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Financial Mathematics

Answer 1 -

$$APR = \frac{\text{average annual interest}}{\text{average loan amount}} = \frac{\text{average annual interest}}{\frac{1}{2} \times \text{original loan}} = \frac{2 \times \text{avg. annual interest}}{\text{original loan}}$$

$= 2 \times$ Flat rate

$\therefore$ Original loan amount = Rs 20,000

$$20000 = 12 \times 427.90\, a_{\overline{5}|}^{(12)} \Rightarrow a_{\overline{5}|}^{(12)} = 3.8499$$

$$\text{Flat rate} = \frac{\text{Total Interest}}{\text{Loan} \times \text{Total years}} = \frac{(5 \times 12 \times 427.90 - 20000)}{20000 \times 5} = 5.67\%$$

$APR = 2 \times 5.67 = 11\%$

$i = 11\% \quad a_{\overline{5}|}^{(2)} = 3.87872$

$i = 10\% \quad a_{\overline{5}|}^{(12)} = 3.96154$

Interpolation →

$$\frac{i - 10\%}{3.8499 - 3.90154} = \frac{11\% - 10\%}{3.87872 - 3.96514}$$

$i = 10.8\%$

At 10.8% → $12 \times 427.90\, a_{\overline{5}|}^{(12)} = 20000.2$

At 10.9% → $12 \times 427.90\, a_{\overline{5}|}^{(12)} = 19958.2$

⇒ APR is 10.8%

(ii) APR = 10.8%

After 1 year, capital left to pay $= 12 \times a_{4}^{(12)}$ @ $10.8 \times 427.90 = 16775.98$

Let t denote the time of new loan

$$16775.98 = 12 \times 274.49\, a_{t}^{(12)}$$

$t = 7.25$ years

(iii) The original loan had 4 years of payment remaing

Total Interest $= 4 \times 12 \times 427.90 - 16775.68$

$= 3763.22$

Total Interest = $7.25 \times 12 \times 274.9 - 16775.68 = 7104.65$

Answer 2 - I. Discounted Payback Period

The time for which the present value of the up to time t exceeds the present value the cost up to time t.

II. Payback Period

The payback period is the same as discount payback period, except the present value calculation is carried out using an interest rate of 0. Therefore, it is the time for which the monetary value of the return exceeds the monetary value of the costs.

(ii) Unlike the NPV, neither the PPP nor the PP give an indication how profitable a project is, as they ignore cash flows after the accumulated value zero is attained.

There may not be a single point of time when the balance on the investors account changes from negative to positive.

The PP can give misleading results, as it does not take in account the time value of money.

Hence, NPV is a superior measure as compared to PPP or PP.

(iii) Interest rate of return

The IRR of project A is the rate of interest that satisfies the equation of value.

$$17000 - 20000v - 10000v^2 + 20000v + 20000v^2 + 200000v^3$$

$$10v^2 + 200v^3 = 170$$

$$v = 0.9309$$

$$i = 0.074229 \approx 7.4\%.$$

(b) Net Present Value

Found by discounting the payments at 6%.

Project A NPV $= -170000 + 10000v^2 + 200000v^3 = 6823.82$

Project B NPV $= -200000 + 14000a_{\overline{6|}} + 200000 \cdot v^6 = 9834.645$

(c) Maximum interest rate beyond which projects are unprofitable. Each project will be profitable if the rate of interest at which funds can be borrowed is less than the internal rate of return.

Project A requires a borrowing rate less than 7.4229% and project B requires a rate lower than 7%

Moreover,

- Project A does not provide any net income during the first year
- The lower internal rate of return for Project B applies for a longer period
- Rate of interest available in years 4 to 6 (i.e. after Project A) finished will against the comparison between the accumulated profits at the end of year.

Answer 3- Value of 1st Nov 1985 is

Annuity 1 $\Rightarrow 200 \times v^{1/4} \times \ddot{a}_{22}$ at 8%.

$= 200 \times v^{1/4} \; a_{\overline{22}|} \times \frac{i}{d}$

$= 2161.366301$

Annuity 2 $\Rightarrow 320 (1+i)^{1/12} \times a^{(4)}_{\overline{16.25}|}$ at 8%. $= 2957.870081$

Annuity 3 $\Rightarrow 180 \; a^{(12)}_{\overline{18.75}|} = 1780.664166$

let the revised annuity be A

Value of revised annuity is $X(1+i)^{1/4} \times a^{(2)}_{\overline{21.5}|} (1+i)^{1/4}$

$A = 656.56$

Answer 4- $(I\ddot{a}_n)$ for $i>0$

$(I\ddot{a})_{\overline{n}|} = 1+2v+3v^2+4v^3 \cdots nv^{n-1}$ — ①

$(I\ddot{a})_{\overline{n}|} = (1+i)(Ia)_{\overline{n}|}$

$= \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

Answer 5- Time weighted rate of return for each fund for the year 2014

Property Fund $16.4/12.4 - 1 = 32.26\%$

Equity Fund $15.5/12.1 - 1 = 28.10\%$

(a) Assuming that the investor bought 100 units per quarter in property fund the equation of value is -

$$1240\,(1+i) + 1310(1+i)^{3/4} + 1420\,(1+i)^{1/2} + 1580\,(1+i)^{1/4} = 4\times 1640$$

$$i = 29.32\%$$

(b) Assuming that the investor purchases ₹100 worth of units

$$400 \times \ddot{s}_{1}^{(4)} = 100 \times 16.4\left(\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{16.8}\right)$$

$$\ddot{s}_{1}^{(4)} = 1.1801$$

(ii) $$1210\,(1+i) + 920\,(1+i)^{3/4} + 1030\,(1+i)^{1/2} + 1310\,(1+i)^{1/4} = 4\times 1550$$

$$i = 67.31\%.$$

(b) $$400 \times \ddot{s}_{n}^{(4)} = 100 \times 15.5\left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right)$$

$$\ddot{s}_{\overline{1|}}^{(4)} = \cancel{1.4}\ 1.413458$$

$$\frac{(1+i)-1}{d^{(4)}} = 1.413458$$

$$\Rightarrow i = 70.89\%.$$

Answer 6 - X : Initial amount repayment

$$160000 = X\, a_{\overline{10|}} \ @\ 8\%.$$

$$X = 23,844.65$$

(ii) The loan outstanding immediately after 4th payment is

$$23844.65\, a_{\overline{6|}} \ @\ 8\%.$$

$$= 110.231.442$$

Y : Revised annual investment

$$Y a_{\overline{6|}} = 110231.44\ @\ 10\%.$$

$$Y = 25309.72$$

(iii) The loan outstanding immediately after 7th payment

$$25309.72\, a_{\overline{3|}} \ @\ 10\%.$$

$$= 62942.74$$

Z : Revised annual installment

$$Z a_{\overline{3|}} = 62\cancel{3}942.74\ @\ 9\%.$$

$$Z = 24865.77$$

Answer 7 - Investing in property may not be preferred alternative to investing in securities. The return on property is lower that that in investment securities.

It is difficult to invest in property because -

(i) Less flexible because of the large unit size

(ii) The marketability for property is poor

(iii) Large buying and selling expenses

Answer 8 - The present value of outlay is

$2.7 + a_{\overline{3}|}$ @ i

The expected PV of income is

$1.03 v^3 + \bar{a}_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3)$ @ i

$2.7 + 0.2\, a_{\overline{3}|} = 0.1 v^2 + \bar{a}_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3)$ @ i

(ii) Based on the information PV of the company is

$0.1 v^3 + 0.85\, \bar{a}_{\overline{10}|} v^3$

$= 1.0089$

+ve NPV ⇒ Worthy Investment

TWRR

$$(1+i)^7 = \frac{33}{30.50} \times \frac{41.05 - 4.5}{33+6} \times \frac{45.6}{41.05} \times \frac{47}{45.6 - 2.50}$$

$i = 0.070951281 \approx 7.0951281\%$

MWRR

$30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} - 47$

$X = 1.07204$

$i = 1.07204 - 1 = 7.204\%$

(iii) Linked rate of return

Year 2011 - Eqn of value $30.5(1+i) + 6(1+i)^{0.25} = 38.50$

$X = 1.0626$

$i = 0.0626 \approx 6.26\%$

Year 2012 . $38.50(1+i) + 4.50(1+i)^{0.5} = 45$

$i = 0.0492 \approx 4.92\%$

$\cdot 22 \times 12 \times \cancel{274} \cdot 9$ -

Year 2015 $\quad 1.5(1+i) - 2.5(1+i)^{0.5} = 47$

$i = 0.10279 \quad = 10.279\%$

Linked rate of return

$(1+i)^3 = (1.0626)(4.0492)(1.102791)$

$i = 7.1289303\%$

(iii) TWRR eliminates the effect of the cash flow amount

Answer 10 - Let loan amount be L

$L = 180000\, a_{\overline{15}|} + 20000\, (Ia)_{\overline{15}|}$ @ 12%

$L = 2040588$

Total cost of house = 2550733

Loan outstanding at the beginning of the year →

$L = (180000v + 180000v^2 \ldots 180000v^7) + [(\cancel{8} \times 20000v) + (10 \times 20000v^2) \ldots (15 \times 20000v^7)]$

$L(9) = 1875850.33$

Ans 12 - The discounted payback period of an investment

$-800(1+i)^t - 50(1+i)^{t-1} \cdot + 100\, \bar{s}_{\overline{t-2}|}$ @ 7% $= 0$

$t = 17.767$ yrs

(b) $100\, \bar{s}_{\overline{41.279}|}$ @ 6% $= 100 \times \dfrac{(1+0.06)^{4.283} - 1}{\ln(1.06)}$

Accumulated amt $= 480074.2066$

Ans 13 - Let $\frac{x}{12}$ be the monthly repayment

$X a^{(12)}_{\overline{25}|} = 9880$ @ 7%

$x = 821.7669096$

$x/12 = 68.4805758$

Hence, the monthly repayment is ~~68.4805~~ 6848.05758

Loan o/s immediately after repayment on 10th March 2029

$$X a^{(12)}_{\overline{24/3}|} = 64\ 821.706906\ \frac{\left(1-\left(1/1.07\right)^{24/3}\right)}{0.067850}$$

$$= 648574.5315$$

(iii) The loan o/s just after the repayment on 10th Sept 2020 is $X a^{(12)}_{83/6}$ @ 7%.

10th Oct 2020 $X a^{(12)}_{85/4}$ @ 7%.

capital repayment on 10th Oct 2028 is

$$X\left[a^{(12)}_{83/16} - a^{(4)}_{55/4}\right] @ 7\%.$$

$$X\left[\frac{v^{13.75} - v^{83/6}}{i^{(2)}}\right] @ 7\%.$$

$$X\left[\frac{(1/1.07)^{13.75} - (1/1.07)^{83/4}}{0.067850}\right] = 26.85902099$$

(iv) The capital repayment contained in these 12 installments

$$a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{14/3}|} @ 7\%.$$

$$= 516.1973861$$

Total interest = 821.7069096 − 516.1973861 = 305.5695233