

Homework K 1

Lagrange's Interpolation

Q.1 The function $y = f(x)$ is given by points $(7, 3)$, $(8, 1)$, $(9, 1)$ and $(10, 9)$. Find the value of y at $x = 9.5$ using ~~lagrange~~ lagrange's interpolation formula

x	$f(x)$	$n=1$	value of $y/f(x)$ at $x = 9.5$
7	3		$u = \frac{9.5 - 10}{n} = -0.5$
8	1		
9	1		
$x_n \leftarrow 10$	$9 \rightarrow f(x_n)$		

x	$f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
7	3			
8	1	-2	+2	6
9	1	0	8	$\nabla^3 f(x)$

$x_n \leftarrow 10$	$9 \rightarrow f(x_n)$	8	$\nabla_3 f(x)$
		$\downarrow$	
		$\nabla^2 f(x)$	

$$f(x) = f(x_n) + u \nabla f(x_n) + \frac{4(4-1)}{2!} \nabla^2 f(x) + \frac{4(4-1)(4-2)}{3!} \nabla^3 f(x)$$

$$= 9 + (-0.5)(9) + \frac{(-0.5)(-1.5)}{2} (8) + \frac{(-0.5)(-1.5)(-2.5)}{6} (6)$$

$$= 9 - 4 + 3 - 1.875 = 6.125$$

Q.2 The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 & 20 of the independent variable. Find the estimate of the junction at the position 6 of the independent

x	y	$x_k = 6$	$f(b) = \frac{(6-7)(6-9)(6-10)(168)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)}$
3	168		
7	120		
9	72	$\frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)}$	
10	63		

$$S(b) = 168 \frac{(-1)(+3)(+4)}{(+4)(+6)(+7)} + 120 \frac{(+3)(-3)(+4)}{4(+7)(-3)} + 72 \frac{(3)(+1)(-4)}{6(2)(+1)} + 63 \frac{(3)(+1)(+3)}{(7)(3)(1)} = 12 + 180 - 72 + 27 = 147$$

Q.3 Following are the values of function $f(x)$ for values $x=1, 2, 7, 8$
 $f(1)=4$, $f(2)=5$, $f(7)=5$, $f(8)=4$.

Find the value of $f(6)$ using Lagrange's interpolation formula.

x	$f(x)$	$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)}$
1	4	
2	5	$+ \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)}$
7	5	
8	4	$= \frac{4(-1)(-2)4}{(-1)(-6)(-7)} + \frac{5(-1)(-2)(5)}{(1)(-5)(-6)} + \frac{5(4)(-2)5}{(6)(5)(-1)} + \frac{5 \times 4(-1)(4)}{7 \times 6 \times 1}$

$$= \frac{-16}{21} + \frac{10}{6} + \frac{10}{3} - \frac{40}{21}$$

$$= 2.333$$

Q.4 Apply Lagrange's interpolation formula to find $f(5)$ given that $f(1)=2$, $f(2)=4$, $f(3)=8$, $f(4)=16$ & $f(7)=128$

x $f(x)$ find $f(5)$

$$\begin{array}{rcl}
 1 & 2 & f(5) = \frac{(5-1)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} \\
 2 & 4 & \\
 3 & 8 & + \frac{(5-3)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(12)}{(4-1)(4-2)(4-3)(4-7)} \\
 4 & 16 & \\
 7 & 128 & + \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}
 \end{array}$$

$$= \frac{(4)(3)(1)(-2)(8)}{(2)(1)(-1)(-4)} + \frac{(4)(3)(2)(+2)}{(3)(2)(1)(+3)} + \frac{(128)(4)(3)(2)}{(6)(5)(4)(3)}$$

$$= 30.6$$

Q.5 $\rightarrow$ Find $f(15)$

$$\begin{aligned}
 f(15) &= \frac{(15-20)(15-14)(15-16)}{(10-12)(10-14)(10-16)} \cdot 0.25 + \frac{(15-10)(15-14)}{(12-10)(12-14)(12-16)} \cdot 0.2 \\
 &+ \frac{(15-10)(15-12)(15-16)}{(14-10)(14-12)(14-16)} \cdot 0.15 \\
 &+ \frac{(15-10)(15-12)(15-14)}{(16-10)(16-12)(16-14)} \cdot 0.10
 \end{aligned}$$

$$= \frac{0.25}{16} + \left(\frac{-0.5}{8} \right) + \left(\frac{2.25}{16} \right) + \left(\frac{2.5}{48} \right)$$

$$= 0.125$$