ASSIGNMENT 1

1. To find : $\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

Let $x = \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$ — ①

squaring both sides :

$x^2 = 7 + \sqrt{7+\sqrt{7+\ldots}}$ — ②

substituting ① in ② :

$x^2 = 7 + x$

$x^2 - x - 7 = 0$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-1)+\sqrt{(-1)^2-4(1)(-7)}}{2(1)}, \quad \dfrac{-(-1)-\sqrt{(-1)^2-4(1)(-7)}}{2(1)}$

$= 3.192582404, \ -2.192582404$

$\therefore$ since we know that x has to be positive (greater than 0), the value $x = -2.1925824$ is invalid

$\therefore \quad x = 3.192582404$

$\therefore \quad \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}} = 3.192582404$

2. $7x^2 - 6y^2 = -11x^2 - 4y^2$

$7x^2 + 11x^2 = -4y^2 + 6y^2$

$18x^2 = 2y^2$

$9x^2 = y^2$

$\frac{x^2}{y^2} = \frac{1}{9}$

$\therefore \frac{x}{y} = \frac{1}{3}$

3. $x + 3y = 4$, $x^2 + y^2 - 4y = 12$

$x + 3y = 4$

$\therefore x = 4 - 3y$ —— ①

Substituting ① in the equation :

$x^2 + y^2 - 4y = 12$

$(4-3y)^2 + y^2 - 4y = 12$

$16 - 24y + 9y^2 + y^2 - 4y = 12$

$10y^2 - 28y + 4 = 0$

$5y^2 - 14y + 2 = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{14 + \sqrt{(-14)^2 - 4(5)(2)}}{2(5)} , \frac{14 - \sqrt{(-14)^2 - 4(5)(2)}}{2(5)}$$

$$= \frac{7 + \sqrt{39}}{5} , \frac{7 - \sqrt{39}}{5}$$

$$= 2.6489996 , 0.1510004$$

When $y = 2.6489996$:

$x = -3.9469988$

When $y = 0.1510004$:

$x = 3.5469988$

$\therefore$ The ordered pairs that satisfy the system are $(-3.9469988, 2.6489996)$ and $(3.5469988, 0.1510004)$

4. $d = 2x^2 - 16x + 34$

$d \rightarrow$ density of smoke

$x \rightarrow$ speed of the engine

a) When $x = 3.5$:

$d = 2(3.5)^2 - 16(3.5) + 34$

$= 2.5$ million particles per cubic foot

b) For minimum smoke, $\frac{d}{dx}$ should be greater than 0

$\frac{d}{dx} = 4x - 16 > 0 \rightarrow 4x > 16$

$\therefore x > 4$

$\therefore$ The no. of revolutions for minimum smoke = 4 per minute

$\therefore$ Minimum output $= 2(4)^2 - 16(4) + 34$

$= 2$

c) When $d = 100$:

$100 = 2x^2 - 16x + 34$

$2x^2 - 16x - 66 = 0$

$x^2 - 8x - 33 = 0$

$x^2 - 11x + 3x - 33 = 0$

$(x-11)(x+3) = 0$

$\therefore x = 11$ OR $x = -3$ (Invalid)

$\therefore$ Speed of the engine is 1100 revolutions per minute

5. Company 1 : 1500 per day plus 25 per km
Company 2 : 2100 per day plus 22 per km
Let n be the total no. of km driven in a day

a) Total cost of renting a car for 1 day from company 1 = $1500 + 25n$
b) Total cost of renting a car for 1 day from company 2 = $2100 + 22n$

c) For it to be less expensive to rent from company 2 :
cost of company 1 > cost of company 2
$$1500 + 25n > 2100 + 22n$$

d)
$$1500 + 25n > 2100 + 22n$$
$$25n - 22n > 2100 - 1500$$
$$3n > 600$$
$$n > 200$$

$\therefore$ It is less expensive to rent from company 2 when the no. of km driven in a day is greater than 200.

6. $f(x) = x^3 - 7x^2 + 14x - 6 = 0$

a) considering $a = 0$, $b = 1$:
$f(a) = f(0) = -6$
$f(b) = f(1) = 2$
since $f(a)$ and $f(b)$ have opposite signs, this interval is appropriate

∴

a	b	$f(a)$	$f(b)$	$(a+b/2)$	$f[(a+b/2)]$
0	1	-6	2	0.5	-0.625
0.5	1	-0.625	2	0.75	0.984375
0.5	0.75	-0.625	0.984375	0.625	0.2597656
0.5	0.259765	-0.625	0.2597656	0.5625	-0.16186523
0.5625	0.625	-0.161865	0.2597656	0.59375	0.05404663
0.5625	0.59375	-0.161865	0.0540466	0.578125	-0.05262375
0.578125	0.59375	-0.0526238	0.0540466	0.5859375	0.0010313987
0.578125	0.5859375	-0.0526238	0.0010313987	0.58203125	-0.025716007
0.58203125	0.5859375	-0.025716007	0.0010313987	0.58398437	

∴ $x = 0.58$

b) Considering $a = 1$, $b = 3.2$

$f(a) = f(1) = 2$

$f(b) = f(3.2) = -0.112$

since $f(a)$ and $f(b)$ have opposite signs, this interval is appropriate

a	b	f(a)	f(b)	(a+b)/2	f[(a+b)/2]
1	3.2	2	-0.112	2.1	1.791
2.1	3.2	1.791	-0.112	2.65	0.552125
2.65	3.2	0.552125	-0.112	2.925	0.08582813
2.925	3.2	0.08582813	-0.112	3.0625	-0.054443359
2.925	3.0625	0.08582813	-0.05444336	2.99375	0.006327881
2.99375	3.0625	0.006327881	-0.05444336	3.028125	-0.02652072
2.99375	3.028125	0.006327881	-0.02652072	3.0109375	-0.010696934
2.99375	3.0109375	0.006327881	-0.010696934	3.00234375	-0.00233275
2.99375	3.00234375	0.006327881	-0.00233275	2.9991211	0.001960747
2.9991211	3.00234375	0.001960747	-0.00233275	3.00019531	-1.95236198

$\therefore \quad x = 3.00$

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c) Considering $a = 3.2$, $b = 4$

$f(a) = f(3.2) = -0.112$

$f(b) = f(4) = 2$

since $f(a)$ and $f(b)$ have opposite signs, this interval is appropriate

a	b	f(a)	f(b)	(a+b)/2	f[(a+b)/2]
3.2	4	-0.112	2	3.6	0.336
3.2	3.6	-0.112	0.336	3.4	-0.016
3.4	3.6	-0.016	0.336	3.5	0.125
3.4	3.5	-0.016	0.125	3.45	0.046125
3.4	3.45	-0.016	0.046125	3.425	0.01301563
3.4	3.425	-0.016	0.01301563	3.4125	-0.00199805
3.4125	3.425	-0.00199805	0.01301563	3.41875	0.005381592
3.4125	3.41875	-0.00199805	0.005381592	3.415625	0.001660065

$\therefore$ $x = 3.42$

7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$\therefore f'(x) = 920x^3 + 54x^2 + 18x - 221$

Since $f(x)$ has a real zero in $[-1, 0]$, we consider the midpoint -0.5 as x_0

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -0.5 - \frac{f(-0.5)}{f'(-0.5)} = -0.1504524887$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.041816814$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.0406593435$

$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.04065928832$

Since $f(x)$ has a real zero in $[0, 1]$, we consider the midpoint 0.5 as x_0

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = -0.7050898204$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.3237911142$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.06460313103$

$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.04068615115$

$\therefore x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = -0.04065928835$

8. Grades of four exams : 70, 86, 81, 83

Grade on the final exam : x

a) Course average $= \frac{70+86+81+83+x}{5} = \frac{320+x}{5}$

b) since you score a B if the average is greater than or equal to 80 and less than 90 :

$$80 \leq \frac{320+x}{5} < 90$$

c) $80 \leq \frac{320+x}{5} < 90$

$400 \leq 320 + x < 450$

$80 \leq x < 130$

If we assume that all marks are out of 100 :

$80 \leq x \leq 100$

$\therefore$ The grade on the final exam should be greater than or equal to 80 and less than or equal to 100 to obtain a B.

9. $x_1 = 8.54$

$x_2 = -11.81$

$x = 0$

$y_1 = 4\% = 0.04$

$y_2 = 20\% = 0.20$

Using linear interpolation :

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0.04 = \frac{0.20 - 0.04}{-11.81 - 8.54} (0 - 8.54)$$

$$y - 0.04 = 0.0671449631 4$$

$\therefore$ $y = 0.1071449631 4$

$= 10.7144963\%$

$\therefore$ IRR $= 10.7145\%$

10. Revenue = $R(x) = 32x - 0.21x^2$

Cost = $C(x) = 195 + 12x$

To remain profitable, the revenues must exceed the cost:

$R(x) > C(x)$

$32x - 0.21x^2 > 195 + 12x$

$20x - 0.21x^2 - 195 > 0$

$0.21x^2 - 20x + 195 < 0$

Equating the inequality to 0:

$0.21x^2 - 20x + 195 = 0$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-20) + \sqrt{(-20)^2 - 4(0.21)(195)}}{2(0.21)}, \dfrac{-(-20) - \sqrt{(-20)^2 - 4(0.21)(195)}}{2(0.21)}$

$= 84.21142753, 11.02666771$

Since $x < 11.02666771$ and $x > 84.211428$ do not satisfy the original inequality:

$11.02666771 < x < 84.2114275 \longrightarrow$ satisfies the original inequality

$12 \leq x \leq 84$

The no. of orders needed for the app to remain profitable should be greater than 12 and less than 84, with the minimum number being 12 orders.

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