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Algebra Assignment

① $r = 12.5$ cm

$r' = 12.65$ cm

Actual area: $\pi r^2 = \pi(12.5)^2$

$= \underline{490.874 \text{ cm}^2}$

Calculated area $= \pi r'^2$

$= \pi(12.65)^2 = \underline{502.725 \text{ cm}^2}$

i. Absolute error $= 502.725 - 490.874$

$= \underline{11.851 \text{ cm}^2}$

ii. Relative error $= \dfrac{11.851}{490.874} = \underline{0.02414}$

iii. Percentage error $= \underline{2.414\%}$

②

x	$f(x) = \log(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513

a. $y = 0.60206 + (x-4)\left(\dfrac{0.7781513 - 0.60206}{6-4}\right)$

$y = 0.60206 + (5-4)\left(\dfrac{0.1760913}{2}\right)$

$\underline{y = 0.69010565}$

b. $y = 0.6532125 + (5-4.5)\left(\dfrac{0.7403627 - 0.6532125}{5.5 - 4.5}\right)$

$y = 0.6532125 + (0.5)(0.0871502) = \underline{0.696788}$

③ $x^4 - x - 10 = 0$

$a = 1.5$

$b = 2$

$\Rightarrow \quad x_1 = \dfrac{a+b}{2} = \dfrac{3.5}{2} = 1.75$

$y_1 = (1.75)^4 - (1.75) - 10 = -2.371094$

$\Rightarrow \quad x_2 = \dfrac{x_1+b}{2} = \dfrac{1.75+2}{2} = 1.875$

$y_2 = (1.875)^4 - (1.875) - 10 = \cancel{0.0} \; 0.48462$

$\Rightarrow \quad x_3 = \dfrac{x_1+x_2}{2} = \dfrac{1.75+1.875}{2} = 1.8125$

$y_3 = (1.8125)^4 - (1.8125) - 10 = -1.020249$

$\Rightarrow \quad x_4 = \dfrac{x_2+x_3}{2} = \dfrac{1.875+1.8125}{2} = 1.84375$

$y_4 = (1.84375)^4 - 1.84375 - 10 = -0.2877$

$\Rightarrow \quad x_5 = \dfrac{x_2+x_4}{2} = \dfrac{1.875+1.84375}{2} = 1.859375$

$y_5 = (1.859375)^4 - (1.859375) - 10 = 0.093375$

$\Rightarrow \quad x_6 = \dfrac{x_4+x_5}{2} = \dfrac{1.84375+1.859375}{2} = 1.8515625$

$y_6 = (1.8515625)^4 - (1.8515625) - 10 = -0.098433$

$\therefore \quad x = 1.859375$ is the required solution.

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(4) $f(x) = x^3 - x - 1$ $\qquad f'(x) = 3x^2 - 1$

x	$f(x)$	$f'(x)$
0	-1	-1
1	-1	2
2	5	11

$x_0 = 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 1 - \left(\frac{-1}{2}\right)$

$x_1 = \frac{3}{2}$ $\qquad y_1 = 0.875$

$f'(x_1) = 5.75$

$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1.3478$ $\qquad f'(x_2) = 4.4496$

$y_2 = 0.10056$

$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3252$

$y_3 = 0.002056$ $\qquad f'(x_3) = 4.268$

$\Rightarrow x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.3247$

$y_4 = -0.00007658$

(5) $A^2 = A$,

$\Rightarrow 7A - (I+A)^3 = 7A - (I^3 + A^3 + 3IA^2 + 3I^2A)$

$= 7A - (I + A + 3A + 3A) = -I$

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(6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$ $|A| = -6 + 4(+1) + 0$

$= -2$

$\text{Adj } A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 1 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj } A}{|A|} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1/2 \\ -2 & 1/2 & -2 \end{bmatrix}$

(7) $A = 8i - 9j$ $B = 7i - 9j$

$\vec{A} = \sqrt{64+81} = \sqrt{145}$

$\vec{B} = \sqrt{49+81} = \sqrt{130}$

$\cos\theta = \dfrac{137}{\sqrt{145}\sqrt{130}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{137}{\sqrt{145}\sqrt{130}}\right)$

(8) $\vec{R} = \vec{a} + \vec{b}$

$= |\vec{a}| + |\vec{b}| + 2\vec{a}\vec{b}$

$= 75^2 + 25^2 + 2(75)(25)\dfrac{\sqrt{3}}{2}$

$R^2 = 9497.5952$

$\vec{R} = 97.56\,m$

(9) $a = 101$ $a_n = a + (n-1)d$

$a_n = 998$ $n = 300$ $S_n = \dfrac{n}{2}[a + a_n]$

$d = 3$

$= 150[101 + 998]$

$= 164850$

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(10) $ax^5 = 32$ $\frac{ax^7}{ax^5} \Rightarrow \underline{x = 2}$

$ax^7 = 128$

(11) $(4+3x)^5$

Using binomial expansion:

$$(4+3x)^5 = {}^5C_0 (4)^0 (3x)^5 + {}^5C_1 (4)^1 (3x)^4 + {}^5C_2 (4)^2 (3x)^3 + {}^5C_3 (4)^3 (3x)^2 + {}^5C_4 (4)^4 (3x) + {}^5C_5 (4)^5 (3x)^0$$

$$(4+3x)^5 = 243x^5 + 162x^4 + 4320x^3 + 7560x^2 + 3840x + 1024$$

(12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow A - \lambda I = 0 \Rightarrow \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$$(2-\lambda)\begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} - (-3)((3-\lambda)(2)-0) = 0$$

$\Rightarrow$ $\lambda^3 - 13\lambda + 12 = 0$

$a = 1$ $b = -13$ $c = 12$

$a + b + c = 0$

$\therefore \underline{\lambda = 1}$

1	0	-13	12
1	1	1	-12
1	1	-12	0

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda+4) - 3(\lambda+4) = 0$

$(\lambda-3)(\lambda+4) = 0$

$\lambda = 1, \quad \lambda = 3, \quad \lambda = -4$

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(13) $f(x) = x^3 - 7x^2 + 8x - 3$ $\quad x_0 = 5$

$\Rightarrow f(x_0) = -13 \quad f'(x_0) = 13$

$x_1 = 5 - \left(\frac{-13}{13}\right) = 6$

$f(x_1) = 9 \quad f'(x_1) = 32$

$\Rightarrow x_2 = 6 - \left(\frac{9}{32}\right) = \underline{5.71875}$

(14) $(1-x+x^2)^4$, Let $1-x = y$

$(y+x^2)^4$

$\Rightarrow {}^4C_0\, y^0 (x^2)^4 + {}^4C_1\, y^1 (x^2)^3 + {}^4C_2 (y^2)(x^2)^2 + {}^4C_3\, y^3 (x^2) + {}^4C_0\, y^4$

$= (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

(15) $e^{-x} = 3\log x$

$f(x) = e^{-x} - 3\log x$

x	$f(x)$
0.5	1.50962
1	0.36757
1.5	-0.305143

$x_1 = 1.25$

$y_1 = -0.0042252$

$\Rightarrow x_2 = \frac{1+1.25}{2} = 1.125$

$y_2 = 0.1711949$

$x_3 = 1.1875$

$y_3 = 0.0810919$

$\Rightarrow x_4 = 1.21875$

$y_4 = 0.0378555$