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# Probability & Statistic Assignment 2

Q1.

Let X be the claim size of home insurance
Let Y be the claim size of car insurance

$X \sim N(800, 100^2)$ , $Y \sim N(1200, 300^2)$

$P(Y_1+Y_2+Y_3 > X_1+X_2+X_3+X_4+800)$
$= P((Y_1+Y_2+Y_3)-(X_1+X_2+X_3+X_4) > 800)$

$Y_1+Y_2+Y_3-(X_1+X_2+X_3+X_4) \sim N((3\times1200)-4\times800), (300^2\times3+100^2\times4)$
$\sim N(400, 310000)$

$P\left(Z > \frac{800-400}{\sqrt{310000}}\right) = P(Z > 0.71842)$
$= 1 - P(Z < 0.71842)$
$= 1 - 0.76424$
$= 0.23576$

Q2. ~~Le~~ Let X be the claim amount
$X \sim Gamma(6, 2)$
$P(N=n) = \binom{n+2}{n}(0.9)^3(0.1)^n$

1) $Y = X_1 + X_2 + \dots + X_n$

$M_Y(t) = E(e^{tY}) = E(e^{t(X_1+X_2+\dots+X_n)})$
$= E(e^{tX_1} \cdot e^{tX_2} \dots e^{tX_n}) = E(e^{tX_1}) \cdot E(e^{tX_2}) \dots E(e^{tX_n})$
$= [E(e^{tX})]^n = [M_X(t)]^n = E(e^{(\ln M_X(t))^n})$
$= E(e^{N \ln M_X(t)}) = M_N(\ln M_X(t))$
$= \left(\frac{p e^{\ln M_X(t)}}{1 - q e^{\ln M_X(t)}}\right)^n$

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$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$

$p = 0.9, x = 3, n = 2$

$\therefore M_Y(t) = \left( \frac{0.9 e^{\ln M_X(t)}}{1 - 0.1 e^{\ln M_X(t)}} \right)^2$

Also, $M_X(t) = \left(1 - \frac{t}{2}\right)^{-6} = \left(\frac{2-t}{2}\right)^{-6}$

$\therefore M_Y(t) = \left( \frac{0.9(2-t)^{-6}}{2^6 - 0.1(2-t)^{-6}} \right)^2$

ii) lets take

$S[Y] = S[X_1] + S[X_2] + \dots + S(X_n)$

$= nS[X]$

$= 2 \times \frac{6}{2^2} = 3$

$\therefore$ SD of $Y = \sqrt{3}$

Q3. i) $M_X(t) = E(e^{tx})$ — Given)

$f_X(x) = \lambda e^{-\lambda(x-\alpha)}$ — ①

$f_Y(y) = \lambda e^{-\lambda y}$ — ② — Since its Exponential Distribution

Comparing ① & ②

$y = x - \alpha$

$\therefore X \sim exp(\lambda)$ and $\alpha = 1$

ii) $\bar{x} = \frac{1}{\lambda}$

var $= \frac{1}{\lambda^2}$

Q5) X and Y have joint Distribution PDF

$$\int_{x=0}^{5}\int_{y=10}^{20} f_{X,Y}(x,y)\,dy\,dx=1$$

$$\int_{x=0}^{5}\int_{y=10}^{20} ax(x+y)\,dy\,dx=1$$

$$\int_{x=0}^{5}\int_{y=0}^{20} (ax^2+axy)\,dy\,dx=1$$

$$\int_{x=0}^{5}\left[ax^2y+\frac{axy^2}{2}\right]_{10}^{20}dx=1$$

$$\int_{x=0}^{5}\left(20ax^2+\frac{400}{2}ax-10ax^2-\frac{100}{2}ax\right)dx=1$$

$$\int_{x=0}^{5}(10ax^2+150ax)\,dx=1$$

$$\left[\frac{10ax^3}{3}+150\frac{ax^2}{2}\right]_0^5=1$$

$$\frac{10a}{3}\times5^3+\frac{150}{2}a\times5^2=1$$

$$\frac{1250}{3}a+1875a=1$$

$$\therefore \frac{6875}{3}a=1$$

$$a=\frac{3}{6875}$$

Q6. X & Y has Poisson Distribution

$X \sim Poi(\lambda)$

$Y \sim Poi(\mu)$

Let us take $Z = X - Y$

$$M_Z(t) = E(e^{tZ}) = E(e^{t(X-Y)}) = E(e^{tX} \cdot e^{-tY})$$
$$= \frac{E(e^{tX})}{E(e^{tY})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$$
$$= e^{(\lambda-\mu)(e^t-1)}$$

$Z \sim Poi(\lambda - \mu)$

Hence, $Z = X - Y$

$$M_Z(t) = E(e^{tZ}) = E(e^{t(X+Y)}) = E(e^{tX} \cdot e^{tY})$$
$$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$$
$$= e^{(\mu+\lambda)(e^t-1)}$$

$\therefore Z \sim Poi(\lambda + \mu)$

Hence the statements by Arthur and Christine are examp...

Q7

X and Y are two discrete random variables following joint probability function.

i) $var(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$

$$E(X^2/Y)=2) = \sum x^2 \cdot \frac{P(X=x, Y=2)}{P(Y=2)}$$
$$= \frac{1 \times 0}{1.6} + \frac{4 \times 0.3}{0.6} + \frac{9 \times 0.1}{0.6} + \frac{16 \times 0.2}{0.6}$$
$$= 8.8333$$

$$E(X/Y=2) = \sum x \cdot \frac{P(X=x, Y=2)}{P(Y=2)}$$

$$= \frac{1\times 0}{0.6} + \frac{2\times 0.3}{0.6} + \frac{3\times 0.1}{0.6} + \frac{4\times 0.2}{0.6}$$

$$= 2.8333$$

$$\therefore Var(X/Y=2) = 0.8657111$$

ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$

$$f_v(v) = \int_{u=0}^{1} \frac{48}{67}(2uv - u^2)\,du = \frac{48}{67}\left(\frac{2u^2v}{2} - \frac{u^3}{3}\right)_0^1$$

$$= \frac{48}{67}\left[u^2v - \frac{u^3}{3}\right]_0^1 = \frac{48}{67}\left[v - \frac{1}{3}\right] = \frac{48}{67}\frac{(3v-1)}{3}$$

$$= \frac{16}{67}(3v-1)$$

$$\frac{f(u,v)}{f(v)} = \frac{48}{67}\frac{(2uv-u^2)}{16(3v-1)} \times 67$$

$$= \frac{3}{3v-1}(2uv - u^2$$

$$E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv - u^2)\,du$$

$$= \frac{3}{3v-1}\int_0^1 (2u^2v - u^3)\,du$$

$$= \frac{3}{3v-1}\left[\frac{2vu^3}{3} - \frac{u^4}{4}\right]_0^1$$

$$= \frac{3}{3v-1}\left(\frac{2v}{3} - \frac{1}{4}\right) = \frac{3}{3v-1}\left(\frac{8v-3}{12}\right)$$

$$= \frac{8v-3}{4(3v-1)}$$

Q8) $X \sim Gamma(3, 2)$

$E(Y/X = x) = 3x + 1$ — Given

$Var(Y/X = x) = 2x^2 + 5$ — Given

$E[Y] = E[E(Y/X = x)] = E(3x+1) = 3E[X] + 1 = (3 \times \frac{3}{2}) + 1 = \frac{11}{2}$

$V[Y] = E[V(Y/X = x)] + V(E[Y/X = x])$

$= E(2x^2+5) + V(3x+1)$ ---- Substituting

$= 2E[x^2] + 5 + \cancel{9V[x]}\ 9V[X]$

$V[X] = \frac{3}{4} = E[X^2] - [E(X)]^2 = \frac{3}{4} = E[x^2] - \frac{9}{4}$

$E[X^2] = \frac{12}{4} = 3$

$\therefore V[Y] = 2 \cdot 6 + 5 + \frac{27}{4}$

$= \frac{71}{4}$

Q9) i) $\cancel{cov(X, Z) = cov(X, Z + Y)}$

$\cancel{= cov(X, Z) + cov(X, Y)}$

i) $cov(X, Z) = cov(X, X + Y)$

$= cov(X, X) + cov(X, Y)$

$= var(X) + cov(X, Y)$

$\rho = -0.3 = \frac{cov(X, Y)}{\sqrt{var[X] Var[Y]}}$

$= -0.3 = \frac{cov(X, Y)}{2}$

$= cov(X, Y) = -0.6$

$cov(X, Z) = 4 - cov(X, Y)$

$= 4 - 0.6$

$= \cancel{3.4} = 3.4$

ii) $Var(z) = Var(X+Y) = var(X) + var(Y) + 2cov(X,Y)$

$= 4 + 1 - (0.6 \times 2)$

$= 5 - 1.2$

$= 3.8$

Q10] $\int_{x=1}^{\infty}\int_{y=1}^{\infty} kx^{-\alpha} e^{-y/\beta}\, dy\, dx = 1$

$= \int_{x=1}^{\infty} k \int_{y=1}^{\infty} x^{-\alpha} e^{-y/\beta}\, dy\, dx = 1$

$= \int_{x=1}^{\infty} kx^{-\alpha} \int_{y=1}^{\infty} e^{-y/\beta}\, dy\, dx = 1 \qquad = \int_{x=1} k x^{-\alpha} \left[\frac{e^{-y/\beta}}{-y^2/2\beta}\right]_1^{\infty} dx = 1$

$= \int_{x=1}^{\infty} kx^{-\infty} \left(\frac{e^{-\infty}}{-\infty} - \frac{e^{-1/\beta}}{-1/2\beta}\right) dx = 1$

$= \int_{x=1}^{\infty} kx^{-\alpha} \times \frac{-e^{-1/\beta}}{-1/2\beta}\, dx = 1$

$= \int_{x=1}^{\infty} kx^{-\alpha} \times 2\beta \times e^{-1/\beta}\, dx = 1$

$= k \times 2\beta \times e^{-1/\beta} \int_{x=1}^{\infty} x^{-\alpha}\, dx = 1$

$= 2\beta k e^{-1/\beta} \times \left[\frac{x^{-\alpha+1}}{-\alpha+1}\right]_1^{\infty} = 1$

$= \cancel{2k}\beta k e^{-1/\beta} \times \left(\frac{x^{-\infty}}{-\infty} - \frac{1^{-\alpha+1}}{-\alpha+1}\right) = 1$

$= 2\beta k e^{-1/\beta} \times \left(\frac{-1}{1-\alpha}\right) = 1$

$= k = \frac{-(1-\alpha)}{2\beta e^{-1/\beta}}$

$= \frac{\alpha - 1}{2\beta e^{-1/\beta}}$

$= \frac{(\alpha-1)e^{1/\beta}}{2\beta}$