CALCULUS ASSIGNMENT

Q1) $\displaystyle\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw.$

let $e^{2/w} = t$

$\dfrac{-2e^{2/w}}{w^2} = \dfrac{dt}{dx}$

$= \displaystyle\int_{1/50}^{2} \frac{t\cancel{w^2}}{-2\cancel{w^2}e^{2/w}}\, dt$

$\dfrac{-1}{2}\displaystyle\int_{1/50}^{2} dt$

$\dfrac{-1}{2}\left[x\right]_{1/50}^{2}$

$\dfrac{-1}{2}\left(2 - 0.02\right)$

Ans $= -0.99$

Q2) $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

let $(t^4+2t)^3 = x$

$3(t^4+2t)(4t^3+2) = \dfrac{dx}{dt}$.

$\displaystyle\int \frac{2t^3+1}{x\; 3(t^4+2t)(4t^3+2)} \frac{dx}{dt}$

Multiply / divide by 2

$$\int \frac{\cancel{(4t^3+2)}}{x(3)\cancel{(4t^3+2)}(t^4+2t)} dt$$

$$= \frac{1}{6}\int x^{-5/3}\,dx$$

$$\frac{1}{6}\left[\frac{x^{-5/3+1}}{-5/3+1}\right]$$

$$\frac{1}{6}\left[\frac{x^{-2/3}}{-2/3}\right]$$

$$-\frac{3}{2}\times\frac{1}{\cancel{6}2}\left[x^{-2/3}\right]$$

Ans) $-\frac{1}{4}x^{-2/3}+C = -\frac{1}{4}(t^4+2t)^{-2/3}+C$

Q3) $f'(x) = x^4+3x-9$ $\quad f(x) = \int f'(x)\,dx.$

$$f(x) = \int (x^4+3x-9)\,dx$$

$$= \frac{x^5}{5}+\frac{3x^2}{2}-9x+C.$$

Ans $\therefore f(x) = \frac{x^5}{5}+\frac{3x^2}{2}-9x+C$

Q4)

$$F(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$\int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx.$$

$$[x^3]_{-2}^{1} + 6[x]_{1}^{3}$$

$$1 + 8 + 18 - 6 = 4$$

$$\boxed{\text{Ans} = 4}$$

Q5) $\int 3(8y-1)e^{4y^2-y}\,dy$

let $e^{4y-y^2} = x$.

$$\frac{dx}{dy} = (e^{4y^2-y})(8y-1).$$

$$\int \frac{3\cancel{(8y-1)}\,x}{\cancel{e^{(4y^2-y)}}\,\cancel{8y-1}}\,dx$$

$$3\int dx$$

$$3x + C$$

Ans) $3(e^{4y^2-y}) + C$

Q6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int (15\sqrt{x} + 5x^3 + 6)\,dx$

$\frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k\,dx.$

$F(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + C$

$F(1) = -\frac{5}{4} = 4 + \frac{1}{4} + 3 + k + C$

$k + C = -8.5$

$f(4) = 4(4)^{5/2} + \frac{1^4}{} + 3(4)^2 + 4k + C. = 404$

$4k + \cancel{C} = 164$

$-\ \ k + \cancel{C} = +8.5$

$3k = 172.5$

$k = 57.5$

$C = -66$

Ans) $\therefore F(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$.

Q9) $\frac{dv}{dt} = 9.8 - 0.196V$

$\int \frac{dv}{9.8 - 0.196v} = \int dt$

$\log(9.8 - 0.196v) = -0.196t + C$

Camlin

Q12) Taylor series:-

$$f(x) = x^3 - 10x^2 + 6 \quad \text{at } x = 3$$

$F(x) = x^3 - 10x^2 + 6$ $\qquad$ $F(3) = -57$

$F'(x) = 3x^2 - 20x$ $\qquad$ $F'(3) = -33$

$F''(x) = 6x - 20$ $\qquad$ $F''(3) = -2$

$F'''(x) = 6$ $\qquad$ $F'''(3) = 6$

$$F(x) = F(3) + (x-3)F'(x) + \frac{(x-3)^2}{2!}F''(x) + \frac{(x-3)^3}{3}F'''(3)$$

$$= -57 + (x-3)-33 + \frac{(x-3)^2(-2)}{2} + \frac{6(x-3)^3}{3!} \ldots$$

Q14) $f(x) = \sqrt{1+x}$.

$F'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad$ $F'(0) = 1/2$

$F''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $\qquad$ $F''(0) = -1/4$

$F'''(x) = \frac{-3}{8}(1+x)^{-5/2}$ $\qquad$ $F'''(0) = \frac{-3}{8}$

Ans) $F(x) = \sqrt{1+x}$ $\quad + \frac{x}{1!}\left(\frac{1}{2}\right) + \frac{x^2}{2!}\left(\frac{-1}{4}\right) + \frac{x^3}{3!}\left(\frac{-3}{8}\right)$

Q13) $F'(x) = u(1+x)^{u-1}$

$F''(x) = u(u-1)(1+x)^{u-2}$

$F'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

$$F(x) = F(0) + \frac{x^1}{1!}F'(0) + \frac{x^2}{2!}F''(0) + \frac{x^3}{3!}F'''(0) + \ldots$$

$$= 1 + u + U(u-1) + u(u-1)(u-2) \ldots$$

Q15) $F(x) = e^{-6x}$

$F(x) = e^{-6x}$

$F'(x) = -6e^{-6x}$

$F''(x) = 36e^{-6x}$

$F'''(x) = -216\, x^{-6x}$

$F(-4) = e^{+24}$

$F'(-4) = -6e^{+24}$

$F''(-4) = 36e^{24}$

$F'''(-4) = -216x^{+24}$

Taylor series :-

$e^{24} + (x-4)e^{24}(6) + \frac{(x-4)^2}{2!} 36e^{24} \ldots$

Q16) $F(x) = \ln(4x+3) \qquad x = 0$

$F'(x) = \frac{4}{4x+3}$

$F''(x) = (-4)(4x+3)^{-2}(4)$

$F'''(x) = 8(4x+3)^{-3}(4)^2$

$F'(0) = 4/3$

$F''(0) = (-1)(4)^2(3)^{-2}$

$F'''(0) = (-1)(-2)(-3)^{-2}(4)^3$

Ans) $= \ln(3) + \frac{x}{1!}\frac{4}{3} + \frac{x^2}{2!}(-1)(4^2)(\cancel{3}^{2}) + \ldots$

-3)^3 F'''(3)

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