Assignment - 2 [Calculus]

Ans: $$\int_{\frac{1}{50}}^{2} \frac{e^{\frac{2}{w}}}{w^2}\, dw$$

let $\frac{2}{w} \to t$ $\Rightarrow$ $w \to \frac{1}{50}, t = 100$

$w \to 2 \to t = 1$

$\frac{-2dw}{w^2} \to dt$ $\Rightarrow$ $dw = -w^2$

$$\frac{dw}{w^2} = \frac{-1}{2} dt.$$

$$\frac{-1}{2}\int_{100}^{1} e^t\, dt \Rightarrow \frac{-1}{2}\left[e^t\right]_{100}^{1} \Rightarrow \frac{[e - e^{100}]}{2}$$

$$\Rightarrow \frac{-1}{2}\left[e^{100} - e\right] \Rightarrow \frac{e - e^{100}}{2}$$

$$\Rightarrow \boxed{\frac{e^{100} - e}{2}}$$ Ans

Ans: $$\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$$

let $t^4 + 2t \to \alpha$

$(4t^3+2)\,dt \to d\alpha$

$(2t^3+1)\,dt \to \frac{d\alpha}{2}$

$$\frac{1}{2}\int \frac{d\alpha}{\alpha^3} \Rightarrow \frac{1}{2}\int \alpha^{-3}\, d\alpha \Rightarrow \frac{1}{2}\left[\frac{\alpha^{-2}}{-2}\right]$$

$$\Rightarrow \frac{-1}{4\alpha^2} \Rightarrow \frac{-1}{4(t^4+2t)} + C$$

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Ques ③ $f'(x) = x^4 + 3x - 9$

$$f(x) = \int (x^4 + 3x - 9)\, dx$$

$$\Rightarrow \cancel{x^4} \quad \frac{x^5}{5} + \frac{3x^2}{2} - 9x \quad + C$$

Ques ④ $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \le 1 \end{cases}$

$$\int_{-2}^{1} (3x^2)\, dx + \int_{1}^{3} 6\, dx$$

$$\Rightarrow \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{-2}^{1} + 6[x]_{1}^{3}$$

$$\Rightarrow 1 - (-2)^3 + 6[3-1]$$

$$\Rightarrow 1 + 8 + 12 \Rightarrow \boxed{21}$$

Ques ⑤ $\int 3(8y-1)\, e^{4y^2 - y}\, dy \Rightarrow$ let $4y^2 - y \to t$

$(8y-1)\, dy \to dt$

$$3\int e^t\, dt \Rightarrow \cancel{3e^t}$$

$$\Rightarrow 3e^t + C$$

$$\Rightarrow 3e^{4y^2-y} + C \quad \text{Ans}$$

Ques (6) $f''(x) = 15\sqrt{x} + 5x^3 + 6.$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6$

$\Rightarrow \frac{2 \times \cancel{15}^{5} x^{3/2}}{\cancel{3}} + \frac{5x^4}{4} + 6x + c$

$f'(x) \Rightarrow 10x^{3/2} + \frac{5}{4}x^4 + 6x + C$

$f(x) = \int 10x^{\frac{3}{2}} + \frac{5}{4}x^4 + 6x + C$

$\Rightarrow \frac{2 \times \cancel{10}^{2} x^{5/2}}{\cancel{5}} + \frac{\cancel{5}x^5}{4 \times \cancel{5}} + \frac{6x^2}{2} + Cx + D$

$f(x) \Rightarrow 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + D.$

$f(1) = \frac{-5}{4}$

$\frac{-5}{4} = 4 + \frac{1}{4} + 3 + C + D$

$-5 = 16 + 1 + 12 + 4C + 4D$

$-5 \Rightarrow 29 + 4C + 4D$

$\frac{-34}{8} = \underline{4C} + \underline{4D}$ ...... (1)

$-34 + 8 = 4c$

$\frac{-26}{4} = \frac{13}{2}$

$f(4) = 404$

$404 = 4(2)^{\cancel{2}\times\frac{5}{\cancel{2}}} + \frac{4^5}{4} + 3(4)^2 + C(4) + D$

$404 \Rightarrow 128 + 256 + 48 + 4C + D$

$-28 = 4C + D$

$-28 = -34 - 4D + D \Rightarrow -28 + 34 = -3D$

$6 = -3D$

$\boxed{D = -2}$

$\boxed{C = -13/2}$

So, $f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x + 2$ Sol

Ques (7) $z = f(x+iy) + g(x-iy)$

$P = \frac{\partial z}{\partial x} = f'(x+iy) + g'(x-iy)$

$Q = \frac{\partial z}{\partial y} \Rightarrow if'(x+iy) - ig'(x-iy)$

$\frac{\partial z}{\partial y} = Q \Rightarrow i[f'(x+iy) - g'(x-iy)]$.

$r = \frac{\partial^2 z}{\partial x^2} = f''(x+iy) + g''(x-iy)$.

$t = \frac{\partial^2 z}{\partial y^2} = i^2[f''(x+iy) + g''(x-iy)]$

so, $\boxed{r + t = 0}$ Required PDE.

Ques (8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $r(1) = 2$

$$\int r^{-2}\,dr = \int \theta^{-1}\,d\theta$$

$$\frac{r^{-2+1}}{-2+1} = \ln|\theta|$$

$$\frac{-1}{r} = \ln|\theta| + c$$

$$\frac{1}{r} = -\ln|\theta| - c$$

$$r(\theta) = \frac{1}{-\ln|\theta| - c}$$

$$2 = \frac{1}{-\ln|1| - c} \Rightarrow \boxed{c = \frac{-1}{2}}$$

$$r = \frac{1}{-\ln|\theta| - c} \Rightarrow \frac{1}{-\ln|\theta| + \frac{1}{2}}$$

$$\boxed{r = \frac{2}{1 - 2\ln|\theta|}}$$

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Ques 9

$$\cancel{\frac{dy}{d}} \quad \frac{dv}{dt} + 0.196v = 9.8$$

$$I.f = e^{\int 0.196\,dt} \Rightarrow e^{0.196t}$$

$$V(If) = \int (If \times 9.8)\,dt + C$$

$$V \Rightarrow \frac{1}{e^{0.196t}} \int 9.8\,e^{0.196t} + C$$

$$\Rightarrow \cancel{9.8}\,e$$

$$V = \frac{1}{e^{0.196t}} \left[\frac{9.8\,e^{0.196t}}{0.196} + C \right]$$

$$\boxed{V \Rightarrow 50 + Ce^{-0.196t}}$$

Ques 10-

$$ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2$$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} + \left(\frac{2}{t}\right)y = \left(t - 1 + \frac{1}{t}\right)$$

$$I.f = e^{2\int \frac{1}{t}dt} \Rightarrow e^{2\ln|t|} = e^{\ln|t|^2}$$

$$\Rightarrow t^2.$$

$$y(t^2) = \int t^2\left(t - 1 + \frac{1}{t}\right) + c$$

$$y(t)^2 \Rightarrow \int t^3 - t^2 + t + c$$

$$y(t)^2 \Rightarrow \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

$$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{c}{t^2}$$

$$\frac{1}{2} = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + \frac{c}{}$$

$$\frac{1}{2} \Rightarrow \frac{3 - 4 + 6 + 12c}{12\ 6}$$

$$6 \Rightarrow 5 + 12c.$$

$$\boxed{\frac{1}{12} = c}$$

$$y = \cancel{\frac{t^2}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}}$$

$$\boxed{y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{12t^2} + \frac{1}{2}}$$

Required sol.

Ques (1)

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1$$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$y^{-3}\frac{dy}{\cancel{y^3}} = \frac{x\,dx}{\sqrt{1+x^2}}$$

let $\to 1+x^2 \to t$

$2x\,dx \to dt$

$$\int y^{-3}\,dy \Rightarrow \frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

$$\frac{y^{-2}}{-2} \Rightarrow 2\times\frac{1}{2}\sqrt{t}$$

$$\frac{-1}{2y^2} = \sqrt{t} + C$$

$$\frac{-1}{2y^2} \Rightarrow \sqrt{1+x^2} + C \Rightarrow \frac{-1}{y^2} = 2\sqrt{1+x^2} + 2C$$

$$-\frac{1}{2} \Rightarrow 1 + C$$

$$C = -\frac{1}{2} - \frac{1}{1} \quad \frac{-1-2}{2}$$

$$\boxed{C = -3/2}$$

$$\frac{1}{y^2} = \cancel{2\sqrt{1}+2C} \Rightarrow -$$

Ques (12) $f(x) = x^3 - 10x^2 + 6$ | $f(3) = (3)^3 - 10(3)^2 + 6 = -57.$

$f'(x) = 3x^2 - 20x.$ | $f'(3) = 3(9) - 20(3) = -33$

$f''(x) = 6x - 20$ | $f''(3) = 6(3) - 20 = -2$

$f'''(x) = 6$ | $f'''(3) = 6$

$f''''(x) = 0$ | $f''''(3) = 0$

Now:: all other high order derivatives are zero

$$f(x) = f(3) + f'(3)(x-3) + f''(3)\frac{(x-3)^2}{2!} + f'''(x)\frac{(x-3)^3}{3!}.$$

$$f(x) \Rightarrow -57 - 33(x-3) + \frac{(-2)(x-3)^2}{2!} + \frac{(6)(x-3)^3}{3!}$$

$$f(x) = -57 - 33(x-3) - (x-3)^2 - (x-3)^3.$$ sol.

Ques (14) $f(x) = \sqrt{1+x}$ $(1+x)^{\frac{1}{2}}$ | $f(0) = \sqrt{1} = 1$

$f'(x) = \frac{1}{2(1+x)^{1/2}} \Rightarrow \frac{1}{2}(1+x)^{-\frac{1}{2}}$ | $f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$ | $f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-\frac{5}{2}}$ | $f'''(0) = \frac{3}{8}$

So, $f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}$

$$f(x) = 1 + \frac{1}{2}x + \frac{-1}{4}\frac{x^2}{(2!)} + \frac{3}{8}\frac{x^3}{3!} \Rightarrow 1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{x^3}{16} + \cdots$$

Ques (15) $f(x) = e^{-6x}$; $f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ | $f'(4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ | $f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ | $f'''(-4) = -216e^{24}$

Taylor series : $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!}$

$f(x) = e^{-6x} = f(-4) + f'(-4)(x+4) + \frac{f''(-4)(x+4)^2}{2!} + \dots$

$e^{-6x} = e^{24} - 6e^{24}(x+4) + \frac{36e^{24}(x+4)^2}{2!} - \frac{216e^{24}(x+4)^3}{3!} + \dots$

$e^{-6x} = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3 + \dots$

Ques (16) $f(x) = \ln(3+4x)$ | about $\boxed{x=0}$ $f(0) = \ln(3)$

$f'(x) = \frac{1(4)}{3+4x}$ | $f'(0) = \frac{4}{3}$

$f''(x) = (-4)(3+4x)^{-2}$
$= -16(3+4x)^{-2}$ | $f''(0) = -\frac{16}{9}$

$f'''(x) = 128(3+4x)^{-3}$ | $f'''(0) = \frac{128}{27}$

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$$f(x) = f(0) + f'(0)(x) + f''(0)\frac{x^2}{2!} + f'''(x)\frac{(x^3)}{3!} + \cdots$$

$$\ln(3+4x) = \ln(3) + \frac{4}{3}x - \frac{16}{9}\frac{x^2}{2!} + \frac{128}{27}\frac{x^3}{3!} + \cdots$$

$$\ln(3+4x) = \ln(3) + \frac{4}{3}x - \frac{8}{9}x^2 + \frac{128}{162}x^3 + \cdots$$ Ans.