# Assignment

Q1]

$R = 12.65$, $r = 12.5\,cm$

Absolute error $= |r - R|$

$= |12.5 - 12.65|$

$= |-0.15|$

$= \boxed{0.15}$

Relative Error $= \dfrac{\Delta r}{r}$

$= \dfrac{0.15}{12.5} = \boxed{0.012}$

Relative Error =

% Error $= \dfrac{\Delta r}{r} \times 100$

$= \dfrac{0.15}{12.5} \times 100$

% Error $= 1.2\%$

Q2]

| $x$ | $f(x)$ |
|---|---|
| 4 | 0.60206 |
| 4.5 | 0.6532125 |
| 5.5 | 0.7403627 |
| 6 | 0.7781513 |

| $x$ | $y$ | $\Delta_1 y$ | $\Delta_2 y$ |
|---|---|---|---|
| 4 | 0.60206 | | |
| | | 0.050613 | |
| | | | 0.036537 |
| 4.5 | 0.653212 | | |
| | | 0.08715 | |
| | | | -0.04362 |
| 5.5 | 0.740363 | | |
| | | 0.037788 | |
| 6 | 0.778151 | | |

Q2) $x^4 - x - 10 = 0$,

Let $a = 1.5$, $b = 2$.

$f(a) = (1.5)^4 - 1.5 - 10$

$= \cancel{13.5625} = \boxed{-6.9375} < 0$

$f(b) = 2^4 - 2 - 10$

$= 16 - 2 - 10$

$= 4 > 0$

Thus, let our first iterator be:

$x_1 = \frac{a+b}{2} = \frac{1.5+2}{2} = \frac{3.5}{2} = \boxed{1.75}$

Then, $f(1.75) = (1.75)^4 - 1.75 - 10$

$= -2.62109$

Again, $x_2 = \frac{x_1 + b}{2} = \frac{1.75 + 2}{2} = \boxed{1.875}$

$f(1.875) = (1.875)^4 - 1.875 - 10$

$= \underline{0.48462}$

Again, $x_3 = \frac{x_2 + x_1}{2} = \boxed{1.8125}$

$f(1.8125) = -1.020248$

Again, $x_4 = \frac{x_2 + x_3}{2} = \frac{1.875 + 1.8125}{2}$

$= 1.84375$

$f(1.84375) = -0.28773$

$x_5 = \frac{x_4 + x_2}{2} = \frac{1.875 + 1.84375}{2}$

$= 1.859125$

$f(1.859125) = \underline{0.08720105}$

Q4) $f(x) = x^3 - x - 1$

In this method, we use the formula:-

$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$

Now

| $x$ | $y$ |
|---|---|
| 0 | $-1$ |
| 1 | $-1$ $\rightarrow$ start |
| 2 | 5 |

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = f'(1)$

$= [3x^2 - 1]$ at $x = 1$

$= 3 - 1 = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1 - \frac{(-1)}{(2)}$

$= 1 + \frac{1}{2} = \frac{3}{2} = 1.5$

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$f(x_1) = f(1.5)$

$= (1.5)^3 - 1.5 - 1$

$= 0.875$

$x_1 = 1.5$

$f(x_1) = 0.875$

$f'(x_1) = (3x^2 - 1)$ at $(1.5)$

$= 3(1.5)^2 - 1 = 5.75$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5 - \frac{0.875}{5.75}$

$= 1.5 - 0.1521739$

$= \boxed{1.347826}$

$f(x_2) = 0.10068$

$f'(x_2) = 3(1.347826)^2 - 1$

$= 4.44991$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.347826 - \frac{0.10068}{4.44991}$

$x_3 = 1.325201$

$f(x_3) = 0.0020609$

$f'(x_3) = 3(1.325201)^2 - 1$

$= 4.268473$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.325201 - \frac{0.0020609}{4.268473}$

$= 1.324718$

$f(x_4) = 0.00000018235$

$\boxed{SO}$

$\begin{bmatrix} \cdot & 0 \\ \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$

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Q5) $A^2 = A$

Pre-multiplying by inverse of A, [assuming that $|A| \neq 0$, i.e there exists a ~~matr~~ inverse matrix of A], then:-

$A^{-1}(A \times A) = A^{-1}$

$(A^{-1} \times A) \times A = A^{-1} \times A$ [Associativity of Matrix]

$I \times A = I$

$\boxed{A = I}$ [since $A^{-1}A = AA^{-1} = I$ and by identity $I \times A = A \times I = A$]

Value of $7A - (I+A)^3$

$\longrightarrow 7I - (2I)^3$

$\longrightarrow 7I - 8I^3$

$\Longrightarrow 7I - 8I$ [since $I^n = I \ \forall \ n \in \mathbb{N}$]

$\Rightarrow -I$

$\Rightarrow -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Q6) Since, to prove that the inverse of the matrix exists:-

$|A| \neq 0$, thus

$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{vmatrix} = 1(0-6)+1(-4-0)+2(4-0)$

$= -6-4+8$

$= -10+8 = \boxed{2 \neq 0}$

Thus, the matrix exists.

Cofactors of an elements is $= (-1)^{i+j} \begin{vmatrix} a_{i+1\,j+1} & a_{i+2\,j+1} \\ a_{i+2\,j+1} & a_{i+2\,j+2} \end{vmatrix}$

calculated by

Cofactor of $a_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix}$

$= \boxed{-6}$

Cofactor of $a_{12} = (-1)^{1+2} \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} = \boxed{-4}$

Cofactor of $a_{13} = (-1)^{1+3} \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = \boxed{4}$

Cofactor of $a_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = \boxed{-1}$

" of $a_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = \boxed{-1}$

Cofactor of $a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = \boxed{1}$

Cofactor of $a_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & 6 \end{vmatrix} = \boxed{-6}$

Cofactor of $a_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = \boxed{-2}$

Cofactor of $a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 4 & 0 \end{vmatrix} = \boxed{4}$

Cofactor Matrix is $= \begin{bmatrix} -6 & -4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix} \Rightarrow \boxed{C}$

Transpose of $C \equiv$ Inverse of $A$.

Thus

$A^{-1} = {}^{T}\begin{bmatrix} -6 & -4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} -6 & -1 & -6 \\ -4 & -1 & -2 \\ 4 & 1 & 4 \end{bmatrix}$$

Q7) $\bar{a} = 4\hat{i} - 9\hat{j}$

$\bar{b} = 7\hat{i} - 9\hat{j}$

Since, $\bar{a}\cdot\bar{b} = \sum^{n} a_i \cdot b_i$

$\bar{a}\cdot\bar{b} = (4\cdot 7) + (-9)(-9)$.

$= 28 + 81$

$\bar{a}\cdot\bar{b} = 109$.

By vector formula,

$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$.

$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$

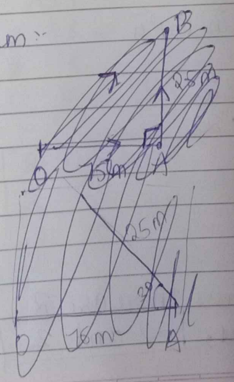
$= \dfrac{109}{\sqrt{4^2+9^2}\sqrt{7^2+9^2}}$

$\boxed{\because |av| = \sqrt{\sum a_i^2} \text{ where } v = a_1\hat{i} + a_2\hat{j}}$

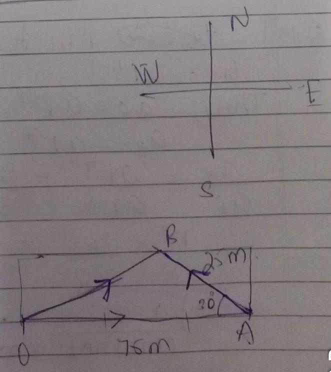
$= \dfrac{109}{\sqrt{97}\sqrt{130}}$ $= 0.97066407$

$= \boxed{13.91249^\circ}$

Q8) Diagram:







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By Vector Addition,

$\overline{OB} = \overline{OA} + \overline{AB}$

By Formula of Magnitude :-

$|\overline{OB}| = \sqrt{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos\theta}$

$= \sqrt{75 + 25 + 2\sqrt{75}\sqrt{25}\cos 30^\circ}$

$= \sqrt{100 + \cancel{2}\sqrt{75}\sqrt{25}\frac{\sqrt{3}}{\cancel{2}}}$

$= \sqrt{100 + \sqrt{75 \times 25 \times 3}}$

$= \sqrt{175}$

$= \boxed{13.22875}$

Q9) The type of numbers are $\Rightarrow 3n+2$

Thus

$\sum 3n+2 = 3\sum n + 2\sum 1$

$= 3\left(\frac{n(n+1)}{2}\right) + 2n$

$= \frac{3}{2}n(n+1) + 2n.$

Q10) ~~tn =~~ nth term in G.P is :-

$t_n = ar^{n-1}$

Thus, $a_6 = ar^5 = 32$

$a_8 = ar^7 = 128$

$\frac{a_8}{a_6} = \frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = \frac{128}{32} = 4$

$\Rightarrow \boxed{r = \pm 2}$, Thus there are 2 common ratios, $\boxed{-2 \text{ and } 2}$

Shot on Y83 Pro
vivo dual camera

Q11] $(4+3x)^5$ By Binomial Theorem,

$\cancel{(4+3x)^5}$

$$(ax+b)^n = \sum_{i=0}^{n} {}^nC_i\, a^{n-i} b^i, \quad \boxed{\begin{array}{l}\text{Here } a=3,\\ b=4,\ n=5\end{array}}$$

$$\begin{aligned}(3x+4)^5 = {} & {}^5C_0 (3)^5 (4)^0 + \\ & {}^5C_1 (3)^4 (4)^1 + \\ & {}^5C_2 (3)^3 (4)^2 + \\ & {}^5C_3 (3)^2 (4)^3 + \\ & {}^5C_4 (3)^1 (4)^4 + \\ & {}^5C_5 (3)^0 (4)^5\end{aligned}$$

$$= (1) \times 3^5 (1) + 5(3^4)(4) + \frac{5 \times 4}{2}(3^3)(4^2) + \frac{5 \times 4 \times 3}{3 \times 2}(3^2)(4^3) + 5 \times 3 \times 4^5 + 1 \times 4^5$$

$$= 3^5 + 20 \times 3^4 + 10 \times 4^2 \times 3^3 + 10(3^2)(4^3) + 15 \times (4^5) + (4^5)$$

$$= \boxed{28,327}.$$

Q12] Eigenvalues :-

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda) \begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} + 3 \begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} + 0 = 0.$$

$$y = \frac{9-28}{5} = 9 - 5.6 = 0.4$$

Q) Find Eigenvalues of $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

Ans) Consider $|A - \lambda I| = 0$

$$\left|\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right| \Rightarrow 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\left[(-5-\lambda)(3-\lambda) - 0\right] + 3 \times 2(3-\lambda) + 0 = 0$$

$$\Rightarrow \cancel{(2-\lambda)[}$$

$$\Rightarrow (2-\lambda)(-5-\lambda)(3-\lambda) + 6(3-\lambda) = 0$$

$$\Rightarrow (3-\lambda)\left[(2-\lambda)(-5-\lambda) + 6\right] = 0$$

$$\Rightarrow \boxed{\lambda = 3} \quad \text{OR} \quad \left[-4 + \lambda^2 + 3\lambda\right] = 0$$

$$\Rightarrow \boxed{\lambda = 3} \quad \text{OR} \quad (\lambda^2 + 3\lambda - 4) = 0$$

$$\Rightarrow \boxed{\lambda = 3} \quad \text{OR} \quad (\lambda + 4)(\lambda - 1) = 0$$

$$\Rightarrow \boxed{\lambda = 3}, \boxed{\lambda = -4}, \boxed{\lambda = 1}$$

Find Eigen Vector:- for $\boxed{\lambda = 3}$

* Each value has a different vector.

for $\lambda = 3$,

$(A - \lambda I)X = 0$.

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ECHLON FORM :- Make as many as zeroes as possible

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow \boxed{z=0}$, $\boxed{x = \text{undeterminate}}$.

When $\lambda = 1$ :-

$$(A - \lambda I)X = 0$$

$$\left( \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\cancel{-\frac{1}{6}R_2, \; R_1 \Rightarrow R_1 + 3R_2}$.

$$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}.$$

$-\frac{1}{2}R_2$

$$\begin{bmatrix} 1 & -3 & 0 \\ -1 & +3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \boxed{z=0}$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\Rightarrow \frac{1}{2}R_2 , \frac{1}{3}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{x=0}$ , $\boxed{y=0}$ , $\boxed{z=0}$

0)

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 3R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2x_1 - x_2 \\ 7x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvector will be : $\begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix}$ OR $\begin{bmatrix} x_2/2 \\ x_2 \\ 0 \end{bmatrix}$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$ if $\boxed{x_0 = 5}$.

By Newton Raphson Method :-

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Thus, here $x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = [3x^2 - 14x + 8]_{x=5}$

$= \boxed{13}$

~~$x_2 = x_1 - f($~~

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 5 - \frac{(-13)}{(13)} = 5 + 1 = \boxed{6}$

$f(x_1) = 9$.

$f'(x_1) = 3x^2 - 14x + 8 = 32$

$x_2 = x_1 - \frac{9}{32}$

$= \boxed{5.7188}$

Q14) $(1 - x + x^2)^4$

$= (1 + x(x-1))^4$

$= {}^4C_0 (1)^4 (x(x-1))^0 + {}^4C_1 (x(x-1))^1$

$+ {}^4C_2 (x(x-1))^2$

$+ {}^4C_3 (x(x-1))^3$

$+ {}^4C_4 (x(x-1))^4$

$= 1 + 4x(x-1) + \frac{4 \cdot 3}{2} [x(x-1)]^2$

$+ 4 (x(x-1))^3$

$+ (x(x-1))^4$

$= 1 + 4x(x-1) + 4(x(x-1))^2 + 4(x(x-1))^3$

$+ (x(x-1))^4$

Q15) $e^{-x} = 3\log x$

$-x = \log(3\log x)$

$\boxed{\log(3\log x) + x = f(x)}$

| x | y |
|---|---|
| 1 | 1 |
| 2 | 1.955731 |
| 3 | 2.035886 |

Q5) $e^{-x} - 3\log x = f(x)$

| x | y |
|---|---|
| 1 | 0.367879 $\longrightarrow$ $\boxed{\text{Start}}$ |
| 2 | −0.767754 |

Here, $a = 1,\ b = 2$

$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$f(x_1) = -0.305147$

Again,

$x_2 = \frac{a + x_1}{2} = \frac{1+1.5}{2} = 1.25$

$f(x_2) = -0.0042252$

$x_3 = \frac{x_2 + a}{2} = \frac{1.25+1}{2} = \frac{2.25}{2} = 1.125$

$f(x_3) = 0.17119$

$x_4 = \frac{x_2 + x_3}{2} = \frac{1.25 + 1.125}{2} = 1.1875$

$f(x_4) = 0.081082$

(4) $x_5 = \frac{x_4 + x_2}{2} = \frac{1.25 + 1.1875}{2} = \boxed{1.21875}$

$\boxed{f(x_5) = 0.0378555}$