# PROBABILITY ASSIGNMENT

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Q.1 i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

a. Mean = $\frac{\text{Sum of all obvs}}{\text{No. of obvs}} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$

$= \frac{134}{10} = 13.4$ $\boxed{\text{Mean} = 13.4}$

b. $n = 10 \rightarrow$ Even No. of Obvs:

Median $= \left(\frac{n+1}{2}\right)^{th}$ obvs. $= \frac{10+1}{2} = \frac{11}{2} = 5.5^{th}$ obvs.

$= 5^{th}$ Obv $+ 0.5[6^{th} - 5^{th}$ Obv$]$

$= 11 + 0.5(15-11) = 11 + 0.5(4) = 13$

* Median = 13

c. Mode = 18

d. $SD = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$ $\frac{\Sigma x}{n} = \text{Mean} = 13.4$

$\Sigma x^2 = 16+49+81+81+121+225+324+324+324+625 = 2170$

$SD = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} = \sqrt{\frac{2170}{10} - (13.4)^2}$

$= \sqrt{217 - 179.56} = \sqrt{37.44}$

* Standard Deviation = 6.1188

e. $Q_3 = \frac{3(n+1)}{4}^{th}$ obv $= 3 \times \frac{11}{4}^{th} = 8.25^{th}$ obv.

$= 18 + 0.25[9^{th} - 8^{th}] = 18 + 0.25(18-18) = \boxed{18 = Q_3}$

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ obv $= \frac{11}{4}^{th} = 2.75^{th}$ obv.

$= 7 + 0.75(9-7) = 7 + 0.75(2) = \boxed{8.50 = Q_1}$

$\therefore IQR = Q_3 - Q_1 = 18 - 8.50 = 9.50$

* $\therefore$ IQR = 9.50

| ii. $x$ | F | $f.x$ | $x^2$ | $f.x^2$ | CF |
|---|---|---|---|---|---|
| 0 | 5 | 0 | 0 | 0 | 5 |
| 1 | 11 | 11 | 1 | 11 | 16 |
| 2 | 26 | 52 | 4 | 104 | 42 |
| 3 | 15 | 45 | 9 | 135 | 57 |
| 4 | 3 | 12 | 16 | 48 | 60 → N |
| | 60 | 120 | 30 | 298 | |

Mean: $\frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$ Mean = 2

Median: $\left(\frac{N+1}{2}\right)^{th}$ obv $= \frac{61}{2} = 30.5^{th}$ obvs ∴ Median = 2

Mode: 2

$SD = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{\cancel{298}^{49.6}}{\cancel{60}_{10}} - \left(\frac{\cancel{120}^{2}}{\cancel{60}}\right)^2}$

$= \sqrt{4.9666 - 4} = 0.9831937 = SD$

$Q_3 = \frac{3(N+1)}{4}^{th}$ obv: $\frac{3 \times \cancel{61}^{15.25}}{\cancel{4}} = 45.75^{th}$ obv ∴ $\boxed{Q_3 = 3}$

$Q_1 = \frac{1(N+1)}{4}^{th}$ obv: $\frac{61}{4} = 15.25^{th}$ obv - $Q_1 = 1$

$IQR = Q_3 - Q_1 = 3-1 = \boxed{2 : IQR}$

Q.2 $m = \lambda =$ Rate $= 0.5$ per year

∴ For Poisson Distribution: $P(X=x) = \frac{e^{-\lambda} \times \lambda^x}{x!}$

$x \to$ No. of claims

i.) $P(X=0) = \frac{e^{-0.5} \times (0.5)^0}{0!} = \frac{0.60653}{1} = 0.6065$

ii.) $P[X=x] = P(X \geq 1) + P(x=0)$

$\{1 - P(x<1)\} + 2 \times P(x=0)$

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iii.) $F_x(x) = 1 - e^{-\lambda x}$

Q3 $\alpha = 2 \quad \lambda = 1/2 \quad , \quad 0 < x < \infty$

i) Mean: $E(x) = \alpha/\lambda = 2/\tfrac{1}{2} = 4$

ii.) $V(x) = \alpha/\lambda^2 = \dfrac{2}{(1/2)^2} = \dfrac{2}{1/4} = 8$

$SD(x) = \sqrt{V(x)} = \sqrt{8} = 2\sqrt{3} = 2 \times 1.732 = 3.464$

ii $\Gamma(\alpha) = \Gamma(2) = (\alpha - 1)! = 1! = 1$

iii. $f_x(x) = \dfrac{\lambda^\alpha \cdot e^{-\lambda x} \times (x^{\alpha-1})}{\Gamma(\alpha)}$

$= \dfrac{(1/2)^2 \cdot e^{-1/2 x} \times x^{2-1}}{\Gamma(2)}$

$= \dfrac{\frac{1}{\cancel{4}2} \times \frac{1}{e^{1/2 x}} \times \cancel{x}}{(2-1)!} = \dfrac{1}{2e^{1/2 x}}$

$F_x(x) = \int_0^x \dfrac{\lambda^\alpha \times e^{-\lambda x} \times (x^{\alpha-1})}{\Gamma(x)} = \left[\dfrac{1}{2e^{1/2 x}}\right]_0^x$

$\cancel{= \dfrac{1}{2e^{1/2 x}} - \dfrac{1}{2e^{0 \times 1/2}}}$

$\cancel{= \dfrac{1}{2e^1} - \dfrac{1}{2e^0}}$

$\cancel{= 0.1839 - 0.5}$

$= \dfrac{1}{2e^{x/2}} - \dfrac{1}{2e^0}$

$= \dfrac{1}{2}\left\{\dfrac{1}{e^{x/2}} - 1\right\}$

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Q.4 A : Prob of 2 qualified females
B : ~~At~~ One female

$$P[A/B] = \frac{P[A \cap B]}{P(B)}$$

$P(B) = P(\text{One qualified female}) = \frac{{}^7C_1}{{}^8C_1}$

$$\frac{P(A \cap B)}{P(B)} = \frac{\frac{{}^7C_2}{{}^8C_2}}{\frac{{}^7C_1}{{}^8C_1}} = \frac{\frac{\cancel{7} \times 6}{\cancel{2}} \times \frac{\cancel{2}}{8 \times \cancel{7}}}{\frac{7}{8}} = \frac{6}{\cancel{8}} \times \frac{\cancel{8}}{7} = \frac{6}{7}$$

$= 0.8571$

Q5 P(Package late via level 2) = Goods sent by level 2 × Prob late goods

$$= \frac{\cancel{50}}{\cancel{100}_2} \times \frac{40}{100} = 0.20$$

Q.6 $X \sim U(1,2)$ $\qquad Y = \frac{3}{6-x}$

$$F_Y(y) = P(Y \leq y) = P\left(\frac{3}{6-x} \leq y\right)$$

$y = \frac{3}{6-x}$, $\quad 6 - x = \frac{3}{y}$

$\therefore \boxed{x = 6 - \frac{3}{y}}$

→ $F_Y(y) = P[Y < y] = P\left(\frac{3}{6-x} < y\right)$

$\therefore P[x \leq 6 - 3/y] = F_X(6 - 3/y)$

$\therefore F_Y(y) = F_X(6 - 3/y)$

→ Derivative of $x = 6 - 3/y$

$$\frac{dx}{dy} = 0 - 3 \times \frac{-1}{y^2} = \frac{3}{y^2}$$

~~$F_Y(y) = F_X(x)$~~ ~~$F_X($~~

$f_x(x) = f_x(1,2) = \frac{1}{2-1} = 1$

$\therefore F_Y(y) = F_x(\omega^{-1}(y)) \times \frac{d}{dy}(\omega^{-1}(y))$

$= 1 \times \frac{3}{y^2}$ $\quad \therefore \boxed{F_Y(y) = 3/y^2}$

$P[Y=y] = 3/y^2$

Q.7

| $x$ | $P(x)$ | CDF |
|---|---|---|
| 1 | 0.05 | 0.05 |
| 2 | 0.15 | 0.20 |
| 3 | 0.35 | 0.55 |
| 4 | 0.4 | 0.95 |
| 5 | 0.05 | 1.00 |

i.) $F(3) = 0.55$

ii.) $E(x)$

| $x$ | $P(x)$ | $xP(x)$ | $x^2.P(x)$ |
|---|---|---|---|
| 1 | 0.05 | 0.05 | 0.05 |
| 2 | 0.15 | 0.30 | 0.60 |
| 3 | 0.35 | 1.05 | 3.15 |
| 4 | 0.4 | 1.60 | 6.40 |
| 5 | 0.05 | 0.25 | 1.25 |
| | | 3.25 | 11.45 |

→ $E(x) = \Sigma x.P(x) = 3.25$

$V(x) = \Sigma x^2.P(x) - [E(x)]^2$

$= (11.45) - 10.5625 = \boxed{V(x) = 0.8875}$

iv. Mean = 3.25 $\quad$ Median $= \frac{N}{2} = 0.5^{th}$ obv $= x = 3$ = Median.

$SD = \sqrt{V(x)} =$

$= \sqrt{0.8875} = 0.94201218.$

$\therefore$ Skewness $= \frac{3(\text{Mean} - \text{Median})}{SD} = \frac{3(3.25-3)}{0.94207218}$

Skewness = 0.7961173421

Q.8 $n = 15$ Success - less than
$= 0, 1, 2$ dowries more than 1000
$p$ = success $= 0.2$.

Binomial:

$P[x<3] = P(x=0) + P(x=1) + P(x=2)$

$= {}^nC_x \cdot p^x \cdot q^{n-x}$

$\rightarrow \{{}^{15}C_0 \times (0.2)^0 \times (0.8)^{15}\} + \{{}^{15}C_1 (0.2)(0.8)^{14}\}$
$+ \{{}^{15}C_2 (0.2)^2 \cdot (0.8)^{13}\}$

$= (1 \times 1 \times 0.0351843) + [15 \times 0.2 \times 0.04398046]$
$+ \left(\frac{15 \times \cancel{14}^{7}}{\cancel{2}} \times (0.04) \times 0.05497558\right)$

$= (0.0351843) + (0.13194138) + (0.230897436)$

$P[x<3] = 0.398023116$

Q.9 $X \sim$ Gamma $(3, 0.01)$
Multiply both by 2:
$2\lambda = 2 \times 0.01 = 0.02$
$2\alpha = 2 \times 3 = 6$

$2\lambda X \sim \chi^2_6 = \chi^2_{2\alpha}$.

$\rightarrow P[x \le 7] = P[2\lambda x \le \lambda \times 2 \times 7]$
$= P[0.02X \le 0.14] = P(\chi^2_6 \le 0.14)$

OR: $\alpha = 3$, $\lambda = 0.01$ $x = 7$
$\therefore P(x<7) = \int_0^7 f_x(x)\, dx = \int_0^7 \frac{\lambda^\alpha \cdot e^{-\lambda x} \cdot x^{\alpha-1}}{\Gamma(\alpha)}$

$= \int_0^7 \frac{(0.01)^3 \times e^{-0.01x} \times x^2}{\Gamma(3)}\, dx$

$$= \frac{0.001}{2!}\int_0^7 x^2 \times e^{-0.01x}\,dx$$

$$= \frac{0.001}{2}\left[\left\{\frac{x^2 \cdot e^{-0.01x}}{-0.01}\right\} - \int \frac{2x \times e^{-0.01x}}{-0.01}\,dx\right]_0^7$$

$$= \frac{0.001}{2}\left[\frac{x^2 \cdot e^{-0.01x}}{-0.01} \cancel{+}\frac{200}{\cancel{(-0.01)}}\left\{\frac{xe^{-0.01x}}{-0.01} - \left[\frac{e^{-0.01x}}{0.0001}\right]\right\}\right]_0^7$$

$$= \frac{0.001}{2}\left[\frac{x^2 \cdot e^{-0.01x}}{-0.01} - \frac{xe^{-0.01x}}{0.01} + \frac{e^{-0.01x}}{0.0001}\right]_0^7$$

$$= (0.0005)\left[(-4568.72 - 652.675 + 9323.938) - \frac{1}{(0.0001)}\right]$$

$$\quad 4102.543$$

$\rightarrow P(\chi^2_6 < 0.14) =$

Q.10 2 claims $\rightarrow$ 5 days $\quad\therefore\quad$ 0.4 claims $\rightarrow$ 1 day

$x \rightarrow$ 1 day $\quad\quad p = 0.4$ (success)

~~X ~ Geometric~~

Q.11 P(Boy) = P(Girl) = 0.5

P[next 3 boys] = ~~0.5~~ P(boy) × P(boy) × P(boy)

= 0.5 × 0.5 × 0.5 = 0.125

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Q.12 10 Claims $\rightarrow$ 1 day $(x=1)$

$\lambda = 10$ per day

~~$P[X < 0.1] = \lambda \times e^{-\lambda x} = 10 \times e^{-10 \times 0.1} = 10 \times e^{-1}$~~

$F_X(x) = F_X(0.1) = 1 - e^{-\lambda x} = 1 - e^{-10 \times 0.1}$

$= 1 - e^{-1}$

$F_X(x) = 0.63212055 = F_X(0.1)$

Q.13 $f_X(a) = ?$ $X \sim U(75, 120)$

$f_X(x) = \dfrac{1}{b-a} = \dfrac{1}{120-75} = \dfrac{1}{45}$

$P[X \sim (80, 100)] = \dfrac{X \sim U(80, 100)}{X \sim U(75, 120)}$

$= \dfrac{\frac{1}{100-80}}{1/45} = \dfrac{1}{20} \times \dfrac{45}{1} = 2.25$

OR: $f_X(x) = \dfrac{1}{100-80} = \dfrac{1}{20} = 0.05 = P[x \sim U(80, 100)]$

Q.14 $X \sim Gamma(5, 0.4)$ $\alpha = 5$ $\lambda = 0.4$

$P[X < 22.89]$

$2\lambda = 2 \times 0.4 = \underline{\underline{0.8}}$. Multiply BS.

$P[0.8X < 22.89 \times 0.8] = P[0.8X < 18.312]$

$\therefore P[\chi^2_{2\alpha} < 18.312] = P[\chi^2_{10} < 18.312]$

Q.15

$\rightarrow$

Q.15 If PDF, then $\int_{0}^{\infty} f(x)\,dx = 1$

$$\rightarrow \int_{0}^{\infty} \frac{k}{(x+5)^4}\,dx = 1 = k\int_{0}^{\infty} (x+5)^{-4}\,dx = 1$$

$$= k\left[\frac{(x+5)^{-3}}{-3}\right]_{0}^{\infty}$$

$$= \cancel{k\left[\frac{(\infty+5)^{-3}}{-3} - \frac{(x+5)^{-3}}{-3}\right]_{0}^{\infty}}$$

$$= k\left\{\frac{(\infty+5)^{-3}}{-3} - \frac{(5)^{-3}}{-3}\right\} = k\left[\frac{1}{\infty(-3)} - \frac{1}{-3(5)^3}\right]$$

$$= k\left[0 + \frac{1}{375}\right] = 1$$

$$\boxed{k = 375}$$

ii.) $F(10) = \int_{0}^{10} f(x)\,dx = \int_{0}^{10} \frac{375}{(x+5)^4}\,dx$

$$= 375\int_{0}^{10} (x+5)^{-4}\,dx = 375\left\{\frac{(x+5)^{-3}}{-3}\right\}_{0}^{10}$$

$$= 375\left[\frac{1}{(15)^3(-3)} + \frac{1}{3(5)^3}\right]$$

$$= \frac{375}{-10125} + \left(\frac{1\times \cancel{375}}{\cancel{375}}\right) = -0.037037 + 1$$

$$F(10) = 0.96296$$

iii.) $E(x) = \int_{0}^{\infty} x \cdot f_x(x)\,dx = \int_{0}^{\infty} x \times \frac{375}{(x+5)^4}\,dx$

$$= 375 \times \int_{0}^{\infty} x \times (x+5)^{-4}\,dx$$

$$= 375 \int_{0}^{\infty} (x+5-5)(x+5)^{-4}\,dx = 375\int_{0}^{\infty} (x+5)^{-3} - 5(x+5)^{-4}\,dx$$

$$375 \times \left[ \frac{(x+5)^{-2}}{-2} - \frac{5(x+5)^{-3}}{-3} \right]_0^{\infty}$$

$$= 375 \left[ \left\{ \frac{1}{(\infty+5)^2(-2)} - \frac{5}{(\infty+5)^3 \times (-3)} \right\} - \left\{ \frac{1}{5^2(-2)} - \frac{5}{5^3(-3)} \right\} \right]$$

$$= 375 \left[ (0 - 0) - \left\{ \frac{1}{-50} + \frac{1}{75} \right\} \right]$$

$$= \frac{\cancel{375}^{\,75}}{\cancel{50}_{\,10}} - \frac{\cancel{375}^{\,5}}{\cancel{75}} = 7.5 - 5 = 2.5$$