FM - Assignment - 1

1. Nominal rate of discount p.a convertible quaterly is 8% $= d^{(4)}$

$$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$\Rightarrow 1 - 0.92236816$

$\Rightarrow 0.07763184$

(i) The equivalent force of interest

$$\cancel{d^{(p)} = p\left(1 - (1-d)^{1/p}\right)}$$

$$\cancel{d^{(3)} \Rightarrow 3\left(1 -\right.}$$

$$\delta = \ln(1+i) = -\ln(1-d)$$

$$\Rightarrow -\ln(1 - 0.07763184)$$

$$\Rightarrow -(-0.080810829) \Rightarrow \boxed{8.0810829\%}$$

(ii) Equivalent effective rate of interest p.a.

$$1+i = \left(1 + \frac{i^{(p)}}{p}\right)^p = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

$$1+i = \frac{1}{\left(1 - \frac{0.08}{4}\right)^4}$$

$$i = 0.0841657849$$

$$\Rightarrow \boxed{8.41657849\%}$$

(iii) equivalent nominal rate of discount p.a. convertible monthly

$$d^{(12)} = \left(1 - (1-d)^{1/p}\right)p$$

$$\Rightarrow 1 - \left(1 - 0.07763184\right)^{\frac{1}{12}} \times 12$$

$$\Rightarrow 0.0805397$$

$$\Rightarrow \boxed{8.05397\%}$$

2. $\delta(t) = 0.004t + 0.0002t^2$

(i) Present value $= C \times V(0,10)$

$$\Rightarrow 1000 \times \frac{1}{e^{\int_0^{10} 0.004t + 0.0002t^2 \, dt}}$$

$$\Rightarrow 1000 \times \frac{1}{e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}}$$

$$1000 \times \frac{1}{e^{3(0.02) + \frac{0.2}{3}}}$$

$$\Rightarrow 1000 \times 0.765928849 \Rightarrow \boxed{765.928849}$$

(ii) Constant annual effective rate of interest.

$AV_{10} = C \cdot A(0,10)$

$1000 \Rightarrow \cancel{100}\ 765.928849 \, (1+i)^{10}$

$1.0270253 = 1 + i$

$\boxed{0.270253} = i$

3.i $d = iv$

d : effective annual rate of discount

i : effective annual rate of interest.

$\cancel{d = iv}$ $v = \frac{1}{1+i}$

$$\therefore d \Rightarrow i \times \frac{1}{1+i} \Rightarrow \boxed{\frac{i}{1+i}}$$

$$v = 1 - d = \frac{1}{1+i}$$

$$d \Rightarrow 1 - \frac{1}{1+i} \Rightarrow \frac{\cancel{1} + i - \cancel{1}}{1+i} \Rightarrow \boxed{\frac{i}{1+i}}$$

Hence proved.

ii Nominal Rate of discount p.a. convertible half yearly.

a) effective rate of discount ⇒ 2.25% per quarter $= \frac{d^{(4)}}{4}$

$$1-d = \left(1-\frac{d^{(p)}}{p}\right)^p$$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^4$$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = (1-0.0225)^4$$

$$= 0.912992193$$

$$d^{(2)} = 0.0889875 \Rightarrow \boxed{8.89875\%}$$

b) nominal rate of discount is 5% convertible every 2 yrs $= d^{(\frac{1}{2})}$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(\frac{1}{2})}}{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = (1-0.1)^{\frac{1}{2}}$$

$$1-\frac{d^{(2)}}{2} = 0.9740037465$$

$$d^{(2)} \Rightarrow 0.051992507 \Rightarrow \boxed{5.1992507\%}$$

iii) effective rate of return ⇒ 5%. $n = 91$ days.

simple discount rate = ?

A.V ⇒ C × 1+i ⇒ 1.05 × C ⇒ $\boxed{105}$

~~Let P.V be 100.~~ Let C be 100

~~P.V ⇒ C (1-d)~~ P.V = C (1-d)

~~100 = C (1-d)~~ 100 = 105 (1-d)

0.0476190 ⇒ (d)

$$\Rightarrow \boxed{4.7619047\%}$$

4.

ii) $i = 0.08$

a $d^{(12)} \Rightarrow \left(1 - \frac{d^{(12)}}{12}\right)^{12} \Rightarrow \left(\frac{1}{1+0.08}\right)$

$\frac{d^{(12)}}{12} \Rightarrow d \Rightarrow 7.407407407$

$d^{(12)} = p\left(\cancel{1-(1-d)^{1/p}}\right)$ $\cancel{p\left[(1+i)^{1/p}\right]}$ $p\left[1-(1+i)^{-1/p}\right]$

$\Rightarrow 12\sqrt{(1-}$ $\Rightarrow 7.671478\%$

b) $i^{(365)}$

$(1+i) = \left(1 + \frac{i^{(365)}}{365}\right)^{365}$

$(1.08)^{1/365} = \left(1 + \frac{i^{(365)}}{365}\right)$

$1.000210874 = 1 + \frac{i^{(365)}}{365}$

$0.07696915S = i^{(365)}$

$\boxed{7.6969155\%}$

c. δ

$(1+i) = e^{\delta}$

$\ln(1.08) = \delta$

$\boxed{7.6961041} = \delta$

d. $i^{(0.5)}$

$1+i = \left(1 + \frac{i^{0.5}}{0.5}\right)^{0.5}$

$(1.08) = (1+i)^{1/2} = 1 + \frac{i^{1/2}}{1/2}$

$2.16 =$

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d) $1+i = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}$

$(1.08)^{\frac{1}{0.5}} = 1 + \frac{i^{(0.5)}}{0.5}$

$0.0832 = i^{(0.5)}$

$\boxed{8.32\%}$

5 i) Rat of discount per annum conv. quarterly = 6% = $d^{(4)}$

a) ~~eq~~ rate of int. p.a. convertible half yearly.

$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^{4}}$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} = \frac{1}{0.94133655}$$

$$i^{(2)} = 0.061377\text{5}162 \Rightarrow \boxed{6.13775162\%}$$

b) Equivalent rate of discount p.a. conv. monthly.

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{4}$$

$$\left(1 - \frac{d^{(12)}}{12}\right) = (0.94133655)^{1/12}$$

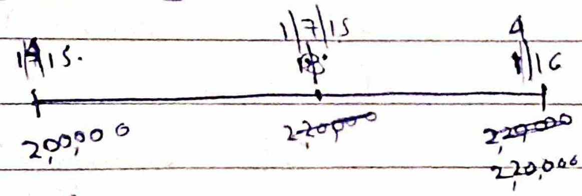
$$d^{(12)} \Rightarrow 0.0603025 \Rightarrow \boxed{6.03025\%}$$

ii $A.V = C(1+i)$

$220000 = 200000(1+i)$

$1.1 = i+1$

$\boxed{0.1} = i$



$$i^{365} \Rightarrow 0.0953226 \rightarrow \boxed{9.53226248\%}$$

$$AV_{17/7/5} = C \times \left(1 + \frac{i^{365}}{365}\right)^{93}$$

$n=9$

$$= 2000000 \left(1 + \frac{0.09532262}{365}\right)^{93}$$

$$\Rightarrow \boxed{₹\, 2049160.3561}$$

6. i

a) cash payment ⇒ 30% below retail price.

credit payment ⇒ ~~30~~ 25% below retail price

Discount offered for cash payment to retailers ⇒ 30%

$$d^{(12)} \Rightarrow 1-d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\Rightarrow 0.351426364 \Rightarrow \boxed{35.1426364\%}$$

$$i^{(2)} \Rightarrow \frac{1}{1-d} = \left(1 + \frac{i^{(2)}}{2}\right)^{2}$$

$$\frac{1}{0.7} = \left(1 + \frac{i^{(2)}}{2}\right)^{2}$$

$$i^{(2)} \Rightarrow \cancel{0.857142857}\ 0.39457218$$

$$\Rightarrow \boxed{39.457218\%}$$

$$\delta = \ln(1+i) = \log\left(\frac{1}{1-d}\right)$$

$$-\delta \Rightarrow \cancel{\log}\ -0.356674943$$

$$\delta \Rightarrow \boxed{0.356674943}$$

Q7.

20,000 $\xleftarrow{\text{17 month}}$ 26,000

i nominal rate of interest p.a. convertible quarterly

$$A.V = C\left(1+\frac{i^{(4)}}{4}\right)^{17/3}$$

$$26000 = 20000\left(1+\frac{i^{(4)}}{4}\right)^{17/3}$$

$$0.18955254 = i^{(4)}$$

$$i^{(4)} \Rightarrow \boxed{18.955254\%}$$

ii nominal rate of discount per annum convertible half yearly

$$PV = C\left(1-\frac{d^{(2)}}{2}\right)^{17/6}$$

$$\frac{20000}{26000} = \left(1-\frac{d^{(2)}}{2}\right)^{17/6}$$

$$d^{(2)} = 0.1940438166 \Rightarrow \boxed{19.40438166\%}$$

iii Simple interest rate p.a.

$$\frac{26000}{20000} = \left(1+\frac{17}{12}i\right)$$

$$0.21176470S = i$$

$$\Rightarrow \boxed{21.176470S\%}$$

iv Result $\Rightarrow$ ~~Simple rate of interest is highest~~ The numerical interest rate increases from nominal rate of interest quarterly to half yearly.

8. ~~Ef~~ When $i = 8\%$

~~Let $P = 2$ and $i^{(P)} \Rightarrow P\left[(1+i)^{1/P}-1\right] \Rightarrow \boxed{9.76+696\%}$~~

8. When $i = 8\%$

$i^{(p)} = i^{(2)} \Rightarrow 7.8460969\%$ which shows, as value of 'p' increases

$i^{(p)}$ decreases.

when 'p' = period i.e. p^{th} times

9.i Force of interest → Force of interest is the instantaneous change in the fund value expressed as an annualised percentage of the current fund value.

ii $i^{(2)} = 0.0775$

$$(1+i) = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$1+i \Rightarrow \left(1 + \frac{0.0775}{2}\right)^2$$

$$i = 0.079001563 \Rightarrow \boxed{7.9001563\%}$$

$$\delta \Rightarrow \ln(1+i)$$

$$\Rightarrow \ln(1+0.079001562) \Rightarrow \boxed{7.6036134\%}$$

$$d^{(12)} \Rightarrow \left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(\frac{1}{1.079001563}\right)$$

$$d^{(12)} = 0.075795747 6$$

$$\Rightarrow \boxed{7.5795746\%}$$

10. $\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$

$$V(0,10) \Rightarrow \frac{1}{e^{\int_0^5 \delta}}$$

$$\Rightarrow e^{\int_0^5}$$

$$\Rightarrow \frac{1}{e^{\int_0^5 0.08\,dt + \int_5^{10} 0.06\,dt + \int_{10}^{t} 0.04\,t}}$$

$$\Rightarrow \frac{1}{e^{[0.08t]_0^5 + [0.06t]_5^{10} + [0.04t]_{10}^{t}}}$$

$$\Rightarrow \frac{1}{e^{\cancel{0.4} + (0.6-0.3) + \cancel{(0.4)}(0.04t - \cancel{0.4})}}$$

$$\frac{1}{e^{0.04t+0.3}} \Rightarrow \boxed{e^{-(0.04t+0.3)}}$$

11.

a) $n = 9$ months

18% p.a.

9 month

50,000

$$PV_0 = C \cdot v(0, 9_m)$$

$$\Rightarrow 50000 \times (1-0.18)^{9/12}$$

$$\Rightarrow 50000 \times 0.86170852 \Rightarrow \boxed{₹43,085.4262}$$

b) (i) $\delta = 6\%$

$$d^{(12)} \Rightarrow \left(1 - \frac{d^{(12)}}{12}\right)^{12} = \frac{1}{e^{\delta}}$$

$$= 0.05985025 \Rightarrow \boxed{5.985025\%}$$

(ii) $d^{(4)} = 6\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{(4)} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$i^{(12)} \Rightarrow$$

c. $d < \delta < i^{(4)} < i$.

Force of interest is the limiting factor which comes from both d and i hence it is in between d being the ~~discount~~ rate of discount is the smallest and i being the highest.

d. $d = iv$

12.

7049 | $6\% = i^{(12)}$ | 1.5% per quarter δ | $d^{(12)} = 6\%$ p.a. con monthly

0 | 2 | $4\frac{1}{2}$ | 6

$$AV_6 \Rightarrow C\,A(0,6)$$

$$\Rightarrow 7049 \times \left(1 + \frac{i^{12}}{12}\right)^{24} \times (1 + 10 \times i) \times \left(1 - \frac{d^{(12)}}{12}\right)^{18}$$

$$\Rightarrow 7049\,(1.127159776) \times (1.15) \times 0.913724886$$

$$\boxed{₹\,8348.84285}$$

13. (i) effective rate of interest $\Rightarrow 10\%$

$$AV = C(1+i)^n$$

$$= \cancel{1(1.01)(1)}$$

$$2 \Rightarrow 1(1.10)^n$$

$\log 2 = n \log(1.1)$ [Taking log on both the sides]

$$\boxed{7.2725 = n}$$

(ii) Nominal dis

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ii Nominal discount rate of 10% p.a convertible monthly

$$A.V = C \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12 \times n}}$$

$$d^{(12)} \Rightarrow p\left(1-(1-d)^{\frac{1}{p}}\right)$$
$$\Rightarrow 12\left(1-(0.9)^{\frac{1}{12}}\right)$$

$$\cancel{2C} \Rightarrow P\ 2 = 1 \cdot \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12 \times n}}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12 \times n} = \frac{1}{2}$$

$$\cancel{\Rightarrow 12n \log} \ 12n \log(0.991667) = \log\left(\frac{1}{2}\right)$$

$$n \Rightarrow 6.904491759 \Rightarrow \boxed{6.9 \text{ yrs}}$$

8. $i = 8\%$, $i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$

$i^{(p)} = i^{(2)} = 7.846096\%$

$i^{(3)} \Rightarrow 7.79567\%$

$\cancel{i^{(12)} \Rightarrow 7.720}$

This shows, ~~when~~ as the value of 'P' increases, $i^{(p)}$ (interest p-thly) decreases where p = ~~peroid~~ period.

$\therefore i^{(p)}$ is inversely proportional to P.