Financial Mathematics

①

1) $d^4 = 8\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = e^{-\delta(1)}$$

$$(1 - 0.02)^4 = e^{-\delta}$$

$$\ln |0.922368160) = -\delta$$

$$\delta = 8.081088\%$$

2) $1 + i = e^{\delta}$

$$i = e^{\delta} - 1$$

$$i = 8.41657\%$$

3) $\left(1 - \frac{d^4}{4}\right)^4 = \left(1 - \frac{d^{12}}{12}\right)^{12}$

$$1 - \frac{d^{12}}{12} = (0.9822868\,2)^{1/12}$$

$$d^{12} = 8.053929$$

② 1) $\delta(t) = 0.004t + 0.0002t^2$

$$PV_0 = 1000 \times e^{-\int_0^{10} \delta(t)\,dt}$$

$$= 1000 \times e^{-\int_0^{10} (0.004t + 0.0002t^2)\,dt}$$

$$= 1000 \times e^{-[0.002(100) + \frac{0.0002}{3}(1000)]}$$

$$= 1000 \times e$$

$$PV_0 = 765.928338$$

2) $AV_{10} \cdot 1000 = 765.928338 \cdot (1+i)^{10}$

$$i = 2.7025404\%$$

③ 1) Since we know $d = \frac{i}{1+i}$

$$\Rightarrow d = iv$$

2) $1-d^4 = \left(1-\frac{d^{(2)}}{2}\right)^2$

$d^{(2)} = 8.89875\%$

⑤ 1) $d^4 = 6\%$

$\left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$

$i^{(2)} = 6.13775\%$

2) $\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^4$

$d^{12} = 6.03025\%$

ii

200000 ——— 220000

$220000 = 200000(1+i)$

$i = 0.1$

$\therefore (1+i)^{3/12} \times 200000 = 204822.7373$

⑥ $\frac{0.70}{0.75} = 1-d$

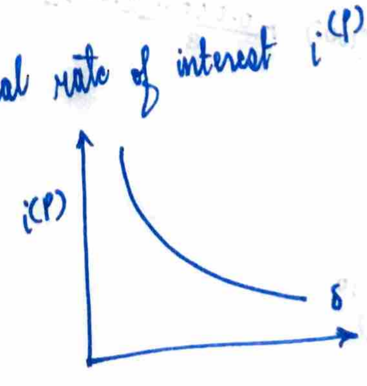
$d = 6.6667\%$

(ii) $d^{12} = 1-(1-d)^{1/2} \times 12 = 6.8798\%$

$i^{(2)} = \left[(1-d)^{1/2}-1\right] \times 2 = 7.02008\%$

$\delta = -\ln(1-d) = 6.8996\%$

⑧ As p increases, the nominal rate of interest $i^{(p)}$ tends to δ, the force of interest.



⑦ 1) $i^4 = ?$

$260000 = 20000\left(1+\frac{i^{(4)}}{4}\right)^{4\times 17/12}$

$i^{(4)} = 18.95525\%$

2) $\frac{26000}{20000} = \left(1-\frac{d^{(2)}}{2}\right)^{-2\times 7/12}$

$d^{(2)} = 17.68123\%$

3) $\frac{26000}{20000} = \left(1+\frac{17}{12}i\right)$

$1.3-1 = \frac{17}{12}i$

$i = 21.17647\%$

(9) 1) Force of interest is the nominal interest or discount rate compounded infinite number of time per time period

$i^{(2)} = 0.0775$

2) $\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i$

$i = 7.9001563\%$

→ $\ln(1+i) = \delta$

$\delta = 7.6086134\%$

→ $(1+i) = \left(1-\frac{d^{(12)}}{12}\right)^{-12}$

$d^{(12)} = 7.57957\%$

→ $\left(1+\frac{i^{(0.5)}}{0.5}\right)^{0.5}$

$i^{(0.5)} = 8.2122186\%$

(10) → $v(t) = e^{-\int_0^t (0.08)ds}$

$= e^{-0.08t} = v(t)$ when $0 < t < 5$

→ $v(t) = e^{-\int_0^5 0.08ds} \times e^{-\int_5^t (0.06)ds}$

$= e^{-0.4} \times e^{-[0.06t-0.3]} = e^{-0.1-0.06t}$ when $5 \le t \le 10$

→ $v(t) = e^{-0.1-0.06(10)} \times e^{-\int_{10}^t (0.04)ds}$ when $t \ge 10$

$= e^{-0.3-0.04t}$

(11) 1) $d = 18\%$

$t = 9/12$

$$PV_0 = 50000 \times (1-0.18)^{9/12} = 43085.42623$$

2) $\delta = 6\%$

$$e^{-\frac{0.06}{12}} = \left(1 - \frac{d^{(12)}}{12}\right)$$

$$d^{(12)} = 5.9850250\%$$

$\rightarrow$ $d^{(4)} = 6\%$ $\quad i^{(12)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$i^{(12)} = 6.0607089\%$$

$\rightarrow$ $d < \delta < i^{(4)} < i$

(12) $$AV_6 = 7049 \times \left(1 + \frac{i^{(12)}}{12}\right)^{24} \times [i + 10 \times 0.05] \times \left(1 - \frac{d^{(12)}}{12}\right)^{-18}$$

$$7049 \times 1.12718978 \times [1.15] \times 1.07442123$$

$$AV_6 = 9999.883762$$

(13) 1) $$2x = x(1+i)^{4t}$$

$$2 = (1+0.1)^{t}$$

$$\log 2 = t \log[1.1]$$

$$t = \frac{\log 2}{\log 1.1} = 7.27254 \text{ years}$$

2) $$2 = \left(1 - \frac{d^{(12)}}{12}\right)^{-12t}$$

$$2 = \left(1 - \frac{0.1}{12}\right)^{-12t}$$

$$\ln 2 = -12t \ln\left(1 - \frac{0.1}{12}\right)$$

$$t = 6.90255 \text{ years}$$