Probability & Statistics Assignment - 1

1.

i. 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Mean $\rightarrow \bar{x} = \frac{\Sigma x}{n} = \frac{134}{10} \Rightarrow \boxed{13.4}$

Mode $\rightarrow$ 18

Median $\rightarrow \frac{5^{th}\text{ obs} + 6^{th}\text{ obs}}{2} \Rightarrow \frac{11+15}{2} \Rightarrow \boxed{13}$

Standard deviation $\rightarrow$

X	$(x-\bar{x})^2$
4	88.36
7	40.96
9	19.36
9	19.36
11	5.76
15	2.56
18	21.16
18	21.16
18	21.16
25	134.56
	$\boxed{374.4}$

$\sigma^2 = \frac{\Sigma(x-\bar{x})^2}{n} \Rightarrow \frac{374.4}{10}$

$\boxed{37.44}$

$\sigma = 6.118823416$

$\Rightarrow \boxed{6.12}$

Interquartile Range $\rightarrow \frac{n}{4}^{th}$ item =

$Q_1 \Rightarrow \frac{n}{4}^{th}$ item $\Rightarrow 2.5^{th}$ item $\Rightarrow \frac{7+9}{2} \Rightarrow 8$

$Q_3 \Rightarrow 7.5^{th}$ item $\Rightarrow \frac{18+18}{2} \Rightarrow 18$

$Q_3 - Q_1 \Rightarrow 18 - 8 \Rightarrow \boxed{10}$

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ii	x No. of household	f No. of people	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
	0	5	0	4	~~0~~ 20
	1	11	11	1	11
	2	26	52	0	0
	3	15	45	1	~~45~~ 15
	4	3	12	4	~~48~~ 12
		60		10	~~58~~ 58

Mean $\Rightarrow \frac{\Sigma fx}{n} \Rightarrow \frac{120}{60} \Rightarrow \boxed{2}$

Mode $\rightarrow$ 2

Median $\Rightarrow \frac{30^{th} + 31^{st} \text{ obs}}{2} \Rightarrow \frac{2+2}{2} \Rightarrow \boxed{2}$

Standard deviation $\Rightarrow \Sigma f(x_i-\bar{x})^2 \Rightarrow 58$

$\sigma^2 \Rightarrow \frac{58}{60} \Rightarrow 0.97$

$\sigma \Rightarrow \boxed{0.98488578}$

Interquartile Range $\rightarrow$

$Q_3 = \frac{3}{4}n^{th}$ term $\Rightarrow \frac{3}{4} \times \overset{15}{60} = 45^{th}$ term $\Rightarrow 3$

$Q_1 \Rightarrow \frac{1}{4}n^{th}$ term $\Rightarrow 15^{th}$ term $\Rightarrow 1$

I $Q_3 - Q_1 \Rightarrow 3-1 \Rightarrow \boxed{2}$

3.

2. $X \sim Poi(\lambda = 0.5)$

i. Probability that no claim arise in a single year

$$P(X=0) \Rightarrow \frac{e^{-\lambda}\lambda^{x}}{x!} \Rightarrow \frac{e^{-0.5}(0.5)^{0}}{0!} = e^{-0.5} \Rightarrow \boxed{0.606530659}$$

ii. Let the success be claims in one or more year $= p$

Let the failure be of no claims. $= q$

$$P(\text{success}) = P(x>0) \Rightarrow 1 - P(X=0)$$

$$\Rightarrow 1 - 0.606530659$$

$$\Rightarrow 0.393469341$$

$$P(\text{failure}) \Rightarrow P(x=0)$$

$$P(X=1) \Rightarrow {}^{n}C_{x} \times p^{x} \times q^{n-x}$$

$$\Rightarrow {}^{3}C_{1} \times (0.393469341)^{1} \times (0.606530659)^{3-1}$$

$$\Rightarrow 3 \times 0.14474928$$

$$\Rightarrow \boxed{0.434247842}$$

iii. $X \sim Exp(\lambda = 0.5)$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$\Rightarrow 1 - \left[1 - e^{-\lambda x}\right]$$

$$\Rightarrow \cancel{1} - \cancel{1} + e^{-\lambda x} = e^{-0.5 \times 2} \Rightarrow$$

$$\Rightarrow \boxed{0.367879441}$$

3. $X \sim Gamma(a = 2, \lambda = 1/2)$ $\qquad 0 < x < \infty$

i. $E(X) = \frac{a}{\lambda} = \frac{2}{1/2} \Rightarrow \boxed{4}$

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$$V(x) = \frac{a}{\lambda^2} \Rightarrow \frac{2}{1/4} \Rightarrow \boxed{8} \qquad S.D \Rightarrow \sqrt{8} \Rightarrow \boxed{2.82842712S}$$

(ii) Skewness

(iii) Cumulative Distribution Function →

$$\Gamma(\alpha) = (\alpha-1)!$$

$$f_X(x) = \frac{\lambda^{\alpha} \cdot e^{-\lambda x} \cdot x^{\alpha-1}}{\Gamma(\alpha)} \Rightarrow \frac{(1/2)^2 \cdot e^{-\frac{x}{2}} \cdot x^1}{1!}$$

$$\Rightarrow \boxed{\frac{1}{4} \cdot x e^{-\frac{x}{2}}}$$

$$F_X(x) = \int_0^x \frac{1}{4} \cdot x e^{-\frac{x}{2}} dx \Rightarrow \frac{1}{4} \int_0^x \underset{u}{x} \cdot \underset{v}{e^{-\frac{x}{2}}} dx$$

$$uv \Rightarrow u \int v dv - \int \frac{d}{dx}(u) \cdot \int v dv$$

$$\Rightarrow \frac{1}{4} \; x \int e^{-\frac{x}{2}} dx - \int \frac{d}{dx}(x) \cdot \int e^{-x/2} dx$$

$$\Rightarrow \frac{1}{4} \cdot x \frac{e^{-\frac{x}{2}}}{-1/2} - \int 1 \frac{e^{-x/2}}{-1/2} dx$$

$$\Rightarrow \frac{1}{4} \left[x \cdot e^{-\frac{x}{2}} \cdot (-2) - \frac{e^{-x/2}}{-1/2} \right]_0^x$$

$$\Rightarrow \frac{1}{4} \left[x \cdot e^{-\frac{x}{2}} \cdot (-2) - e^{-x/2}(-2) - 0 \cdot e^{-\frac{0}{2}} \cdot (-2) - e^{-0/2}(-2) \right]$$

$$\frac{1}{4} \left[-2e^{-\frac{x}{2}} x \quad 2 \qquad + 2e^{-\frac{x}{2}} \right]$$

4. 10 - male & 8 - femal
6 mal & 7 femal are fully qualified actuaries
P(2 Female / atleast 1 femal)

Total No. of ways to select $\Rightarrow {}^{18}C_2 \Rightarrow \frac{18!}{2! \times 16!} \Rightarrow \frac{\cancel{18}^{9} \times 17 \times \cancel{16!}}{\cancel{16!} \times \cancel{2}} \Rightarrow \boxed{153}$

P(atleast 1 is femal) $\Rightarrow$ 1 − P(both are mal)

$\Rightarrow 1 - \frac{{}^{10}C_2}{{}^{18}C_2} \Rightarrow 1 - \frac{45}{153} \Rightarrow \boxed{0.705882}$

P(~~tot~~ No. of ways to select a mal $\Rightarrow {}^{10}C_2 \Rightarrow \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{2!} \times \cancel{8!}} \Rightarrow \boxed{45}$

P(2 qualified females) $\Rightarrow \frac{{}^{7}C_2}{{}^{18}C_2} \Rightarrow \frac{7 \times \cancel{6}^{3} \times \cancel{5!}}{\cancel{2!} \times \cancel{5!}} \Rightarrow \boxed{\frac{21}{153}}$

P(2 qualified femal / atleast 1 femal) $\Rightarrow \frac{P(F \cap Q)}{P(F)}$

$\Rightarrow \frac{\frac{21}{153}}{0.705882352}$

$\Rightarrow 0.1944444$

$\boxed{0.1945}$

5. Level 1 → 40% packages are delivered. → 20% arriving late

Level 2 → 50% packages are delivered → 40% arriving late

Level 3 → 10% → 10% arriving late

~~P(late package | level 2)~~ = ?

$P(Late | L_1) = 20\%$

$P(Late | L_2) = 40\%$

$P(Late | L_3) = 10\%$

$$P(L_2 / Late) = \frac{P(L_2 \cap Late)}{P(Late)} \Rightarrow \frac{P(L_2) \cdot P(Late | L_2)}{\sum P(Level) \cdot P(Late | L_2)}$$

$$\Rightarrow \frac{0.4 \times 0.5}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$\Rightarrow \frac{0.4 \times 0.5}{0.08 + 0.20 + 0.01} \Rightarrow \boxed{\frac{0.20}{0.29}}$$

6. $X \sim U(\overset{a}{1}, \overset{b}{2})$ $Y = \frac{3}{6-X}$, PDF of Y = ?

$$F_X(x) = \frac{1}{\beta - \alpha} \Rightarrow \frac{1}{2-1} \Rightarrow \boxed{1}$$

$$Y = \frac{3}{6-x} \Rightarrow Y(6-x) = 3 \Rightarrow X = \boxed{6 - \frac{3}{Y}}$$

diff w.r.t 'x' $\Rightarrow \frac{dx}{dy} = 0 - 3 \cdot \left(\frac{-1}{y^2}\right) \Rightarrow \boxed{+\frac{3}{y^2}}$

$F_Y(y) \Rightarrow$

7.

X	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

(i) $F(3) \Rightarrow P(x=1)+P(x=2)+P(x=3)$

$\Rightarrow 0.05+0.15+0.35 \Rightarrow \boxed{0.55}$

(ii) $E(x) \Rightarrow \sum x(P(X=x))$

$= 1(0.05)+2(0.15)+3(0.35)+4(0.4)+5(0.05)$

$= 0.05+0.30+1.05+1.6+0.25$

$\boxed{3.25}$

(iii) $V(x) \Rightarrow E(x^2) - [E(x)]^2$

$\Rightarrow 11.45 - (3.25)^2 \Rightarrow \boxed{0.8875}$

(iv) Coefficient of skewness

~~8. Let the~~

8. Let ~~the~~ success be claim amount > 1000.

$P(S) = 0.2 \quad \therefore P(F) = 1-0.2 \Rightarrow 0.8$

$n = 15 \quad X \sim Bin(15, 0.2)$

$P(X<3) \Rightarrow P(x=0)+P(x=1)+P(x=2)$

$\Rightarrow {}^{15}C_0 \cdot (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$\Rightarrow 0.035184372 + 0.131941395 + 0.230897441$

$\Rightarrow$ ~~0.398~~ $\boxed{0.398023208}$

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9. $P(\text{winning}) = 0.01$ $P(F) = 1 - 0.01 \Rightarrow 0.99$

$n = 7$ $x = 3$

$P(x=3) = {}^nC_x \cdot p^x q^{n-x}$

$\Rightarrow {}^7C_3 \cdot (0.01)^3 \cdot (0.99)^4$

$\Rightarrow \cancel{35} \times (3.362056035)^{-05}$

~~10. P(M) = P(F) 5 boys & 2 girls~~

10. No. of claims $\Rightarrow$ 2 per week (5 days)

P(more than 2 claims in 1 day)

$X \sim Poin(\lambda = \frac{2}{5}) = \boxed{0.4}$

$P(X > 2) = 1 - P(X \leq 2)$

$\Rightarrow \cancel{1 - \left(\frac{e^{-2} \times (\frac{2}{5})^2}{2!}\right)}$

$P(x=0) = \frac{e^{-\frac{2}{5}} \cdot \lambda^0}{0!} \Rightarrow \boxed{0.670320046}$

$P(x=1) \Rightarrow \frac{e^{-\frac{2}{5}} \cdot (\frac{2}{5})^1}{1!} \Rightarrow \boxed{0.268128018}$

$P(x=2) = \frac{e^{-\frac{2}{5}} (\frac{2}{5})^2}{2!} \Rightarrow \boxed{0.053625603}$

$P(x \leq 2) \Rightarrow \boxed{0.992073667}$

$P(x > 2) = 1 - 0.992073667$

$\Rightarrow \boxed{0.007926332}$

11. $P(M) = P(F) = \frac{1}{2}$ 5B & 2G.

Birth of a girl and boy are not dependent on each other

$\therefore P\left(\begin{matrix}\text{next 3 children} \\ \text{are boys}\end{matrix}\right) \Rightarrow P(M) \times P(M) \times P(M)$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \Rightarrow \boxed{\frac{1}{8}}$

12. $X \sim$ Pois (rate 10 per day)

P(time between 2 claims is less than 0.1 day)

It follows exponential because 'rate' is specified

where $\alpha = 1$ & $\lambda = 10$

$P(T < 0.1) = \cancel{P(\alpha\lambda)} \Rightarrow 1 - P(T = 0.1)$

$\Rightarrow 1 - \cancel{\alpha}\lambda e^{-\lambda x}$

$\Rightarrow 1 - 1 . e^{-10 \times \frac{1}{10}} \Rightarrow \boxed{0.632120558}$

Doubt?

13.* $X \sim U(a = 75, b = 120)$

$P(80 < X < 100) = ?$

$f_X(x) \Rightarrow \frac{1}{b-a} \Rightarrow \frac{1}{120-75} \Rightarrow \frac{1}{45}$

$P(80 < X < 100) = F(100) - F(80)$

$\Rightarrow \frac{100-75}{120-75} - \frac{80-75}{120-75}$

$\Rightarrow \frac{25}{45} - \frac{15}{45} \Rightarrow \frac{10}{45} \Rightarrow \boxed{0.22222}$

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14. $X \sim Gamma(5, 0.4)$ Calculate $P(X < 22.89)$

$\alpha = 5$; $\lambda = 0.4$

$P(X < 22.89)$

15. $\dfrac{k}{(x+5)^4}$, $x > 0$

(i) $k = \int_0^\infty f_X(x) = 1 \Rightarrow k \int_0^\infty \dfrac{1}{(x+5)^4} \, dx = 1$

Let $x + 5 = t$

diff w.r.t 't' $dx = dt$ ∴ limits ⇒ ∞ & 5.

$k \int_5^\infty \dfrac{1}{t^4} \, dt \Rightarrow k \left[\dfrac{t^{-3}}{-3} \right]_5^\infty = 1$

$k \left[0 - \left(-\dfrac{1}{36125} \right) \right] = 1$

$k \left(\dfrac{1}{36125} \right) = 1$

15. $\dfrac{k}{(x+5)^5}, x > 0$

(i) $k =$

$$\int_0^\infty \frac{k}{(x+5)^4} \, dx = 1$$

$$k \int_0^\infty (x+5)^{-4} \, dx = 1$$

$$k \left[\frac{(x+5)^{-3}}{-3} \right]_0^\infty = 1$$

$$k \left[0 + \frac{(5)^{-3}}{3} \right] = 1$$

$$k \left[\frac{1}{375} \right] = 1$$

$$\boxed{k = 375}$$

(ii) $F(10)$

$$\Rightarrow \int_0^{10} \frac{375}{(x+5)^4} \, dx$$

$$\int_0^{10} 375 \cdot (x+5)^{-4} \, dx$$

$$375 \int_0^{10} \frac{(x+5)^{-3}}{-3}$$

$$375 \left[\frac{(x+5)^{-3}}{-3} \right]_0^{10}$$

$$\frac{375}{3} \left[\frac{(15)^{-3}}{3} - \frac{(5)^{-3}}{3} \right] \Rightarrow \frac{375}{3} ($$

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$$375\left[\cancel{x}\frac{(15)^{-3}}{3}-\frac{(5)^{-3}}{-3}\right]\Rightarrow \boxed{0.96296}$$

(iii) $E(x)=\int_0^\infty x \cdot f_X(x)\,dx$

$$=\int_0^\infty \frac{x \cdot k}{(x+5)^4}\Rightarrow 375\int_0^\infty \frac{x}{(x+5)^4}$$

Let $(x+5)=t$ ∴ $dx=dt$ when limits ⇒ $5,\infty$

$$\Rightarrow 375\int_5^\infty \frac{t-5}{t^4}\,dt \Rightarrow 375\int_5^\infty \frac{1}{t^3}-\frac{5}{t^4}\,dt.$$

$$375\left[-\frac{1}{2t^2}+\frac{5}{3t^3}\right]_5^\infty \Rightarrow 375\left[\frac{5}{3t^3}-\frac{1}{2t^2}\right]$$

$$\cancel{375\left[0-0-5\right.}$$

$$375\left[0-0-\frac{5}{375}+\frac{1}{50}\right]$$

$$375\left[\frac{1}{50}-\frac{5}{375}\right]\Rightarrow \frac{375}{50}-5\Rightarrow 7.5-5$$

$$\Rightarrow \boxed{2.5}$$