

Numerical Methods

Assignment 1

1) $x = \sqrt{7 + \sqrt{7 + \sqrt{7}}}$

$$x = \sqrt{7 + x}$$

$$x^2 = 7 + x$$

$$x^2 - x - 7 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-7)}}{2 \times 1}$$

$$= 3.1926 \text{ or } -2.1926$$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$18x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{1}{9}$$

$$\frac{x}{y} = \pm \frac{1}{3}$$

3) $x + 3y = 4$

-(i)

$$x = 4 - 3y$$

Multiplying

$$xy + 3y^2 = 12$$

$$-x^2 + y^2 - 4y = 12$$

$$2y^2 + xy - x^2 = 12$$

Substituting in eq 2

$$(4 - 3y)^2 + y^2 - 4y = 12$$

$$16 - 24y + 9y^2 + y^2 - 4y = 12$$

$$10y^2 - 28y + 4 = 0$$



$$10y^2 - 28y + 4 = 0$$

$$y = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(10)(4)}}{2 \times 10}$$

$$= \frac{28 \pm 24.98}{20}$$

$$y = 0.151 \text{ or } 2.649$$

4) a) $d = 2(3.5)^2 - 16(3.5) + 34$
 $= 2.5 \text{ million particles / cubic ft}$

b) $d = 2r^2 - 16r + 34$
 $\frac{dd}{dr} = 4r - 16$

$$r = 4$$

Minimum $d = 2(4)^2 - 16(4) + 34$
 $= 2 \text{ million particles / cubic ft}$

c) $2r^2 - 16r + 34 = 100$

$$2r^2 - 16r - 66 = 0$$

$$r^2 - 8r - 33 = 0$$

$$(r-11)(r+3) = 0$$

$r = 11$ (speed can't be negative)

5 a) $C = 1500 + 25n$

b) $C = 2100 + 22n$

c) $1500 + 25n > 2100 + 22n$

d) $3n > 600$

$$n > 200$$

$$x^3 - 7x^2 + 14x - 6 = 0$$

a	b	t	(t = a + b/2)		
			f(a)	f(b)	f(t)
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9843
0.5	0.75	0.625	-0.625	0.9843	0.2597
0.5	0.625	0.5625	-0.625	0.2597	-0.1618
0.5625	0.625	0.5938	-0.1618	0.2597	0.054
0.5625	0.9838	0.5781	-0.1618	0.054	-0.6526
0.5781	0.5938	0.5859	-0.0526	0.054	0.001

Answer = 0.59 (Accurate to 10^{-4})

a	b	t	f(a)	f(b)	f(t)
1	3.2	2.1	2	-0.112	1.791
2.1	3.2	2.65	1.791	-0.112	0.552
2.65	3.2	2.925	0.552	-0.112	0.0858
2.925	3.2	3.0625	0.0858	-0.112	-0.054
2.925	3.0625	2.994	0.0858	-0.054	0.006
3.006	3.06	3.028	0.006	-0.054	-0.02652
2.994	3.028	3.011	0.006	-0.02652	-0.0107
2.994	3.011	3.0023	0.006	-0.107	-0.0023

Soln = 3.00

a	b	t	f(a)	f(b)	f(t)
3.2	4	3.6	-0.112	2	0.336
3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.0461
3.4	3.45	3.425	-0.016	0.0461	0.013
3.4	3.425	3.4125	-0.16	0.013	-0.0019

Soln = 3.41

$$07 \quad f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 920x^3 + 54x^2 + 18x - 221$$

Assuming $x_0 = -0.54$

Simulating the equation on the calculator

$$x_1 = -0.0927$$

$$x_2 = -0.0408$$

$$x_3 = -0.04066$$

$\therefore$ One root is -0.04066

Assuming $x_0 = \cancel{0.7} 0.7$

$$x_1 = 1.4326$$

$$x_2 = 1.1599$$

$$x_3 = 1.0138$$

$$x_4 = 0.9671$$

$$x_5 = 0.9624$$

$$x_6 = 0.9624$$

Solutions are -0.04066 and 0.9624

18a $\frac{\sum x}{n} \quad (n = 4 \text{ here})$

b $80 \leq \frac{\sum x}{n} < 90$

$$80n \leq \sum x < 90n$$

c $n = 4$

$$320 \leq \sum x < 360$$

$$\sum x = 320$$

$\therefore$ B grade is earned

Q9 $(4, 8.54)$ $(20, -11.81)$ $(x, 0)$

$$\frac{8.54 - 0}{4 - x} = \frac{8.54 - (-11.81)}{4 - 20}$$

$$\frac{8.54}{4 - x} = \frac{20.35}{-16}$$

$$4 - x = \frac{8 \times -16 \times 8.54}{20.35}$$

$$x = 10.7145$$

Q10 Profit $[x]$ = $32x - 0.21x^2 - (195 + 12x)$

$$= -0.21x^2 + 20x - 195$$

$$-0.21x^2 + 20x - 195 > 0$$

Using the quadratic formula

$$x = \frac{-20 \pm \sqrt{20^2 - 4(-0.21)(-195)}}{2 \times (-0.21)}$$

$$= \frac{-20 \pm 15.3688}{-0.42}$$

$$x = 11.0267 \text{ or } 84.2114$$

$$(x - 11.0267)(x - 84.2114) > 0$$

Either both positive or both negative

$$\therefore x > 11.0267 \text{ and } x > 84.2114$$

OR

$$x < 11.0267 \text{ and } x < 84.2114$$

$$\therefore \text{Solution } x > 84.2114 \text{ or } x < 11.0267$$