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Financial Engineering

Assignment 2

Q1. $S_0 = ₹ 65$

$\sigma = 25\%$ p.a

$r = 2\%$ p.a

$K = ₹ 55$

$T = 6$ months $= \frac{6}{12} = \frac{1}{2}$

To find :- $C_0 = ?$

(i) $C_t = S_0 \Phi(d_1) - K e^{-rT} \Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}}$$

$$\therefore d_1 = \frac{\ln\left(\frac{65}{55}\right) + \left[0.02 + \frac{1}{2} \times (0.25)^2\right] 0.5}{0.25 \times \sqrt{0.5}}$$

$$\boxed{\therefore d_1 = 1.089957}$$

$d_2 = d_1 - \sigma\sqrt{T}$

$= 1.089957 - 0.25\sqrt{0.5}$

$$\boxed{\therefore d_2 = 0.91318}$$

$C_t = 65\, \Phi(1.0891) - 55\, e^{-0.02 \times 0.5}\, \Phi(0.91318)$

$$\boxed{\therefore C_t = 11.4646}$$

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(ii) $\Delta_c = \dfrac{\text{Change in price of call option}}{\text{Change in price of underlying}}$.

(iii) $\Delta c = \Phi\, d_1$

$\Delta p = -\Phi(-d_1)$

$\Delta c = 0.86214$

(iv) $\Delta p = -0.13786$

Q2.

(i) $\Delta = \dfrac{\text{change in price of derivative}}{\text{Change in price of underlying asset}}$

Rate of change of option price with respect to change in price of underlying asset.

$\nu = \dfrac{\text{Change in price of option.}}{\text{Change in volatility of underlying asset.}}$

Rate of change of option price w.r.t. change in volatility of underlying asset.

(ii) Put call parity :-

$$C - P = S - Ke^{-rT}$$

$$\frac{d}{d\sigma}(C - P) = \frac{d}{d\sigma}(S - Ke^{-rT})$$

$$\nu_c - \nu_p = 0 \qquad (\because S \text{ and } K \text{ are constants})$$

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$$\boxed{V_c = V_p}$$

8 (iii)

$S_0 = \$55$
$X = \$50$
$\sigma = 25\%$ p.a
$r = 5\%$
$T = 1$

$$C_t = S_0\, \Phi(d_1) - Ke^{-rT}\, \Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{55}{50}\right) + \left[0.05 + 0.5 \times (0.25)^2\right] \times 1}{0.25\sqrt{1}}$$

$$\therefore \boxed{d_1 = 0.706241}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\therefore \boxed{d_2 = 0.456241}$$

$$C_t = 55 \times (0.706241)\Phi - 50\, e^{-0.05}\, (0.456241)\Phi$$

$$\therefore C_t = 55 \times (0.76115) - 50\, e^{-0.05}\, (0.67724)$$

$$\therefore C_t = \$9.65$$

$$C - P = S - Ke^{-rT}$$

$$9.65 - P = 55 - 50\, e^{-0.05 \times 1}$$
$$\therefore 9.65 - P = 7.43853$$

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$\therefore P = \$ 2.211$

(iv) Delta Hedging.

A portfolio for which the overall delta is equal to 0 is described as delta hedged or delta neutral. The portfolio is immune to changes in the price of the underlying asset.

Vega Hedging :-

A portfolio for which the overall vega is equal to 0 is described as vega hedge or vega neutral. The portfolio is immune to small changes in the assumed level of volatility.

(v) X units of call option
Y units of put option
Z units of forward.

	C.F @ PV	Δ	ν
Call	9.65	Δc	ν_c
Put	2.211	Δp	ν_p
Forward	-	1	0

Δ of forward = 1
~~Δ~~ ν of forward = 0

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Vega Hedging

$$x V_c + y V_p + z V_F = 0$$

$$\boxed{\because V_F = 0}$$

$$x V_c + y V_p = 0$$

$$V_c = V_p$$

$$V_c (x + y) = 0$$

$$x + y = 0$$

$$\therefore \boxed{y = -x} \quad \text{— (i)}$$

Delta Hedging.

$$x \Delta_c + y \Delta_p + z \Delta_F = 0$$

Now,

$$(\Delta_F = 1, \; y = -x, \; \Delta_p = \Delta_c - 1)$$

$$x \Delta_c + (-x)(\Delta_c - 1) + z = 0$$

$$x + z = 0$$

$$\therefore \boxed{x = -z} \quad \text{— (ii)}$$

$$\boxed{x = -y = -z}$$

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Portfolio = 1000

$x \cdot C + y \cdot P + z \cdot F = 1000$

$(\because F = 0)$

$x C + y P = 1000$

$\therefore x(9.65) + y(2.211) = 1000$

$9.65x + (2.211 - x) = 1000$

$\therefore 9.65x - 2.211x = 1000$

$7.439x = 1000$

$\boxed{x = 134.43}$

$\boxed{y = z = -134.43}$

$\therefore$ Portfolio :-

1) Long position on 134 call options
2) Short position on 134 put options
3) Short position on 134 forwards

Q3.

i) $C_t = E\left[e^{-r(T-t)} \frac{C_T}{F_t}\right]$

$F_t \rightarrow$ Filteration at $t > 0$

$C_T \rightarrow$ Payoff under derivative at T

$C_t \rightarrow$ derivative value at time t.

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(ii) $S_0 = ₹\,50$
$X = ₹\,49$
$r = 5\%$ p.a
$\sigma = 25\%$ p.a
$T = 0.5$

$$C_t = S_0\,\Phi(d_1) - K e^{-rT}\,\Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{50}{49}\right) + \left(0.05 + 0.5 \times 0.25^2\right)0.5}{0.25\sqrt{0.5}}$$

$\therefore d_1 = 0.3441$

$\therefore d_2 = d_1 - \sigma\sqrt{T}$
$= 0.3441 - 0.25\sqrt{0.5}$
$\therefore d_2 = 0.167317$

$\Phi(d_1) = 0.6346$

$\Phi(d_2) = 0.5664$

$$C_T = S_0\,\Phi(d_1) - K e^{-rT}\,\Phi(d_2)$$

$$= 50 \times 0.6346 - 49\, e^{-0.05 \times 0.5}(0.5664)$$

$$\boxed{C_T = 4.66}$$

(iii)

Same as european call $= 4.66$

(iv) $S_t + P = C + Ke^{-rt}$

$$P = 4.66 + 49\, e^{-0.05 \times 0.5} - 50$$

$$\therefore \quad P = 2.45$$

(v) If the stock pays a dividend the price of the underlying would fall. This will lead to a fall in the price of the European call and a a rise in price of the European put.
Due ~~of~~ to the potential early exercise opportunity, the American call would be more expensive than the European call.

Q4. $Z_t \sim$ SBM under P
$\gamma_t \rightarrow$ previsible process.

$\therefore$ There exists a measure Q equivalent to P where,

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$$

$\tilde{Z}_t \longrightarrow$ SBM under Q.

If Z_t is a SBM under P and if Q is equivalent to P then there exists a previsible process γ_t such that:

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$$

$\tilde{Z}_t \rightarrow$ SBM under Q

(iii) The discounted value of asset prices are martingales.

Q5.

(i) $\Delta = \Phi(d_1)$

$= \dfrac{\text{Change in price of underlying asset}}{\text{Change in price of call option}}$

(ii) $d_1 = \dfrac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$

$0.3 = \dfrac{\ln\left(\frac{40}{45.91}\right) + (0.02 + 0.5 \times \sigma^2)5}{\sigma\sqrt{5}}$

$0.3 \times \sqrt{5} = \dfrac{-0.138456 + (0.1 + 2.5\sigma^2)}{\sigma}$

$\sigma - 2.5\sigma^2 = -0.05634$

$2.5\sigma^2 - 0.670836\sigma = 0.03780$

$\therefore \quad \sigma = \dfrac{0.67083 \pm \sqrt{0.67083^2 - 4 \times 2.5 \times (-0.03780)}}{2 \times 2.5}$

$\therefore \sigma = 32\%$

(iii) $R_0 = \$30$

$\sigma = \sqrt{15}\ \%$ p.a

Option payoff $= C$ at time T if

$\dfrac{S_1}{S_0} > KS, \quad \dfrac{R_1}{R_0} < KR$

Payoff $= Ce^{-rT} Q\left(\dfrac{S_1}{S_0} < K_S\right) Q\left(\dfrac{R_1}{R_0} < K_R\right)$

(iv) Perfect Correlation:-

$\left(\dfrac{R_1}{R_0} = \dfrac{S_1}{S_0}\right)$

Price $= Ce^{-rT} Q\left(\dfrac{R_1}{R_0} < K_S\right) = Ce^{-rT} Q\left(\dfrac{S_1}{S_0} < K_S\right)$

$= Ce^{-rT} Q\left[\dfrac{R_1}{R_0} < \min(K_S, K_R)\right]$

$= Ce^{-rT} Q\left[\dfrac{S_1}{S_0} < \min(K_S, K_R)\right]$

(v) Price $= Ce^{-rT} Q\left(\dfrac{S_1}{S_0} < K_S\right) \cdot Q\left(\dfrac{R_1}{R_0} < K_R\right)$

$= 50e^{-0.02 \times 1} Q(S_1 < 32) \cdot Q(R_1 < 18)$

$= \$1.61$

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Q6.

(i) a. $\Delta = -\Phi(-d_1)$

$\Delta = \Phi(d_1) - 1$

b. $\Delta = 0$

$100{,}000\ \Delta + 24830 = 0$

$\Delta = \frac{-24830}{100000}$

$\therefore \Delta = -0.24830$

(ii)

$\Phi(-d_1) = 0.2483$

$1 - \Phi(d_1) = 0.2483$

$\Phi(d_1) = 0.7517$

$\therefore d_1 = 0.68$

$0.68 = \frac{\ln\left(\frac{0.64}{0.63}\right) + \left(0.03 + \frac{1}{2}\sigma^2\right)}{\sigma}$

$\boxed{\therefore \sigma = 7.1\%}$

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(iii)

(a) $X e^{-rT}\Phi(-d_2) - S_0 [\Phi(-d_1) = 0.24830$

$= 0.63 \times e^{-0.03}(1 - \Phi(0.609)] - 0.64 \times 0.2483$

$= 6.894$

(b) Total Cash holding.

$= 24830 \times 0.64 + 100000 \times 0.6894$

$=$ £ $165{,}806$

Q7. (i) $\Delta = \frac{\partial f}{\partial s} = \frac{\partial f}{\partial s}(t, S_t)$

$f \rightarrow$ derivative

$S \rightarrow$ underlying asset.

$\gamma = \frac{\partial^2 f}{\partial s^2}$

$\nu = \frac{\partial f}{\partial \sigma_s}$

(ii) $X = 50$ $\sigma = 25\%$ p.a $\nu = \$29$

$C = 17.91$ $T = 3$ years

$S_0 = 60$ $r = 3\%$ p.a

$\Delta = 0.801$

(iii) 0.801 shares

$C - \Delta \times S_0$ short cash $=$ 30.15 short in cash

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(iv) $f(S, \sigma + \delta) = f(S, \sigma) + \delta \dfrac{df}{d\sigma}$

$= 12.91 + 29 \times 0.02$

$= 18.49$

Q8. $\Delta = \dfrac{\partial f}{\partial S}$

(ii) $S_0 = \$ 100$

$r = 3\%$

$X = 109.42$

$T = 1$

$\Delta = 0.42074$

$\Phi(d_1) = 0.42074$

$\therefore d_1 = -0.2$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$-0.2 = \frac{\ln\left(\frac{100}{109.42}\right) + \left(0.03 + \frac{\sigma^2}{2}\right)1}{\sigma}$$

$-0.2\sigma + 0.09 - 0.03 - \dfrac{\sigma^2}{2} = 0$

$-\dfrac{\sigma^2}{2} - 0.2\sigma + 0.06 = 0$

$\therefore \boxed{\sigma = 0.2}$

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Q9. (i)

PDE for g:

$$\frac{\partial g}{\partial t} + (r-q) S_t \frac{\partial g}{\partial S_T} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 g}{\partial S_t^2} = rg$$

Boundary condition

$$D_T = g(T, S_T) = f(S_T)$$

(ii)

Price of derivative @ time t and value for μ

$$D_T = g(t, S_t) = \frac{S_t^h e^{\mu(T-t)}}{S_0^{h-1}} \quad (h > 1)$$

Then,

$$D_T = g(T, S_T)$$

$$= f(S_T) = \frac{S_T^h}{S_0^{h-1}}$$

$$\frac{\partial g}{\partial t} = -\mu \frac{S_t^h}{S_0^{h-1}} e^{\mu(T-t)} = -\mu g$$

$$\frac{\partial^2 g}{\partial S_t^2} = \frac{h(h-1) S_t^{h-2}}{S_0^{h-1}} e^{\mu(T-t)} = \frac{h(h-1)}{S_t^2} g$$

$$-\mu + (r-q)h + \frac{1}{2}\sigma^2(h-1) = r$$

$$\mu = (r-q)h - r + \frac{1}{2}\sigma^2 h(h-1)$$

If $q = 0$

$$\mu = rn - r + \frac{1}{2}\sigma^2 h(n-1)$$

$$= r(n-1) + \frac{1}{2}\sigma^2 h(n-1)$$

$$= \left(r + \frac{1}{2}\sigma^2 h\right)(n-1)$$

Q10.

(i) Put Call parity for stocks paying no dividends.

$$C_t + Ke^{-r(T-t)} = S_t + P_t$$

(ii) $X = 120$

$T = 1$

$C = 10.09$

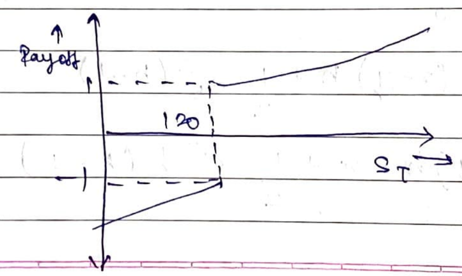
$r = 2\%$ p.a

$S_0 = 110$

$$C = S_0\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$$

$$d_2 = d_1 - \sigma$$

(iii)



(iv) $S_1 - 121 \le D \le S_1 - 120$

$$S_0 - 121e^{-r} \le V \le S_0 - 120e^{-r}.$$