Calculus Assignment 2

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Q1) $\displaystyle\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

$\rightarrow$ Let $t = e^{2/w}$

$\therefore \frac{dt}{dw} = \frac{-2}{w^2}$

$\therefore \frac{dw}{w^2} = \frac{-dt}{2}$

$\therefore w = 1/50 \Rightarrow t = 100$

$w = 2 \Rightarrow t = 1$

$\therefore I = \displaystyle\int_{100}^{1} -e^t \cdot \frac{dt}{2}$

$= \frac{-1}{2}[e^1 - e^{100}]$

$= \frac{1}{2}[e^{100} - e]$

Q2) $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

$\rightarrow \Rightarrow t^4 + 2t = x$

$4t^3 + 2 = dx/dt$

$(2t^3+1)\, dt = \frac{dx}{2}$

$\therefore \displaystyle\int \frac{dx}{2x^3} = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right]$

$= \frac{-1}{4(t^4+2t)^2}$

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(Q3) $f'(x) = x^4 + 3x - 9$

$\therefore f(x) = \int x^4 + 3x - 9$

$= \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + C$

(Q4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

then $\int_{-2}^{3} f(x)\,dx$

$= \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \int_{-2}^{1} 3x^2 \cdot dx + \int_{1}^{3} 6 \cdot dx$

$= [x^3]_{-2}^{1} + [6x]_{1}^{3}$

$= 9 + 12$

$= 21$

(Q5) $\int 3(8y-1)e^{4y^2-y}\,dy$

$\longrightarrow \Rightarrow$ Let $4y^2 - y = t$

$(8y-1)\,dy = dt$

$= 3\int e^t \cdot dt$

$= 3e^{4y-y^2} + C$

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Q6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f(1) = -\frac{5}{4}$; $f(4) = 404$

$\therefore f(x) = \iint (15\sqrt{x} + 5x^3 + 6)\, dx \cdot dx$

$= \int \left(15 \cdot x \cdot \frac{x^{3/2}}{3/2} + \frac{5}{4} x^4 + 6x \, dx \right.$

$= \frac{2}{5} \times 10 \cdot x^{5/2} + \frac{5}{4} \times \frac{x^5}{5} + \frac{6x^2}{2} + c$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c$

Q7) $f(x+iy) + g(x-iy) = z$

$\longrightarrow i^2 \frac{d^2z}{dx^2} - (i+i)\frac{d^2z}{dxdy} + \frac{d^2z}{dy^2} = 0$

$\therefore i^2 \frac{d^2z}{dx^2} + 2i \frac{d^2z}{dxdy} + \frac{d^2z}{dy^2} = 0$

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Q8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$; $r(1) = 2$

$\rightarrow \int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$\therefore \frac{-1}{r} = \log\theta + c$

$\therefore r = 1 ; \theta = 2$

$\frac{-1}{(1)} = \log 2 + c$

$\therefore c = -3.32192$

$\therefore \frac{-1}{r} = \log\theta - 3.3219$

Q9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\rightarrow \int \frac{dv}{9.8 - 0.196v} = \int dt$

$\Rightarrow \int \frac{1}{(9\sqrt{8/5})^2 - (7\sqrt{10v}/50)^2} dv = t + c$

$\log\left[\frac{\sqrt{50}+\sqrt{v}}{\sqrt{50}-\sqrt{v}}\right] = \frac{14}{\sqrt{5}} t + c$

Q10) $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\int \frac{dy}{y^3} = \int \frac{x \cdot dx}{\sqrt{1+x^2}}$ let $1+x^2 = t$

$x\,dx = dt$

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$$\frac{-1}{2y^2} = \frac{1}{2}\int \frac{t}{t^{1/2}}$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + c \quad ; y(0) = -1$$

$$\frac{-1}{2(-1)^2} = 1 + c$$

$$\therefore c = -3/2$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

Q12) $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$\rightarrow$ $f(x) = x^3 - 10x^2 + 6 \quad \therefore f(3) = -57$

$f'(x) = 3x^2 - 20x \quad \therefore f'(3) = -33$

$f''(x) = 6x - 20 \quad f''(3) = -2$

$f'''(x) = 6 \quad f'''(3) = 6$

$$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) - (x-3)^2(2) + (x-3)^3 \ldots$$

Q13) $(1+x)^u$

$\rightarrow$ $f(x) = (1+x)^u \quad f(0) = 1$

$f'(x) = u(1+x)^{u-1} \quad f'(0) = u$

$f''(x) = u(u-1)(1+x)^{u-2} \quad f''(0) = u(u-1)$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

$f'''(0) = u(u-1)(u-2)$

$$f(x) = 1 + ux + u(u-1)\frac{x^2}{2} + u(u-1)(u-2)\frac{x^3}{6} \ldots$$

$$f(x) = 1 + \sum_{i=1}^{n} \frac{ux^n(u-n)}{n!}$$

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Q14) $f(x) = \sqrt{1+x}$

$\longrightarrow$ $f(x) = \sqrt{1+x}$ $\quad f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\quad f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4} \times \dfrac{1}{(1+x)^{3/2}}$ $\quad f''(0) = -1/4$

$f'''(x) = \dfrac{3}{8(1+x)^{5/8}}$ $\quad f'''(0) = \dfrac{3}{8}$

$f(x) = 1 + \dfrac{x}{2} + \dfrac{x^2}{8} + \dfrac{x^3}{16} + \ldots$

Q15) $f(x) = e^{-6x}$ about $x = -4$

$\longrightarrow$ $f(x) = e^{-6x}$ $\quad f(-4) = e^{24}$

$f'(x) = (-6)e^{-6x}$ $\quad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad f''(-4) = 36e^{24}$

$f(x) = e^{24} (x+4) 6e^{24} + (x+4)^2 18e^{24} \ldots$

Q16) $f(x) = \ln(3+4x)$ about $x = 0$

$\longrightarrow$ $f(x) = \ln(3+4x)$ $\quad f(0) = \ln(3)$

$f'(x) = \dfrac{1}{3+4x}$ $\quad f'(0) = \dfrac{1}{3}$

$f''(x) = \dfrac{-4}{(3+4x)^2}$ $\quad f''(0) = \dfrac{-4}{9}$

$f'''(x) = \dfrac{8}{(3+4x)^3}$ $\quad f'''(0) = \dfrac{8}{27}$

$f(x) = \ln(3) + \dfrac{x}{3} - \dfrac{2x^2}{9} + \dfrac{4x^3}{81} + \ldots$