SRMA Assignment - 2 (Roll. No. 404)

Q1)

i)

$$\begin{array}{c|cc} & B & O \\ \hline B & -\lambda & \lambda \\ O & \mu & -\mu \end{array}$$

ii) Holding times are exponentially distributed with parameter λ in state B and μ in state O.

iii) $\frac{d}{dt} P_s BB = -\lambda_t P_s BB + \mu_t P_s BO$

$\frac{d}{dt}{}_t P_s^{BO} = \lambda_t P_s^{BB} - \mu \cdot {}_t P_s^{BO}$

iv) ${}_t P_s BB + {}_t P_s BO = 1$

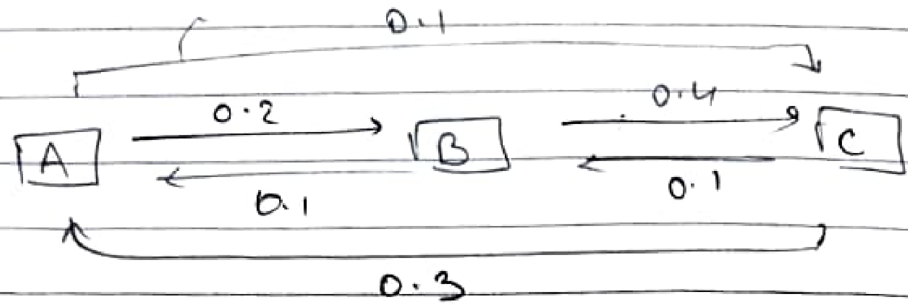
$\frac{d}{dt}{}_t P_s^{BB} = -\lambda \cdot {}_t P_s^{BB} + \mu \cdot (1 - {}_t P_s BB)$

$\frac{d}{dt}\left[\exp((\lambda+\mu)t) \cdot P_s BB\right] = \mu \cdot \exp((\lambda+\mu)t)$

$\frac{\lambda}{\mu+\lambda}$

${}_s P^{BB}$ $P_{BB} = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} \cdot \exp.(-(\lambda+\mu)t)$

2) i)



ii) $\frac{d}{dt} P_{AA}(t) = -0.3 P_{AA}(t) + 0.1 P_{AB}(t) + 0.3 P_{AC}(t)$

$\frac{d}{dt} P_{AB}(t) = 0.2_{AA}(t) - 0.5 P_{AB}(t) + 0.1 P_{AC}(t)$

$\frac{d}{dt} P_{AC}(t) = 0.1 P_{AA}(t) + 0.4 P_{AB}(t) - 0.4 P_{AC}(t)$

iii) For paramets $\rightarrow$ $(0.1 + 0.2) \times 2 = 0.6$

$P(\text{Poi}(0.6) = 0) \Rightarrow \frac{e^{-0.6} \, 0.6^0}{0!}$

$= e^{-0.6} = 0.5488$

(v) All the probabilities of each jumps =>

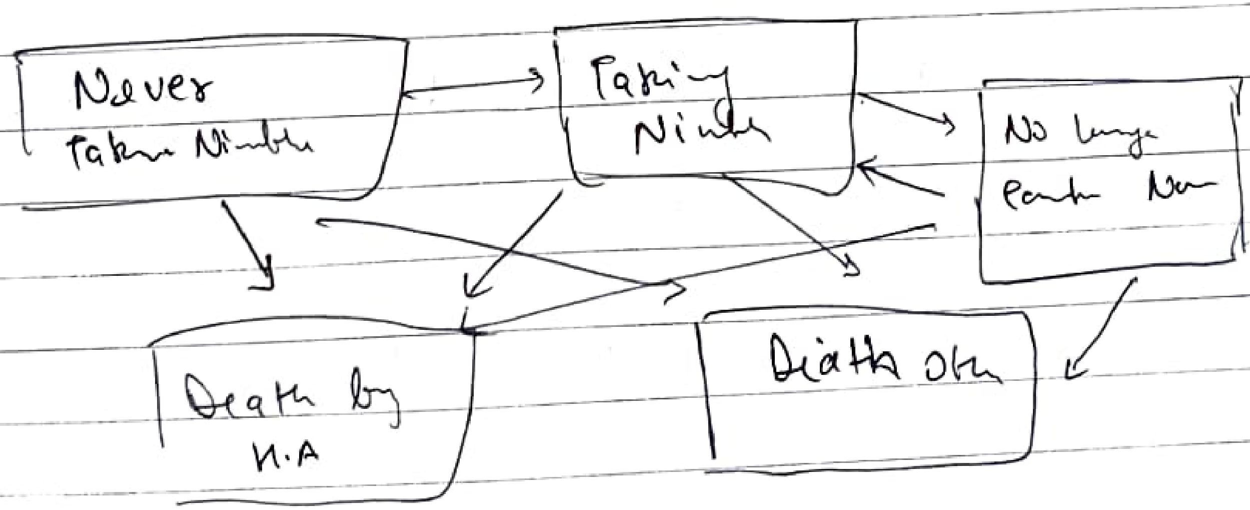
$$CAC = \frac{1}{3} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$BAC = \frac{2}{3} \cdot \frac{1}{5} \cdot \frac{1}{3} = \frac{2}{45}$$

$$CBC = \frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{1}{15}$$

$$\Rightarrow \frac{7}{36} = 0.194$$

3) i)



ii) CKE $\Rightarrow$

$${}_{dt+t}p_x^{34} = {}_tp_x^{32}\, {}_{dt}p_{x+t}^{24} + {}_tp_x^{33}\, {}_{dt}p_{x+t}^{34} +$$

$${}_tp_x^{34}\, {}_{dt}p_{x+t}^{44}$$

$\hookrightarrow = 1$

$${}_{dt}p_{x+t}^{ij} = \mu_{x+t}^{ij}\, dt + o(dt)$$

$$\lim_{dt \to 0} \frac{o(dt)}{dt} = 0$$

so, $${}_{dt+t}p_x^{34} - {}_tp_x^{34} = {}_tp_x^{32}\,\mu_{x+t}^{24}\, dt + {}_tp_x^{33}\,\mu_{x+t}^{34}\, dt + o(dt)$$

4) i) Mean = parameter, $\rightarrow$ 3 calls per hr

ii) Memoryless property. Exp. time for next call is ...

iii) ~~$P_j(t) = e$~~

$$P_0(0.5) = \frac{e^{-1.5}(1.5)^0}{0!} = e^{-1.5}$$

$$= 0.2231$$

iv) Ans.

3 call in 7 min

$$P(\text{Engaged phone}) = \frac{21}{60} = 0.35$$

5) i) $P_{ii}(t, t+dt) = 1 - \sum_{j=1} \lambda_{ij}(t)\, dt + o(dt)$

$$\lim_{dt \to 0} \frac{o(dt)}{dt} = 0$$

$$P_{HH}(x, t+dt) = P_{HH}(x,t)(1 - \sigma(t)dt - \mu(t)dt) + P_{HS}(x,t)\rho(t, c_t) + o(dt)$$

$$P_{HH}(x, t+dt) - P_{HH}(x,t) = P_{HH}(x,t)(-\sigma(t) - \mu(t))dt + P_{HS}(x,t)\rho(t,c)dt + o(dt)$$

$$\Rightarrow P_{HH}(x,t)(-\sigma(t) - \mu(t)) + P_{HS}(x,t)\rho(t, c_t)$$

ii) $\frac{d}{dt} P_{HH}(0,t) = -(\sigma(t) + \mu(t)) P_{HH}(t)$

$$\frac{1}{P_{HH}(0,t)} \cdot \frac{d}{dt} P_{HH}(0,t) = -(\sigma(t) + \mu(t))$$

$$= \frac{d}{dt} \ln P_{HH}(t)$$

$$\left[\ln P_{HH}(0,t)\right]_0^t = \int_{s=0}^{t} -(\sigma(s) + \mu(s))\,ds$$

as $P_{HH}(0) = 1$

$$P_{HH}(0,t) = \exp\left(-\int_{s=0}^{t} (\sigma(s) + \mu(s))\,ds\right)$$

6) All process have discrete s.s. Markov jump operates in cont. time

The Markov Jump chain and Markov chain are expressed in terms of prob. whereas the Markov Jump process is expressed in terms of rates.

The Markov chain can have loops in each state. the Markov jump process and Markov Jump chain only has loops on absorbing states

7) i) The MLE of transition intensity from $i \to j$ is number of transitions from $i \to j$ divide by the total waiting time in state i.

ii) $$\frac{d}{dt} P_{AA}(s,t) = -P_{AA}(s,t)\,\mu$$

$$\frac{d}{dt} P_{AT}(s,t) = P_{AA}(s,t)\,\mu$$

$$P_{AT}(s,t) = \int_{u=0}^{t} P_{AA}(s,u)\cdot\mu\cdot du$$

iii) $$\frac{1}{P_{AA}(t)} \cdot \frac{d}{dt} P_{AA}(t) = -\mu$$

$$\frac{d}{dt}\left(\ln(P_{AA}(t))\right) = -\mu$$

$$P_{AA}(t) = \exp(-\mu t + c)$$

$P_{AA}(0) = 1$, $c = 0$, so,

$$P_{AA}(t) = \exp(-\mu_x t)$$

iv)

$$\begin{tikzcd} & \boxed{\text{Active policy}} \arrow[dl, "\lambda"'] \arrow[dr, "\mu"] & \\ \boxed{\text{Lapsed}} & & \boxed{\text{Theft claim}} \end{tikzcd}$$

v) $$\frac{d}{dt} P_{AA}(t) = -P_{AA}(t)(\mu+\lambda)$$

$$P_{AA}(t) = \exp(-(\mu+\lambda)t)$$

$$\frac{d}{dt} P_{AT}(t) = P_{AA}(t)\,\mu$$

$$= \mu \exp(-(\mu+\lambda)t)$$

$$\text{Claim} \rightarrow \frac{\mu}{\mu+\lambda} c(1-\exp(-(\mu+\lambda)T))$$

8) ~~a~~

i) $\{0, 1, 2, 3 \ldots\}$

ii)

ii)

$$\begin{tikzcd} \boxed{0} & \boxed{1} \arrow[l, "\mu"'] \arrow[r, shift left, "\lambda"] & \boxed{2} \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & \boxed{3} \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & \cdots \arrow[l, shift left, "\mu"] \end{tikzcd}$$

iii) Generator matrix

$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & \cdots \\ \hline 0 & 0 & 0 & 0 & 0 & \\ 1 & \mu & -(\mu+\lambda) & \lambda & 0 & \\ 2 & 0 & \mu & -(\mu+\lambda) & \lambda & \\ 3 & 0 & 0 & \mu & (-\mu+\lambda) & \lambda \\ \vdots & & & & & \end{array}$$

iv) A Jump chain is each distinct state visited in the ~~order set~~ where the time bet when states are moved there.

v)

$$\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & 1 & 0 & 0 \\
1 & \mu/(\mu+\lambda) & 0 & \lambda/(\mu+\lambda) \\
2 & 0 & \mu(\mu+\lambda) & 0 \\
\vdots & \vdots & &
\end{array} \quad \cdots$$

vi) $\left(\frac{\mu}{\mu+\lambda}\right)^2$

10)

i) $\frac{d}{dt} P_{\bar{A}A}(t) = -2t \times P_{\bar{A}A}(t)$

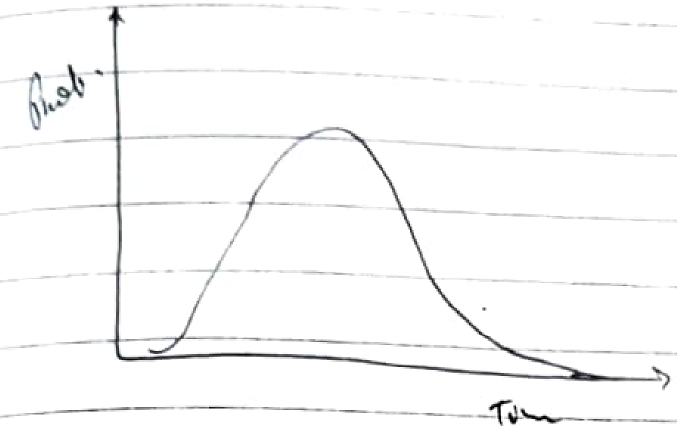
$$\frac{d}{dt}\left[\ln P_{\bar{A}A}(t)\right] = -2t$$

$$\ln P_{\bar{A}A}(s) = -s^2 + \text{constant}$$

$$P_{\bar{A}A}(0) = 1$$

$$P_{\bar{A}A}(s) = \exp -s^2$$

iii)



A → B ↑es

b) Probability ↑es from 0 at T = 0.
Transition increases, it becomes more likely that process has already B or jumped back to A.

c) $\frac{d}{dt}\left[e^{-t^2} \times t^2\right] = 2t \times e^{-t^2} - 2t^3 \times e^{-t^2}$

$\Rightarrow e^{-t} \times 2t \times (1-t^2) = 0$ $t \neq 0$

ii) $P_0 (T_0 \leq y \mid N_s = 1) = \frac{P(N_y = 1, N_s - N_y = 0)}{P(N_s = 1)}$

$\frac{(\lambda y e^{-\lambda y}) e^{-\lambda(s-y)}}{\lambda s e^{-\lambda s}} = \frac{\cancel{y}}{S} \frac{y}{S}$

ii) $\lambda^n e^{-\lambda(t_1 + t_2 + .. + t_n)}$