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1) length (real) = 12.5 cm = r

area (real) = πr^2 = 490.87385 cm^2

length (approximate) = ~~12~~ 12.65 cm

area (approximate) = $\pi (12.65)^2$ = 502.7255 cm^2

(i) absolute error = 502.7255 − 490.87385 = 11.85166 cm^2

(ii) proportional error = $\frac{11.85166}{490.87385}$ = 0.024144

(iii) Percentage error = 0.024144×100 = 2.4144%

2)
(ii)

x	$y = \log(x)$
4.5	0.6532125
5	y_5
5.5	0.7403627

By interpolation,

$$\frac{5-4.5}{5.5-4.5} = \frac{y_5 - 0.6532125}{0.7403627 - 0.6532125}$$

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(i)

x	$y=\log(x)$
4	0.60206
5	y_5
6	0.778513

By interpolation,

$$\frac{5-4}{6-4} = \frac{y_5 - 0.60206}{0.7781513 - 0.60206}$$

$$\Rightarrow \frac{1}{2} = \frac{y_5 - 0.60206}{0.1760913}$$

$$\Rightarrow y_5 = 0.6900565$$

5) Given,

$A^2 = A$

Now,

$$7A - (I+A)^3$$

$$= 7A - [(I)^3 + 3IA(I+A) + A^3]$$

$$= 7A - [(I)^3 + 3A + 3A^2 + A^3]$$

$$= 7A - [I^3 + 3A + 3A + A] \quad (\because A^2 = A)$$

$$= 7A - [I^3 + 7A]$$

$$= -I^3$$

$$= -I.$$

6) Given,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$|A| = 1(0-6) - (-1)(-4) + 2(4)$$

$$= -2$$

Inverse exists as $|A| \neq 0$.

$C_{11} = -6$ $\quad C_{12} = 4$ $\quad C_{13} = 4$

$C_{21} = 1$ $\quad C_{22} = -1$ $\quad C_{23} = -1$

$C_{31} = -6$ $\quad C_{32} = 2$ $\quad C_{33} = 4$

$$\therefore \ \text{adj} A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -0.5 & 3 \\ -2 & 0.5 & -1 \\ -2 & 0.5 & -2 \end{bmatrix}$$

7) Given,

$$\vec{a} = 8\hat{i} - 9\hat{j} \ ; \ b = 7\hat{i} - 9\hat{j}$$

We know that,

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

$$= (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$$

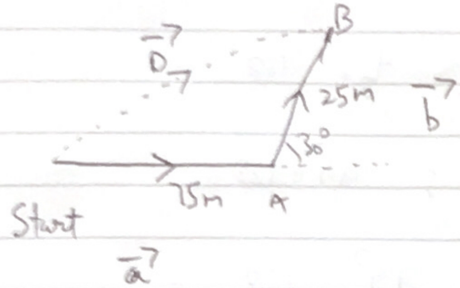
$$= 56 - 81 = 137$$

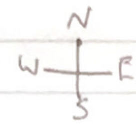
$$|a| = \sqrt{8^2 + 9^2} = \sqrt{145} \ ; \quad |b| = \sqrt{7^2 + 9^2} = \sqrt{130}$$

Then, $\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|a||b|}\right)$

$$= \cos^{-1}\left(\frac{137}{\sqrt{130 \times 145}}\right) = 3.758^\circ$$

8) Given,





Here, $\vec{D} = \vec{a} + \vec{b}$

$\Rightarrow \vec{D}^2 = a^2 + b^2 + 2ab\cos\theta$

$= 75^2 + 25^2 + 2 \times 75 \times 25 \times \cos 30°$

$= 9497.595264$

$\Rightarrow |\vec{R}| = 97.45560663$ m

9) Here,

$a = 101$, $d = 3$, $t_n = 998$.

Then, $t_n = a + (n-1)d$

$\Rightarrow 998 = 101 + (n-1)3$

$\Rightarrow n = 300$

$S_{300} = \frac{300}{2}[a + t_n] = 150(101 + 998) = 164850$

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10) Given,

$t_6 = 32$, $t_8 = 128$

$\Rightarrow ar^5 = 32$; $ar^7 = 128$

Then, $\frac{t_8}{t_6} = \frac{ar^7}{ar^5} = \frac{128}{32} = \frac{4}{1}$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

11) Pascal triangle :-

		1				~~0~~
		1	1			- 1
	1	2	1			- 2
1	3	3	1			- 3
1	4	6	4	1		- 4
1	5	10	10	5	1	- 5

$$(3x+4)^5 = 3x^5 + 5(3x)^4(4) + 10(3x)^3(4)^2 + 10(3x)^2(4)^3 + 5(3x)(4)^4 + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

12) Given,

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (2-\lambda)\left[(5-\lambda)(3-\lambda)\right] - (-3)\left[2\times(3-\lambda)\right]$$

$$= (3-\lambda)(2-\lambda)(5-\lambda) + 6(3-\lambda)$$

$$= (3-\lambda)\left[(2-\lambda)(5-\lambda)+6\right]$$

$$= (3-\lambda)\left[\lambda^2 + 3\lambda - 4\right]$$

$$= -\lambda^3 + 13\lambda - 12$$

Let $\det(A-\lambda I) = 0$.

Then, $\lambda^3 - 13\lambda + 12 = 0$

$\Rightarrow (\lambda-1)(\lambda^2+\lambda-12) = 0$

$\Rightarrow (\lambda-1)(\lambda+4)(\lambda-3) = 0$

$\Rightarrow \lambda = 1, -4, 3$ which are eigen values

14) $(1-x-x^2)^4$

$= \left((x-1)^2 + x\right)^4$

$= \left((x-1)^2\right)^4 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$

$= (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$