = 0$ or $x = -2$

$f''(x) = 6x + 6$

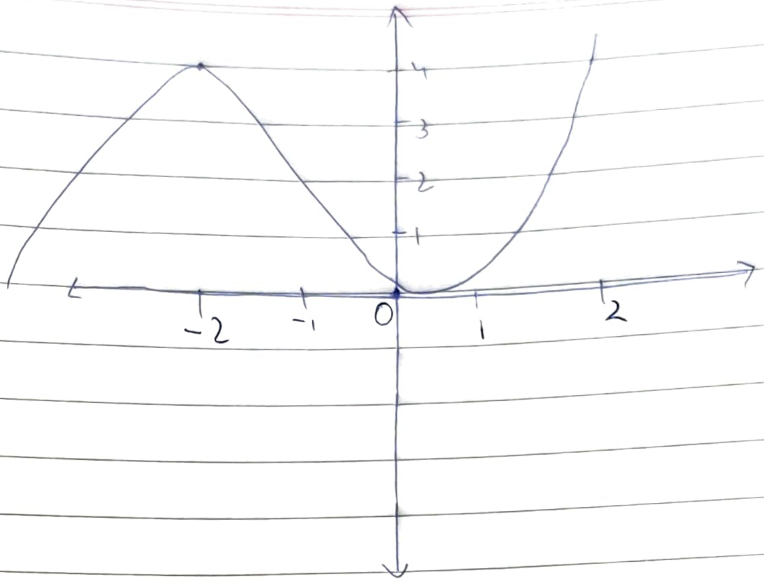
$f''(x) = 6(-2) + 6$

$= -12 + 6 = -6 \leftarrow$ ~~maximum~~ ~~minim~~

$= -6 \rightarrow$ minimum at $x = 0$

$\therefore f(-2) = 4$

$f(0) = 0$




16. $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

$= 6(x-1) = 0$

$x = 1$

$f''(x) = 6$

$f''(1) = 6 \leftarrow$ minima

$f(0) = 0$

$f(1) = 3$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 2$ or $0$