Calculus - Assignment

Question 1 →

$$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

$$\int_{1/50}^{2} e^{-2w^{-1}} \cdot w^{-2}\, dw$$

$$2w^{-1} = t$$

$$dw \cdot -2w^{-2} = dt$$

$$\int_{1/50}^{2} e^{t}\, dt = \left[e^{t}\right]_{1/50}^{2}$$

$$e^{2} - e^{1/50} = e^{100}$$

Question 2 →

$$\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$$

$$t^4+2t = x$$

$$4t^3+2 = dx$$

$$2(2t^3+1) = dx$$

$$\frac{1}{2}\int x^{-3}\, dx$$

$$\frac{-2x^{-2}}{2} = -x^{-2} + C = -(t^4+2t)^{-2} + C$$

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Question 3.

$$f'(x) = x^4 + 3x - 9$$

$$f(x) = \int f'(x)$$

$$f(x) = \int x^4 + 3x - 9$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

$$0 = \frac{1}{5} + \frac{3}{2} - 9 + C$$

$$0 = \frac{2 + 15 - 90}{10} + C$$

$$0 =$$

Question 5.

$$\int 3(8y-1)\, e^{4y^2-y}$$

$$\int 3 \cdot e^x \cdot dx$$

$$4y^2 - y = x$$

$$(8y-1)dy = dx$$

$$\int 3e^x \, dx$$

$$= 3e^x$$

$$= 3e^{4y^2-y} + C$$

Ques 4. $f(x) = \begin{cases} 6, & \text{if } x > 1 \\ 3x^2, & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= [x^3]_{-2}^{1} + [6x]_{1}^{3}$$

$$= 9 + 12$$

$$= 21$$

Question 6.

$$f''(x) = 15\sqrt{x} + 5x^3 + 6$$

$$f'(x) = \int f''(x)$$

$$f'(x) = \int 15x^{1/2} + 5x^3 + 6$$

$$f'(x) = \frac{30}{3}x^{3/2} + 20x^4 + 6x + C_1$$

$$f(x) = \frac{20}{3}x^{5/2} + \frac{1}{4}x^5 + \frac{6}{2}x^2 + C_1 + C_2$$

$$f(x) = \frac{20}{3}x^{5/2} + \frac{1}{4}x^5 + 3x^2 + C_1 + C_2$$

$$f(1) = \frac{20}{3} + \frac{1}{4} + 3 + C_1 + C_2$$

$$\frac{-5}{4} = \frac{200 + 300 + 36}{12} + C_1 + C_2$$

$$\frac{-5}{4} - \frac{356}{3} = C_1 + C_2$$

$$\frac{-15 - 1429}{12} = C_1 + C_2$$

$$-155 - 1429 = 12C_1 + 12C_2 \quad \text{— (1)}$$

Question 8:

$$\frac{dx}{d\theta} = \frac{x^2}{\theta}$$

$$\int \frac{dx}{x^2} = \int \frac{d\theta}{\theta}$$

$$\frac{-1}{x} = \log\theta + c$$

$$x = 1, \quad \theta = 2$$

$$\frac{-1}{1} = \log(2) + c$$

$$c = -3.321928$$

$$\frac{-1}{x} = \log\theta - 3.321928$$

Ques 9. $\frac{dv}{dt} = 9.8 - 0.196v$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$\frac{1}{2\frac{7\sqrt{5}}{\sqrt{2}}} \ln \left[\frac{\frac{7\sqrt{5}}{5} + \frac{7\sqrt{5}}{50}}{\frac{7\sqrt{5}}{5} - \frac{7\sqrt{5}}{50}} \right] = t + c$$

$$\log \left[\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}} \right] = \frac{14}{\sqrt{5}} t + c$$

Ques 11. $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$$\int \frac{dy}{y^3} = 0 \int \frac{x}{\sqrt{1+x^2}} dx$$

$1 + x^2 = t$

$2x \cdot dx = dt$

$x\,dx = \frac{dt}{2}$

$$\int \frac{dy}{y^3} = \frac{1}{2} \int \frac{1}{\sqrt{t}} dt$$

$$\frac{-1}{2y^2} = \frac{1}{2} \cdot 2\sqrt{t} + c$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$$

$$\frac{-1}{2(-1)^2} = 1 + c$$

$$\boxed{c = \frac{-3}{2}}$$

$$\therefore \frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

12. $f(x) = x^3 - 10x^2 + 6$ for $[x=3]$

$f(x) = x^3 - 10x^2 + 6$ $\quad f(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\quad f'(3) = 27 - 60 = -33$

$f''(x) = 6x - 20$ $\quad f''(3) = -2$

$f'''(x) = 6$ $\quad f'''(3) = 6$

$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) + (x-3)^2(-2) + (x-3)^3 6$

(13) $(1+x)^u$

$f(x) = u(1+x)$ $\quad f(x) = (1+x)^u$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

$f(0) = 1$

$f'(0) = u$

$f''(0) = u(u-1)$

$f'''(0) = u(u-1)(u-2)$

$f(x) = 1 + ux + u(u-1)\frac{x^2}{2} + u(u-1)(u-2)\frac{x^3}{6} + \dots$

$$f(x) = 1 + \sum_{i=1}^{n} \frac{ux^n}{n!}(u-n)$$

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(14) $f(x) = \sqrt{1+x}$ $\quad f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\quad f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}}$ $\quad f''(0) = \dfrac{-1}{4}$

$f'''(x) = \dfrac{3}{8(1+x)^{5/2}}$ $\quad f'''(0) = \dfrac{3}{8}$

$$f(x) = 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{16} + \cdots$$

(15) $f(x) = e^{-6x}$ $\quad f(-4) = e^{24}$

$f'(x) = (-6)e^{-6x}$ $\quad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad f''(-4) = 36e^{24}$

$$f(x) = e^{24} - (x+4)6e^{24} + (x+4)^2 \times 36e^{24} + \cdots$$

(16)

(16) $f(x) = \ln(3+4x)$ for $x=0$

$f(x) = \ln(3+4x)$ $\quad f(0) = \ln(3)$

$f'(x) = \frac{4}{3+4x}$ $\quad f'(0) = \frac{1}{3}$

$f''(x) = \frac{-4}{(3+4x)^2}$ $\quad f''(0) = \frac{-4}{9}$

$f'''(x) = \frac{8}{(3+4x)^3}$ $\quad f'''(0) = \frac{8}{27}$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{4x^2}{2\times 9} + \frac{8x^3}{27\times 6} \cdots$$

$$\boxed{f(x) = \ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81} \cdots}$$