

classmate
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Numerical Methods and Algebra

Assignment - 2

Name $\rightarrow$ Divya Sharma Roll No. $\rightarrow$ 70
Class $\rightarrow$ FY BSc.

Ques 1.

Area of circular plate with radius 12.65 = 502.7
Area of circular plate with radius 12.5 = 490.8

$$\text{Absolute error} = |490.874 - 502.726| \\ = \underline{11.852}$$

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{Actual value}}$$

$$= \frac{11.852}{490.874} = \underline{0.024}$$

$$\text{Percentage error} = \text{Relative error} \times 100 \\ = 0.024 \times 100 \\ = 2.4\%$$

Ques 2. (a)

x	y (fx)	$\Delta y (\Delta f(x))$
4	0.60206	0.1760913
6	0.7781573	

$$h = |x_1 - x_2| = |4 - 6| = 2$$

$$U = \frac{x - x_0}{h} = \frac{5 - 4}{2} = +\frac{1}{2}$$

$$f(x) = 1 \cdot f(x_0) + U \Delta f(x_0)$$

$$\log(5) = 0.60206 + \left(\frac{1}{2}\right)(0.1760913)$$

$$= 0.60206 + 0.08805$$

$$\log(5) = \cancel{0.69011} \quad \cancel{0.69011} \quad \underline{\underline{0.69011}}$$

- b)

x	y (f(x))	$\Delta y (\Delta f(x))$
4.5	0.6532125	0.0871502
5.5	0.7403627	

$$h = |4.5 - 5.5| = 1$$

$$U = \frac{5 - 4.5}{1} = 0.5$$

$$\log(5) = (0.6532125) + (0.5)(0.0871502)$$

$$\log(5) = 0.6532125 + 0.0435751$$

$$\log(5) = \cancel{0.6967876} \quad \underline{\underline{0.6967876}}$$

Ques 3.

$$y = x^4 - x - 10 = 0$$

$$a = 1.5$$

$$b = 2$$

x	y
1.5	-6.4375
2	4

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1.5 + 2}{2} = 2.5$$
$$= 1.75$$

$$y_3 = (1.75)^4 - 1.75 - 10 = -2.371$$

$$x_4 = \frac{x_3 + x_2}{2} = \frac{1.75 + 2}{2} = 1.875$$

$$y_4 = (1.875)^4 - 1.875 - 10 = 0.4846$$

$$x_5 = \frac{x_3 + x_4}{2} = \frac{1.75 + 1.875}{2} = 1.8125$$

$$y_5 = (1.8125)^4 - 1.8125 - 10 = -1.02$$

$$x_6 = 1.84375$$

$$x_7 = 1.859375$$

$$y_6 = -0.1972$$

$$y_7 = 0.0939$$

$$\text{Root of } x^4 - x - 10 = 0 \text{ is } x = 1.859375$$

Question 4.

$$f(x) = x^3 - x - 1$$

$$f'(x) = 3x^2 - 1$$

x	0	1	2
$f(x)$	-1	-1	5

$$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_0 = 1, \quad f(x_0) = -1, \quad f'(x_0) = 2$$

$$x_1 = 1 - \left(\frac{-1}{2} \right) = \frac{1.5}{1}$$

$$x_2 = 1.3487$$

$$f(x_1) = (1.5)^3 - (1.5) - 1 = -0.875$$

$$f'(x_1) = 3(1.5)^2 - 1 = 5.75$$

$$x_2 = 0.9765$$

$$x_2 = 1.3487$$

$$f(x_2) = 0.1046$$

$$f'(x_2) = 4.457$$

$$x_3 = 0.9765$$

$$f(x_3) = -1.045$$

$$f'(x_3) = 1.8607$$

$$x_4 = 1.5383$$

$$f(x_4) = 1.1019$$

$$f'(x_4) = 6.0991$$

$$x_5 = 1.35$$

$$\begin{aligned}
 x_5 &= 1.3576 \\
 f(x_5) &= 0.1446 \\
 f'(x_5) &= 4.5292
 \end{aligned}$$

$$\begin{aligned}
 x_6 &= 1.3257 \\
 f(x_6) &= 0.00419 \\
 f'(x_6) &= 4.2724
 \end{aligned}$$

$$\begin{aligned}
 x_7 &= 1.3247 \\
 f(x_7) &= -0.0000766 \\
 f'(x_7) &= 4.2645
 \end{aligned}$$

Root of the equation $f(x) = x^3 - x - 1$ is $x = 1.3247$

Question 5:

$$A^2 = A$$

$$7A - (I + A)^3 = 7A - (I^3 + A^3 + 3I^2A + 3IA^2)$$

$$= 7A - (I^3 + A^3 + 3AI + 3AI)$$

$$= 7A - I^3 - A^3 - 6A$$

$$= A - I^3 - A^2$$

$$7A - (I + A)^3 = \underline{\underline{-I}}$$

Question 6

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{-1}{|A|} C^T$$

$$M_{11} = -6$$

$$M_{12} = -4$$

$$M_{13} = 4$$

$$M_{21} = -1$$

$$M_{22} = -1$$

$$M_{23} = 1$$

$$M_{31} = -6$$

$$M_{32} = -2$$

$$M_{33} = 4$$

$$C = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & -2 & 4 \end{bmatrix}$$

$$C^T = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & -2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$|A| = 1(-6) - (-1)(-4) + 2(4)$$

$$|A| = -6 - 4 + 8$$

$$|A| = -2$$

$$A^{-1} = \frac{-1}{-2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & -2 \\ 4 & -1 & 4 \end{bmatrix}$$

Question 7.

$$A = 8\hat{i} - 9\hat{j} \quad |A| = \sqrt{(8)^2 + (-9)^2} = 12.042$$

$$B = 7\hat{i} - 9\hat{j} \quad |B| = \sqrt{(7)^2 + (-9)^2} = 11.402$$

$$A \cdot B = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$$

$$= (8)(7) + (-9)(-9)$$

$$= 56 + 81$$

$$A \cdot B = 137$$

$$\cos \theta = \frac{A \cdot B}{|A||B|} = \frac{137}{(12.042)(11.402)}$$

$$\cos \theta = 0.998$$

$$\theta = \cos^{-1}(0.998)$$

$$\theta = -3.62^\circ$$

Question 8.

$$\text{Magnitude} = \sqrt{(75)^2 + (25)^2}$$

$$= \sqrt{5625 + 625}$$

$$= \sqrt{6250}$$

$$= \underline{\underline{79.057}}$$

Question 9. 101, 104, ..., 998

$$a = 101$$

$$d = 3$$

$$a_n = 998$$

$$a_n = a + (n-1)d$$

$$998 = 101 + (n-1)3$$

$$\frac{897}{3} = n-1$$

$$299 = n-1$$

$$300 = n$$

$$S_{300} = \frac{n}{2} (2a + (n-1)d)$$

$$= \frac{300}{2} (202 + (299)3)$$

$$= 150 (202 + 897)$$

$$= 150 \times 1099$$

$$S_{300} = 164850$$

Question 10.

$$T_6 = 32, \quad T_8 = 128$$

$$T_n = ar^{n-1}$$

$$ar^5 = 32$$

$$ar^7 = 128$$

$$a \cdot \frac{32}{r^5} = \frac{32}{r^5}$$

$$a = \frac{128}{r^7}$$

$$\frac{32}{r^5} = \frac{128}{r^7}$$

$$r^2 = 4$$

$$\underline{r = 2}$$

Question 11.

$$(4+3x)^5 = {}^5C_0(4)^5(3x)^0 + {}^5C_1(4)^4(3x)^1 + {}^5C_2(4)^3(3x)^2 + {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)(3x)^4 + {}^5C_5(4)^0(3x)^5$$

$$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$$

Question 12.

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} = A$$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix}$$

$$3-\lambda \left((-5-\lambda)(2-\lambda) + 6 \right) = 0$$

$$3-\lambda \left(-10 - 2\lambda + 5\lambda + \lambda^2 + 6 \right) = 0$$

$$\lambda = -3 \quad \text{or}$$

$$\lambda^2 + 3\lambda - 4 = 0$$

$$\lambda^2 + 4\lambda - \lambda - 4 = 0$$

$$\lambda(\lambda+4) - 1(\lambda+4) = 0$$

$$\boxed{\lambda = 1, -4, -3}$$

Question 14.

$$(1 - x + x^2)^4$$

$$(x^2 - x + 1)^4$$

$$\text{let } x^2 - x = a$$

$$(a+1)^4$$

$${}^4C_0 a^4 + {}^4C_1 a^3 + {}^4C_2 a^2 + {}^4C_3 a + {}^4C_4$$

$$a^4 + 4a^3 + 6a^2 + 4a + 1$$

$$(x^2 - x)^4 + 4(x^2 - x)^3 + 6(x^2 - x)^2 + 4(x^2 - x) + 1$$

Question 15.

$$e^{-x} = 3 \log(x)$$

$$e^{-x} - 3 \log(x) = 0$$

$$a = 1$$

$$b = 2$$

$$\text{iteration} = 4$$

$$u$$

$$y$$

$$1$$

$$0.3678794412$$

$$2$$

$$-0.7677547038$$

$$u_1 = \frac{1+2}{2} = 1.5$$

$$y_1 = -0.305143617$$

$$u_2 = \frac{1+1.5}{2} = 1.25$$

$$y_2 = -0.000422524$$

$$u_3 = \frac{1+1.25}{2} = 1.125$$

$$y_3 = 0.1711949$$

$$u_4 = \frac{1.125+1.25}{2} = 1.1875$$

$$y_4 = 0.08108191382$$